University of Babylon

College of Materials Engineering

Department of Engineering of Polymer and Petrochemical Industries

Strength of Materials

B.Sc. Course for Second stage

By

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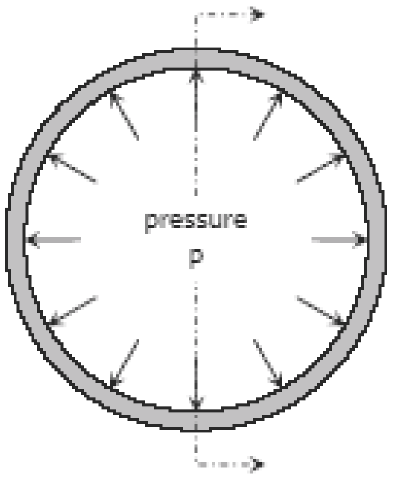
Lecture 2: Simple Stress

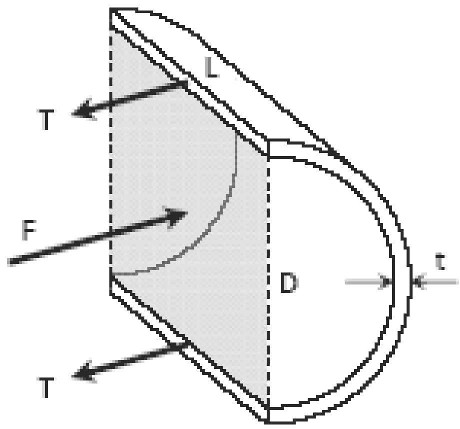
Thin-Walled Pressure Vessels

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

1- **TANGENTIAL STRESS** (Hoop stress or Circumferential Stress), $\boldsymbol{\sigma_t}$

Consider the tank shown being subjected to an internal pressure **p**. The length of the tank is **L** and the wall thickness is **t**. Isolating the right half of the tank:





$$F = pA = pDL$$

$$T = \sigma_t A_{wall} = \sigma_t tL$$

$$[\ \Sigma F_H = 0\ ]$$

$$F = 2T$$

$$pDL = 2(\sigma_t tL)$$

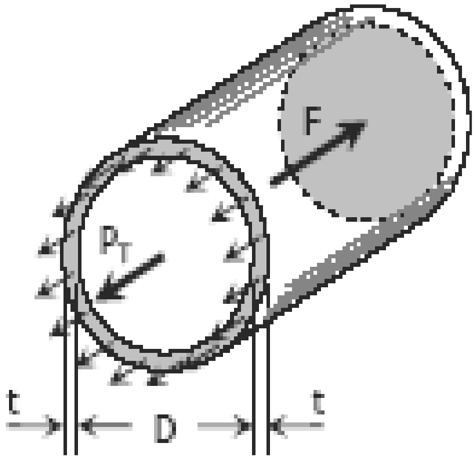
$$\sigma_t = \frac{pD}{2t}$$

If there exist an external pressure $\mathbf{p_o}$ and an internal pressure $\mathbf{p_i}$, the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_0)D}{2t}$$

2- LONGITUDINAL STRESS, σ_L

Consider the free body diagram in the transverse section of the tank:



The total force acting at the rear of the tank **F** must equal to the total longitudinal stress on the wall $P_T = \sigma_L A_{wall}$. Since **t** is so small compared to **D**, the area of the wall is close to πDt

$$F = pA = p\frac{\pi}{4}D^2$$

$$P_T = \sigma_L \pi Dt$$

$$[\ \Sigma F_H = 0\ ]$$

$$P_T = F$$

$$\sigma_L \pi Dt = p\frac{\pi}{4}D^2$$

$$\sigma_L = \frac{pD}{4t}$$

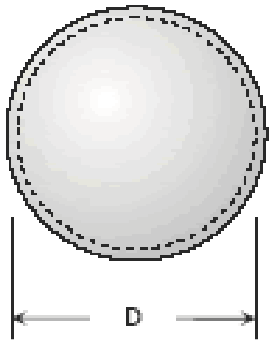
If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

$$\sigma_l = \frac{(p_i - p_0)D}{4t}$$

It can be observed that the tangential stress is twice that of the longitudinal stress; $\boldsymbol{\sigma_t = 2\sigma_l}$

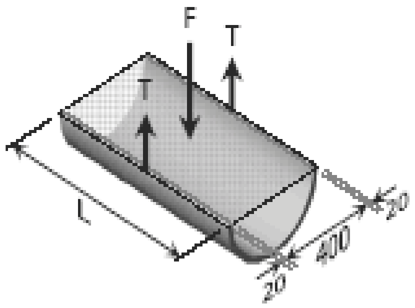
3- SPHERICAL SHELL

If a spherical tank of diameter **D** and thickness **t** contains gas under a pressure of **p**, the stress at the wall can be expressed as:



$$\boldsymbol{\sigma_t = \frac{pD}{4t}}$$

EX: A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m^2. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m^2?



Longitudinal Section

(a) Tangential stress (longitudinal section):

$F = 2T$

$pDL = 2(\sigma_t\, tL)$

$$\sigma_t = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

$\sigma_t = 45$ MPa

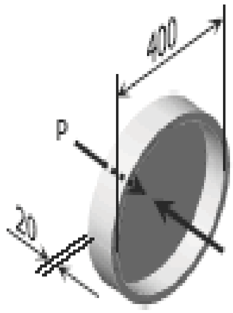
Longitudinal Stress (transverse section):

$F = P$

$\frac{1}{4}\pi D^2 p = \sigma_l\,(\pi D t)$

$$\sigma_l = \frac{pD}{4t} = \frac{4.5(400)}{4(20)}$$

$\sigma_l = 22.5$ MPa



Transverse Section

(b) From (a), $\sigma_t = \dfrac{pD}{2t}$ and $\sigma_l = \dfrac{pD}{4t}$ thus, $\sigma_t = 2\sigma_l$,

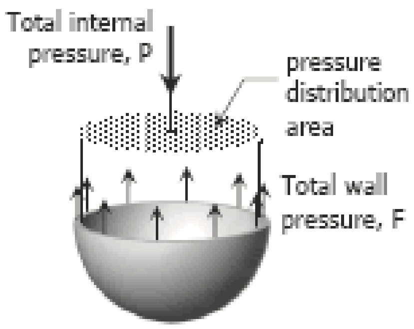
this shows that tangential stress is the critical.

$$\sigma_t = \frac{pD}{2t}$$

$$120 = \frac{p(400)}{2(20)}$$

$$P = 12\text{ MPa}$$

EX: The wall thickness of a 4-ft-diameter spherical tank is 5/16 in. Calculate the allowable internal pressure if the stress is limited to 8000 psi.



Total internal pressure:

$$P = p\left(\tfrac{1}{4}\pi D^2\right)$$

Resisting wall:

$$F = P$$

$$\sigma A = p\left(\tfrac{1}{4}\pi D^2\right)$$

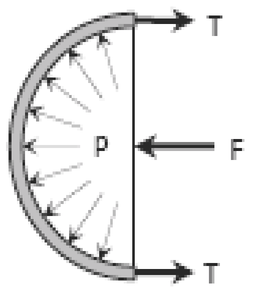
$$\sigma(\pi D t) = p\left(\tfrac{1}{4}\pi D^2\right)$$

$$\sigma = \frac{pD}{4t}$$

$$8000 = \frac{p(4\times 12)}{4(\frac{5}{16})}$$

$$p = 208.33\text{ psi}$$

Ex: A water tank, 22 ft in diameter, is made from steel plates that are ½ in. thick. Find the maximum height to which the tank may be filled if the circumferential stress is limited to 6000 psi. The specific weight of water is 62.4 lb/ft³.



$\sigma_t = 6000 \text{ psi}$

$\sigma_t = \dfrac{6000 \text{ lb}}{\text{in}^2}\left(\dfrac{12 \text{ in}}{\text{ft}}\right)^2$

$\sigma_t = 864\,000 \text{ lb/ft}^2$

Assuming pressure distribution to be uniform:

$p = \gamma h = 62.4h$

$F = pA = 62.4h(Dh)$

$F = 62.4(22)h^2$

$F = 1372.8h^2$

$T = \sigma_t A_t = 864\,000(th)$

$T = 864\,000\,(\frac{1}{2} \times \frac{1}{12})\,h$

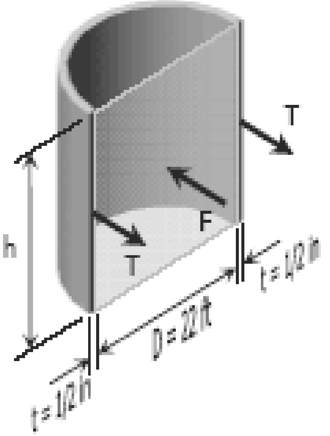
$T = 36\,000h$

$\Sigma F = 0$

$F = 2T$

$1372.8h^2 = 2(36\,000h)$

$h = \mathbf{52.45 \text{ ft}}$



EX: The strength of longitudinal joint in Fig. is 33 kips/ft, whereas for the girth is 16 kips/ft. Calculate the maximum diameter of the cylinder tank if the internal pressure is 150 psi.

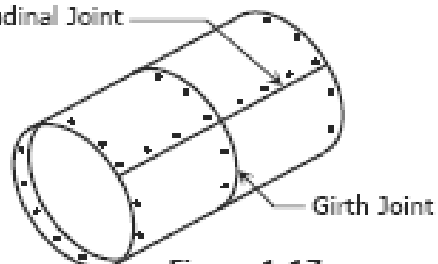
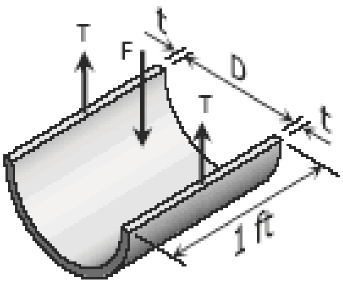


Figure 1-17

Internal pressure, p:

$$p = 150 \text{ psi} = \frac{150 \text{ lb}}{\text{in}^2}\left(\frac{12 \text{ in}}{\text{ft}}\right)^2$$

$$p = 21\,600 \text{ lb/ft}^2$$



For longitudinal joint (tangential stress):

Consider 1 ft length

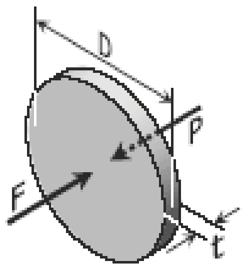
$$F = 2T$$

$$pD = 2\sigma_t t$$

$$\sigma_t = \frac{pD}{2t}$$

$$\frac{33000}{t} = \frac{21600\,D}{2t}$$

$$D = 3.06 \text{ ft} = 36.67 \text{ in}$$



For girth joint (longitudinal stress):

$$F = P$$

$$p\left(\tfrac{1}{4}\pi D^2\right) = \sigma_l\,(\pi D t)$$

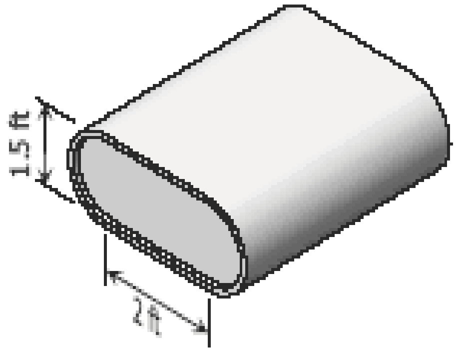
$$\sigma_l = \frac{pD}{4t}$$

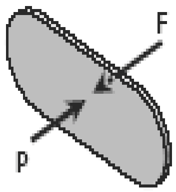
$$\frac{16000}{t} = \frac{21600\,D}{4t}$$

$$D = 2.96 \text{ ft} = 35.56 \text{ in.}$$

Use the smaller diameter, $D = 35.56$ in.

EX: The tank shown in Fig. is fabricated from 1/8-in steel plate. Calculate the maximum longitudinal and circumferential stress caused by an internal pressure of 125 psi.





See dimensions in Fig. P-141, thickness, t = 1/8 in.

Longitudinal Stress:

$F = pA = 125[1.5(2) + \frac{1}{4}\pi(1.5)^2](12)^2$

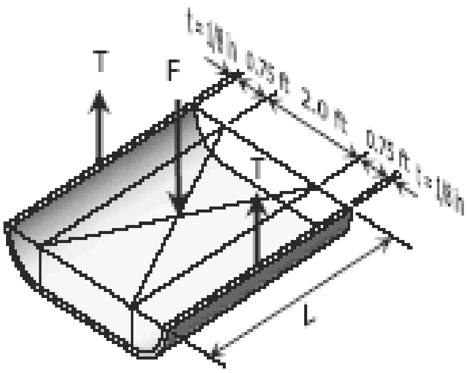
$F = 85\,808.62$ lbs

$P = F$

$\sigma_l\,[2(2 \times 12)(\frac{1}{8}) + \pi(1.5 \times 12)(\frac{1}{8})] = 85\,808.62$

$\sigma_l = 6\,566.02$ psi

$\sigma_l = \mathbf{6.57}$ **ksi**



Circumferential Stress:

$F = pA = 125[(2 \times 12)L + 2(0.75 \times 12)L]$

$F = 5250L$ lbs

$2T = F$

$2[\sigma_t\,(\frac{1}{8})\,L] = 5250L$

$\sigma_t = 21\,000$ psi

$\sigma_t = \mathbf{21}$ **ksi**