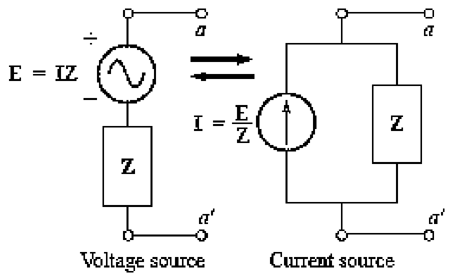
Methods of AC Analysis

SOURCE CONVERSIONS:

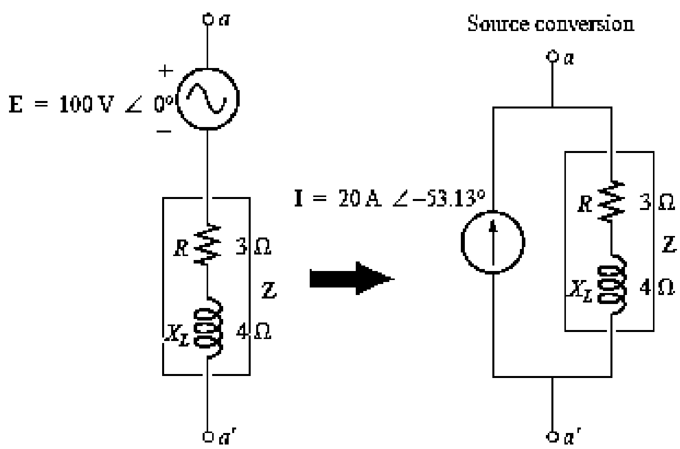
When applying the methods to be discussed, it may be necessary to convert a current source to a voltage source, or a voltage source to a current source. This source conversion can be accomplished in much the same manner as for DC circuits, except now we shall be dealing with phasors and impedances instead of just real numbers and resistors.

Independent Sources:

In general, the format for converting one type of independent source to another is as shown in Figure.



EXAMPLE: Convert the voltage source of Figure to a current source.



Sol/

$$I = \frac{E}{Z} = \frac{100\angle 0}{5\angle 53.13^0} = 20A\angle -53.13^0$$

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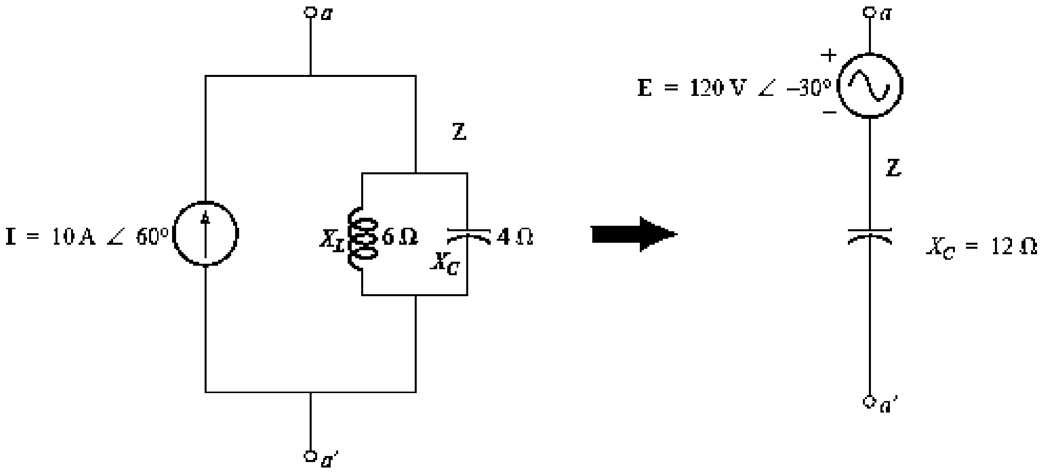
EXAMPLE : Convert the current source of Figure to a voltage source.



Sol/

$$Z=\frac{Z_C\times Z_L}{Z_C+Z_L}=\frac{(4\angle-90^0)(6\angle 90^0)}{-j4+j6}=-j12=12\Omega\angle-90^0$$

$$E=IZ=(10\angle 60^0)(12\angle-90^0)=120V\angle-30^0$$

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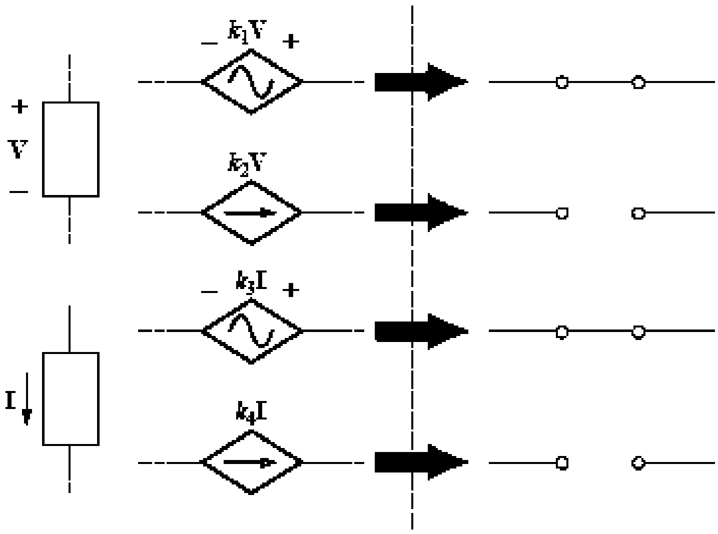
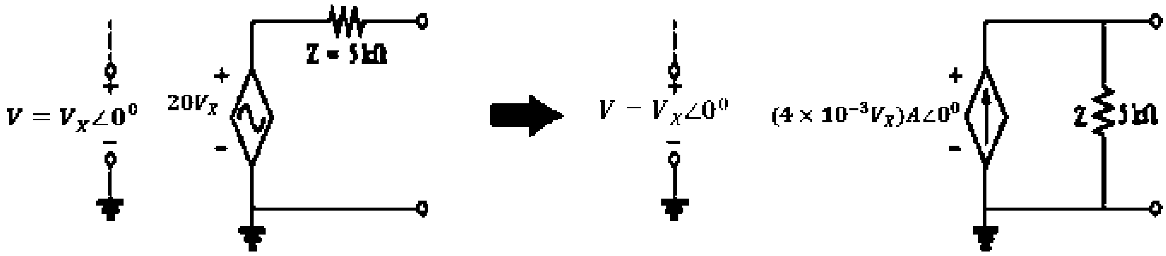
Dependent Sources

For dependent sources, the direct conversion of Figure can be applied if the controlling variable (V or I in Figure) is not determined by a portion of the network to which the conversion is to be applied.



For example, (1&2), V and I, respectively, are *controlled by an external* portion of the network. Conversions of the other kind, where V and I are controlled by a portion of the network to be converted.

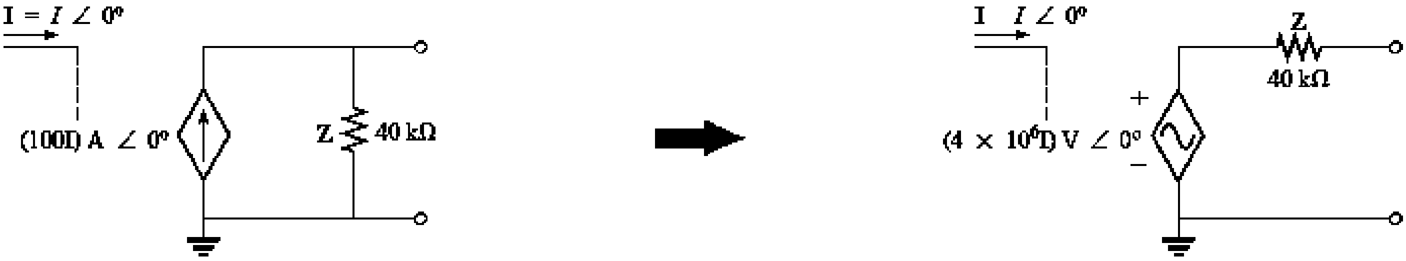
EXAMPLE1 : **Convert the voltage source of Figure to a current source.**



Sol/

$$I = \frac{E}{Z} = \frac{(20)V_X\angle 0}{5K\Omega\angle 0} = (4\times 10^{-3}V_X)A\angle 0$$

EXAMPLE2 : **Convert the current source of Figure to a voltage source**



Sol/

$$E = IZ = [(100I)A\angle 0][40K\Omega\angle 0] = (4\times 10^{6}I)V\angle 0$$

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Mesh (loop) Analysis:

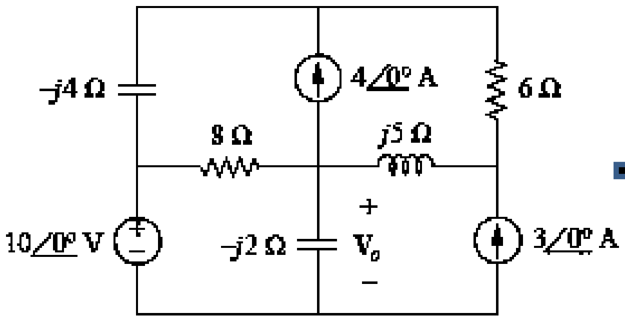
Mesh analysis allows us to determine each loop current within a circuit, regardless of number of sources within the circuit.

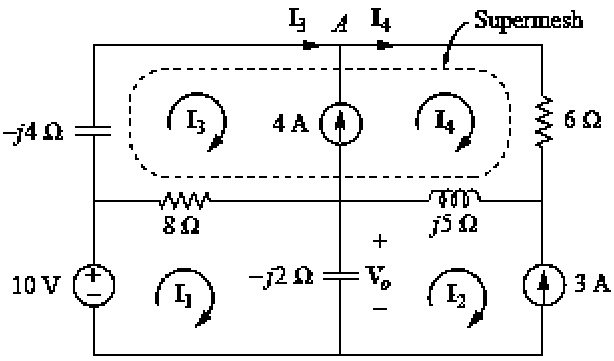
The following steps of using mesh analysis:

1) **Convert all sinusoidal expression into equivalent phasor notation.**
2) **Redraw the given circuit, simplifying the given impedance (Z_1 ,Z_2 , etc)**

3) **Arbitrarily assign clockwise loop currents to each interior closed loop within a circuit.**
4) **Apply KVL to each closed loop in the circuit.**
5) **Solve the resulting simultaneous linear equation.**

<u>Example:</u> Using the format approach to mesh analysis, Solve for V*o* in the circuit of Figure.





Sol/

<u>For the mesh 2:</u>

$$I_2 = -3\ A$$

<u>For the mesh 1:</u>

$$(8 - j2)I_1 + j2I_2 - 8I_3 = 10$$

$$(8 - j2)I_1 - 8I_3 = 10 + j6 \text{ ....(1)}$$

<u>For supermesh 3-4:</u>

$$(8 - j4)I_3 - 8I_1 + (6 + j5)I_4 - j5I_2 = 0$$

$$I_4 = I_3 + 4$$

$$-8I_1 + (14 + j)I_3 = -24 - j35 \text{ ....(2)}$$

$$\begin{bmatrix} (8-j2) & (-8) \\ (-8) & (14+j) \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} (10+j6) \\ (-24-j35) \end{bmatrix}$$

$$\Delta = (50 - j20)$$

$\Delta_1 = (-58 - j186)$

$$I_1 = \frac{\Delta_1}{\Delta} = 3.618A\angle 274.5^0$$

$$V_0 = -j2(I_1 - I_2) = -j2(3.618\angle 274.5 + 3) = 9.756V\angle 222.32^0$$

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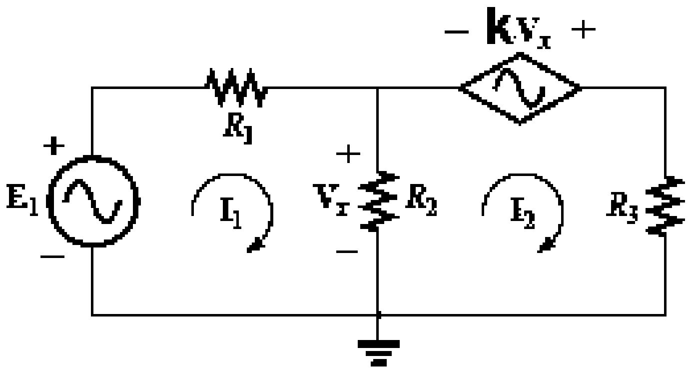
<u>EXAMPLE :</u> Write the mesh currents for the network of Figure having a dependent voltage source.



Sol/

<u>For the mesh 1:</u>

$$R_1 I_1 + R_2(I_1 - I_2) = E_1$$

$(R_1 + R_2)I_1 - R_2 I_2 = E_1$ **……………………(1)**

<u>For the mesh 2:</u>

$$R_2(I_2 - I_1) + R_3 I_2 = KV_X$$

$$-R_2 I_1 + (R_2 + R_3)I_2 = KV_X$$

$$V_X = R_2(I_1 - I_2)$$

$-(R_2 + KR_2)I_1 + (R_2 + R_3 + KR_2)I_2 = 0$ **……..(2)**

$$\begin{bmatrix} (R_1 + R_2) & (-R_2) \\ -(R_2 + KR_2) & (R_2 + R_3 + KR_2) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}$$

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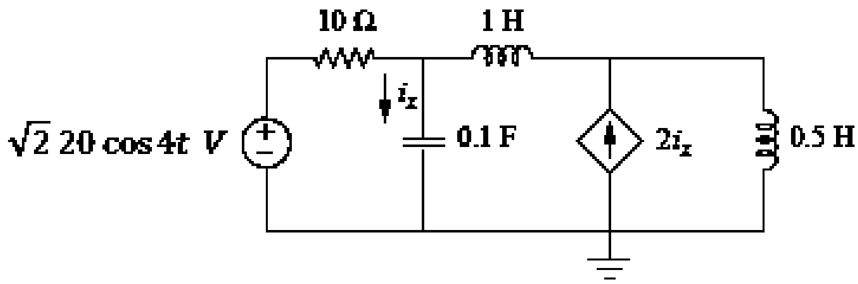
NODAL ANALYSIS:

The fundamental steps are the following:

1) ***Determine the number of nodes within the network.***
2) ***Pick a reference node and label each remaining node with a subscripted value of voltage:*** **V1, V2,** ***and so on.***
3) ***Apply Kirchhoff's current law (KCL) at each node except the reference. Assume that all unknown*** **currents leave** ***the node for each application of Kirchhoff's current law.***
4) ***Solve the resulting equations for the nodal voltages.***

Example: **Find I_x in the circuit of Figure using nodal analysis**



Sol/

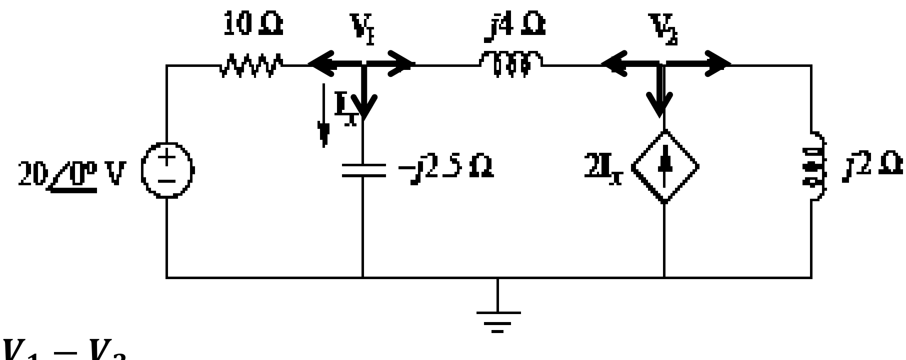
$$\sqrt{2}\ 20cos4t \quad \rightarrow 20\ V\angle 0 \quad \rightarrow \omega = 4\ rad/s$$

$$1H \rightarrow X_L = j\omega L = j4$$

$$0.5H \quad \rightarrow X_L = j2$$

$$0.1F \quad \rightarrow X_C = \frac{1}{j\omega c} = -j2.5$$

The frequency –domain equivalent circuit is shown



At node 1:

$$\frac{V_1 - 20}{10} + \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4} = 0$$

$$0.1(V_1 - 20) + j0.4V_1 - j0.25(V_1 - V_2) = 0 \qquad (\times 100)$$

$(10 + j15)V_1 + j25V_2 = 200$ ……………………..(1)

At node 2:

$$\frac{V_2 - V_1}{j4} + \frac{V_2}{j2} - 2I_X = 0$$

$$I_X = \frac{V_1}{-j2.5}$$

$$\frac{V_2 - V_1}{j4} + \frac{V_2}{j2} + \frac{2V_1}{j2.5} = 0$$

$$-j0.25(V_2 - V_1) - j0.5V_2 - j0.8V_1 = 0 \qquad (\times j100)$$

$55V_1 + 75V_2 = 0$ ……………………(2)

$$\begin{bmatrix} (10 + j15) & (j25) \\ 55 & 75 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 200 \\ 0 \end{bmatrix}$$

$\Delta = (750 - j250)$

$\Delta_1 = 15000$ $\qquad$ $\Delta_2 = -11000$

$V_1 = 18.97V\angle 18.43^0$

$V_2 = 13.91V\angle -161.56^0$

$V_2 = 13.91V\angle 198.44^0$

$I_X = 7.588\ A\angle 108.43^0$

In time domain

$$i_x(t) = \sqrt{2}\,(7.588)\,cos(4t + 108.43^0)$$

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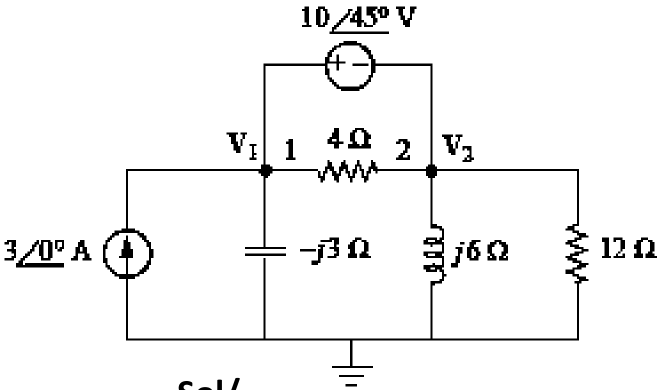
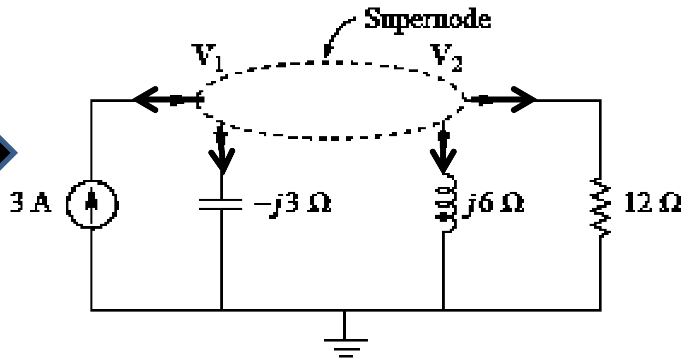
Example: Compute V_1 and V_2 in the circuit of Figure.





Sol/

At supernode 1-2:

$$-3 + \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12} = 0 \qquad (\times 12)$$

$$-36 + j4V_1 - j2V_2 + V_2 = 0$$

$$j4V_1 + (1 - j2)V_2 = 36 \quad \text{......................(1)}$$

Supernode 1-2:

$$V_2 = V_1 - 10\angle 45^0 \quad \text{........................(2)}$$

Sub. eq. (2) in eq (1) we get,

$$j4V_1 + (V_1 - 10\angle 45^0)(1 - j2) = 36$$

$$V_1 = (25.78\,V\angle -70.48^0)$$

$$V_2 = (31.41\,V\angle -87.18^0)$$

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AC Net Work Theorems

Superposition Theorem-Independent sources:

- ✓ Since AC circuits are linear, the superposition theorem applies to AC circuits the same way it applies to DC circuits.
- ✓ The voltage across(or current through) an element is determined by added algebraically the voltage (or current) due to each independent source, to find the total solution for the current or voltage.
- ✓ The theorem becomes important if the circuit has sources operating at *different* frequencies. In this case, since the impedances depend on frequency, we must have a different frequency domain circuit for each frequency. The total response must be obtained by adding the individual responses in the *time* domain.

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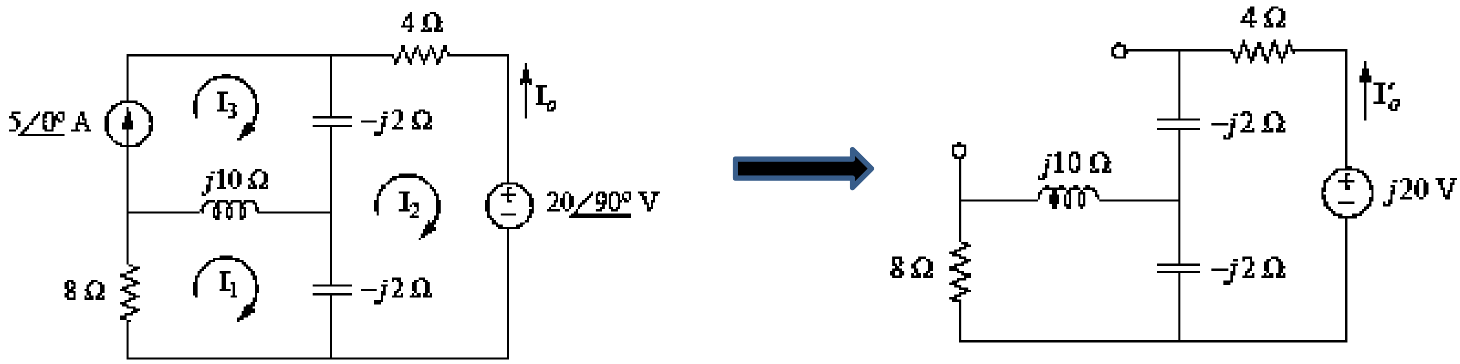
Example: Use the superposition theorem to find I_0 in the figure shown.



Sol/

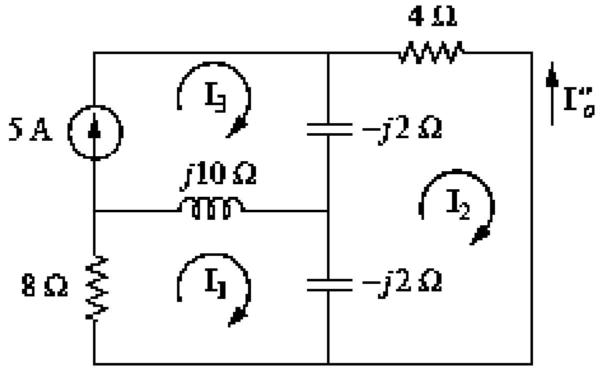
1) Taken voltage source only

$$Z = (8 + j10) \parallel (-j2) = (0.25 - j2.25)\Omega$$

$$Z_T = (4 - j2) + (0.25 - j2.25) = (4.25 - j4.25)\Omega$$

$$I_0^{\backslash} = \frac{20\angle 90^0}{4.25 - j4.25} = 3.327\angle 135^0 A$$

2) Taken current source only



❖ **For mesh 3:**

$$I_3 = 5A$$

❖ **For mesh 1:**

$$(8+8j)I_1 + j2I_2 - j10I_3 = 0$$

$$(8+8j)I_1 + j2I_2 - j50 = 0 \qquad \text{..............(1)}$$

❖ **For mesh 2:**

$$j2I_1 + (4-j4)I_2 + j2I_3 = 0$$

$$j2I_1 + (4-j4)I_2 + j10 = 0$$

$$I_1 = (2+j2)I_2 - 5 \text{ ....(2)}$$

Substituting in eq. (1) we get,

$$I_2 = \frac{90-j40}{34} = 2.896\angle{-23.962^0}A$$

$$I_0^{\backslash\backslash} = -I_2 = 2.896\angle 156.04^0 A$$

$$I_0 = I_0^{\backslash} + I_0^{\backslash\backslash} = 6.11\angle 144.78^0 A$$

It should be noted that applying the superposition theorem is not the best way to solve this problem. It seems that we have made the problem twice as hard as the original one by using superposition

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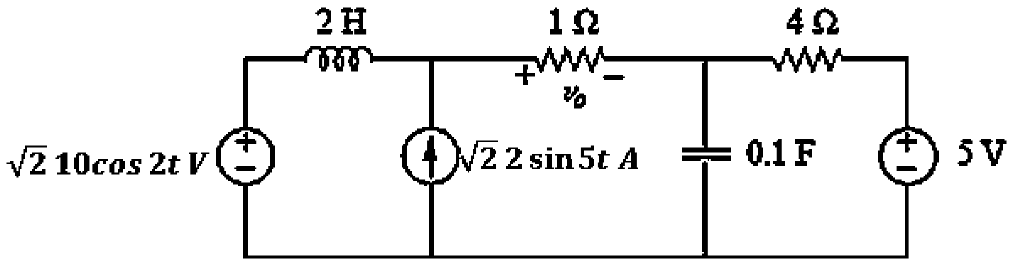
Example: Find V_o of the circuit of Figure using the superposition theorem.

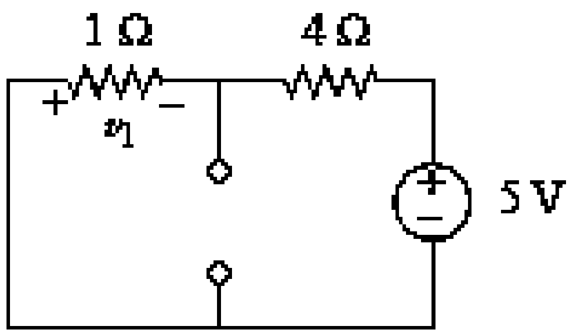


SOL/

Since the circuit operates at <u>three different frequencies</u>, one way to obtain a solution is to use superposition, which breaks the problem into single-frequency problems.

1) Taken DC voltage source 5V only

At steady state, capacitor is open circuit while an inductor is a short circuit.



Since $\omega = 0$

$$X_L = j\omega L = 0\ , \qquad X_C = \frac{1}{j\omega c} = \infty$$

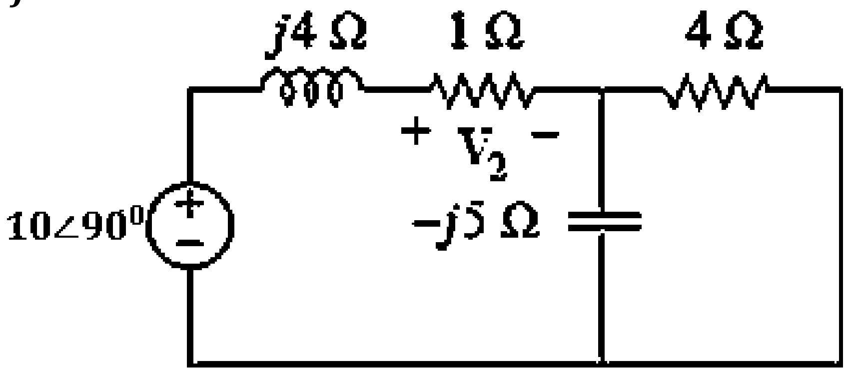
$$V_1 = -\left[5\,\frac{1}{5}\right] = -1V$$

2) Taken voltage source only

$$\sqrt{2}\,10cos\,2t \quad \rightarrow \quad ,10\angle 90^0 V_{rms} \quad \rightarrow \omega = 2\,rad/s$$

$$2H \ \rightarrow X_L = j\omega = j4\Omega$$

$0.1F \quad \rightarrow X_C = \frac{1}{j\omega c} = -j5\Omega$

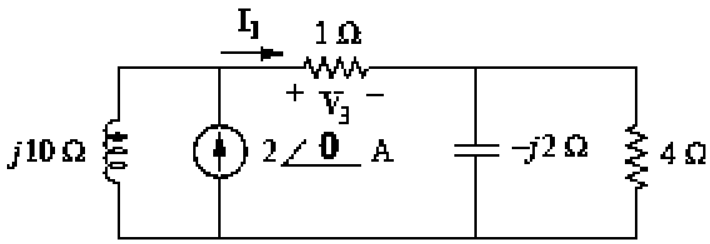


$Z = (-j5) \parallel (4) = (2.439 - j1.95)\Omega$

By voltage division

$V_2 = (10\angle 90^0)\left(\frac{1}{1+j4+2.439-j1.95}\right) = 2.497\, V\angle 59.2^0$

In time domain



$V_2(t) = \sqrt{2}\,(2.497)sin(2t + 59.2)$

3) Taken current source only

$\sqrt{2}\ \ 2\ sin5t \quad \rightarrow 2\angle 0\ I_{rms} \quad \rightarrow \omega = 5\ rad/s$

$X_L = j\omega = 10\Omega\ , \quad X_C = \frac{1}{j\omega c} = -j2\Omega$

$Z = (-j2)(4) = (0.8 - j1.6)\Omega$

By current division

$$I_1 = (2\angle 0)\frac{j10}{(j10 + 1 + 0.8 - j1.6)} = 2.328\angle 12.09^0 A$$

$V_3 = I_1 \times 1 = 2.328\angle 12.09^0$

In the time domain

$V_3(t) = \sqrt{2}\,(2.328)\ sin(5t + 12.09)$

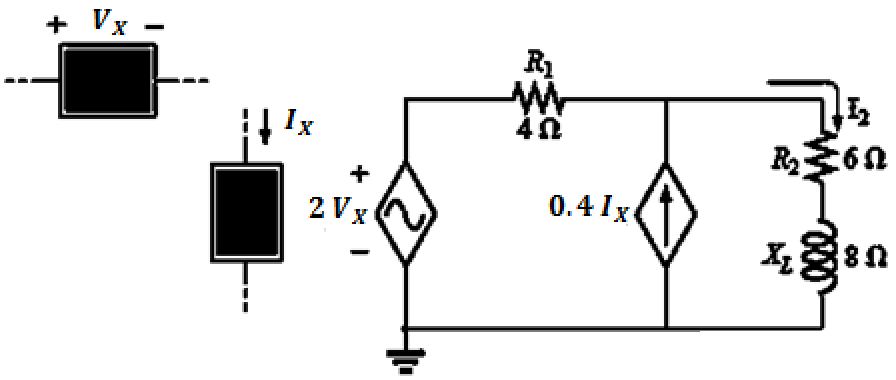
$v_0(t) = -1 + \sqrt{2}\,(2.498)sin(2t + 59.2) + \sqrt{2}\,(2.32)sin(5t + 12.09)$

Superposition theorem-Dependent sources:

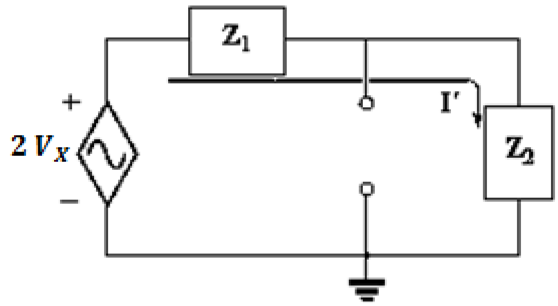
In order to analyze circuits having dependent sources, it is first necessary to determine whether the dependent source is conditional upon a controlling element in its own circuit or whether the controlling element is located in some other circuit.

- If the controlling element is external to the circuit under consideration, the method of analysis is the same for an independent source.
- If the controlling element is in the same circuit, the dependent source cannot be eliminated from the circuit.

Example: Using the superposition theorem, determine the current I_2 for the network of Figure. If $V_X = 12.8205\ V$ and $I_X = 4mA$.



For the voltage source only

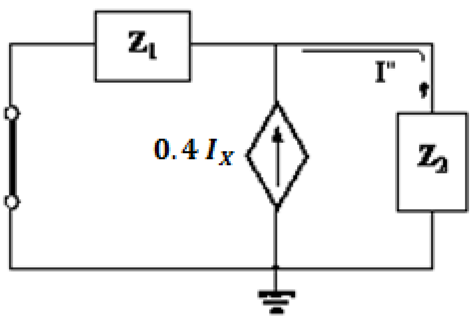


$$Z_1 = R_1 = 4\Omega$$

$$Z_2 = (6 + j8)\Omega$$

$$I^{\backslash} = \frac{2\ V_X}{Z_1 + Z_2} = 0.156\ V_X\angle{-38.66^0}$$

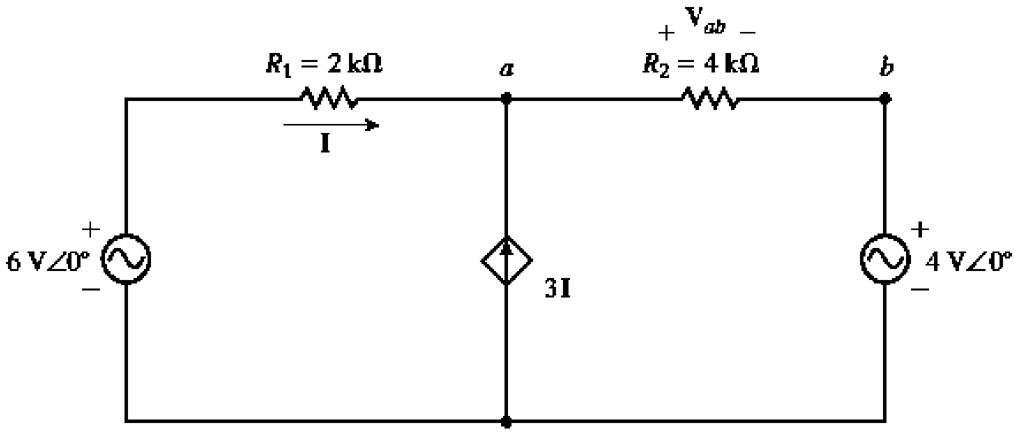
For the current source only



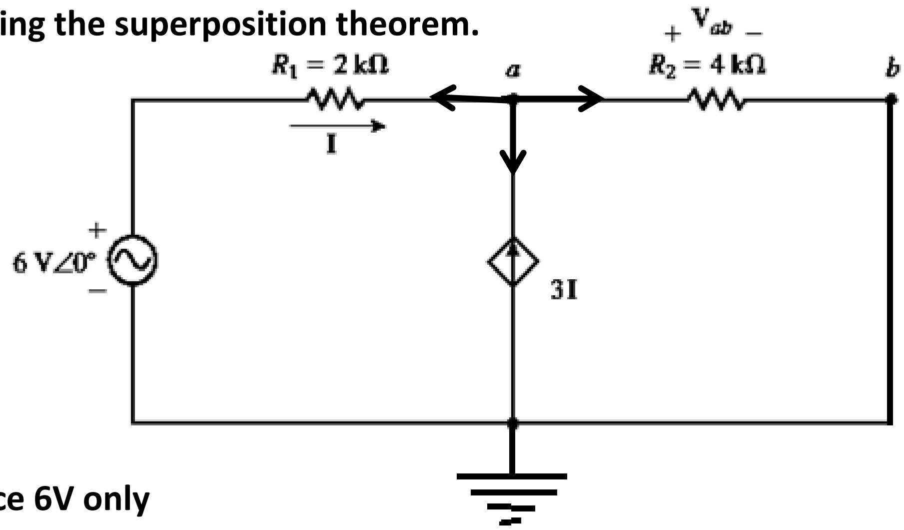
$$I^{\backslash\backslash} = (0.4\ I_X)\frac{Z_1}{Z_1 + Z_2} = 0.125\ I_X\angle{-38.66^0}A$$

$$I_2 = I^{\backslash} + I^{\backslash\backslash}$$

<u>EXAMPLE :</u> Use superposition to determine the voltage V_{ab} for the circuit of Figure below.



Sol/ The dependent current source cannot be set to zero unless I is zero. with the result that neither current source can be eliminated, as is normally done using the superposition theorem.



<u>1st method:</u>

Taken voltage source 6V only

At node (a)

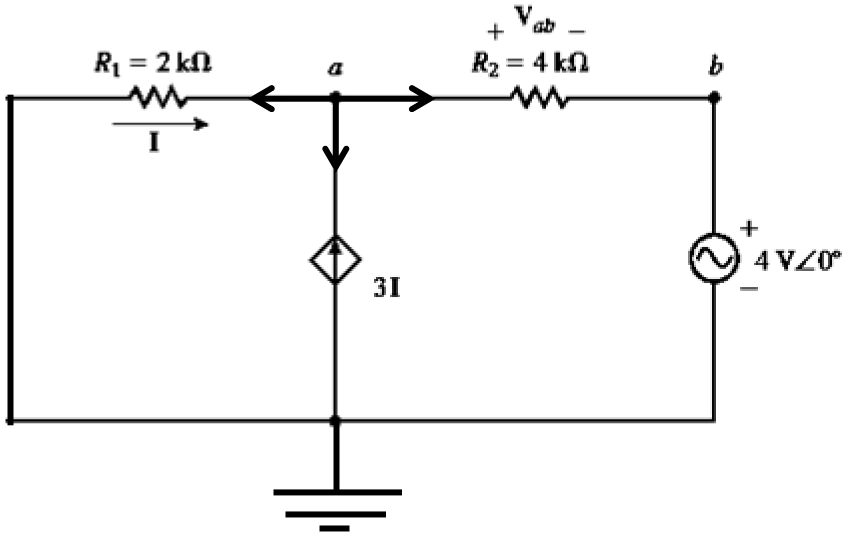
$$\frac{V_a - 6}{2K} + \frac{V_a}{4K} - 3\,I = 0$$

$$I = \frac{6 - V_a}{2K}$$

$$\frac{V_a - 6}{2K} + \frac{V_a}{4K} - 3\left(\frac{6 - V_a}{2K}\right) = 0 \qquad \times (4K)$$

$$9V_a = 48 \quad \Rightarrow \quad V_a = 5.333\ V$$

$$V^{\backslash}_{ab} = V_a = 5.332\ V$$



Taken voltage source 4 V only

At node (a)

$$\frac{V_a}{2K} + \frac{V_a - 4}{4K} - 3\,I = 0$$

$$I = \frac{-V_a}{2K}$$

$$\frac{V_a}{2K} + \frac{V_a - 4}{4K} - 3\left(\frac{-V_a}{2K}\right) = 0 \qquad (\times 4K)$$

$$9V_a = 4 \quad \Longrightarrow \quad V_a = 0.444\,V$$

$$V_{ab}^{\backslash\backslash} = (V_a - 4) = -3.556\,V$$

$$V_{ab} = 5.332 - 3.556 = 1.776\,V$$

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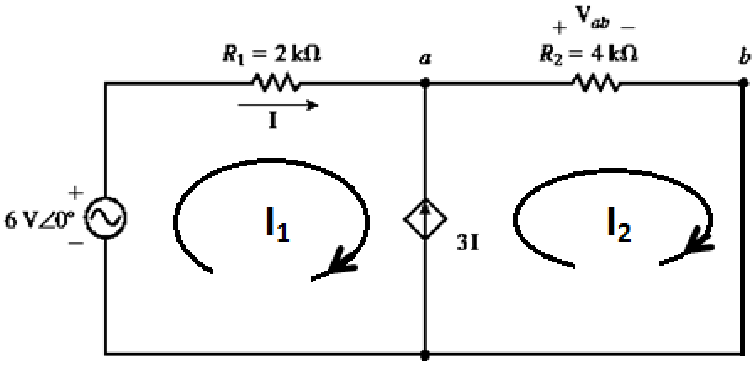
<u>2nd method:</u>



Taken voltage source 6 V only

Supermesh 1-2:

$\boldsymbol{2000I_1 + 4000I_2 = 6}$ ....(1)

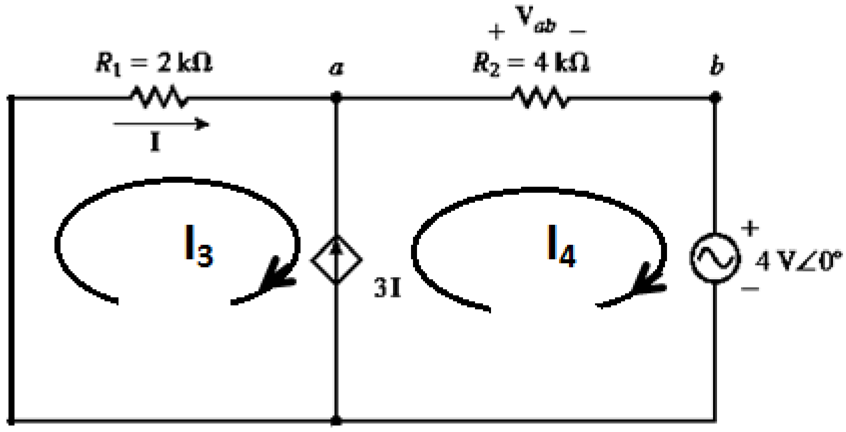
$\boldsymbol{I_2 - I_1 = 3\,I \quad , \quad I = I_1}$

$\boldsymbol{-4I_1 + I_2 = 0}$ ....(2)

$\boldsymbol{I_2 = 1.333 \times 10^{-3}\,A}$

$\boldsymbol{V^{\backslash}_{ab} = 5.332\,V}$

Taken voltage source 4 V only



Supermesh 3-4:

$\boldsymbol{2000I_3 + 4000I_4 = -4}$ ....(3)

$\boldsymbol{I_4 - I_3 = 3\,I \quad , \quad I = I_3}$

$\boldsymbol{-4I_3 + I_4 = 0}$ ....(4)

$\boldsymbol{I_4 = -8.888 \times 10^{-4} A}$

$\boldsymbol{V^{\backslash\backslash}_{ab} = -3.5552\,V}$

$\boldsymbol{V_{ab} = V^{\backslash}_{ab} + V^{\backslash\backslash}_{ab} = 5.332 - 3.5552 = 1.776\,V}$

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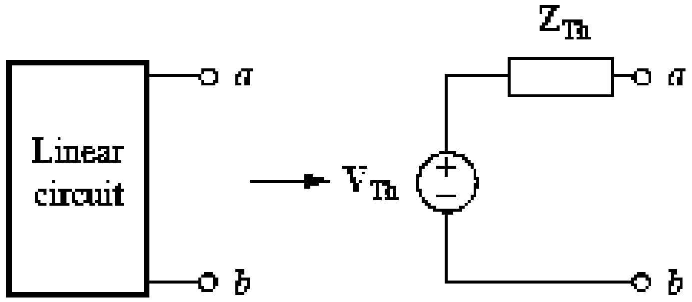
Thevenin's Theorem-Independent sources:

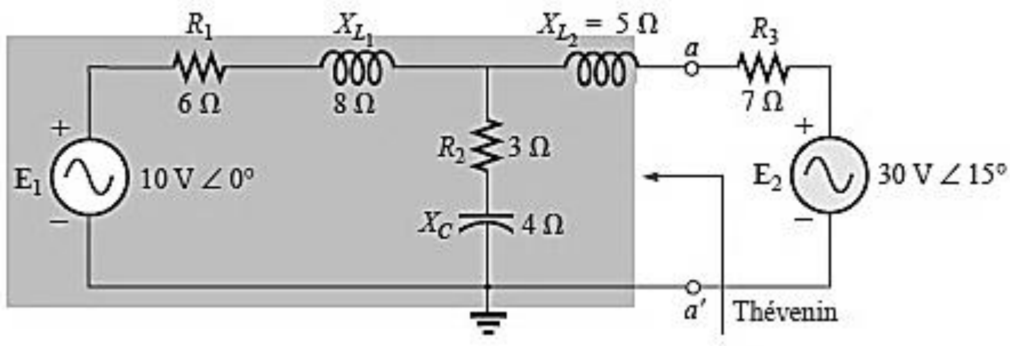
Thévenin's theorem, as stated for sinusoidal AC circuits, is changed only to include the term *impedance* instead of *resistance;* that is,

any two-terminal linear AC network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series, as shown in Figure.



If the circuit has sources operating at different frequencies, the Thevenin or Norton equivalent circuit must be determined at each frequency. This leads to entirely different equivalent circuits, one for each frequency, not one equivalent circuit with equivalent sources and equivalent impedances.

EXAMPLE: Find the Thévenin equivalent circuit for the portion of the network to the left of terminal *a-a'* in Figure.

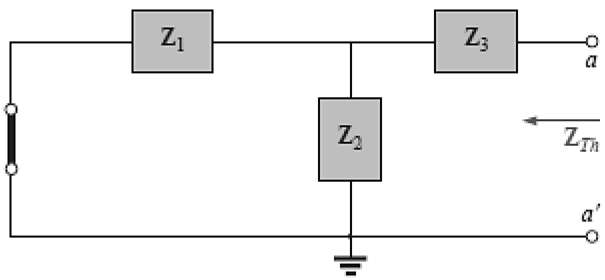


Sol

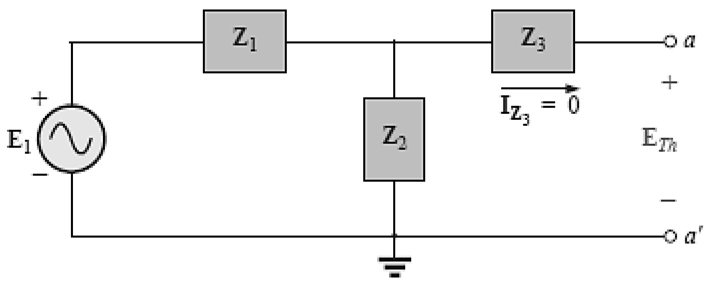
$$Z_1 = R_1 + jX_{L_1} = 6\,\Omega + j\,8\,\Omega$$

$$Z_2 = R_2 - jX_C = 3\,\Omega - j\,4\,\Omega$$

$$Z_3 = +jX_{L_2} = j\,5\,\Omega$$



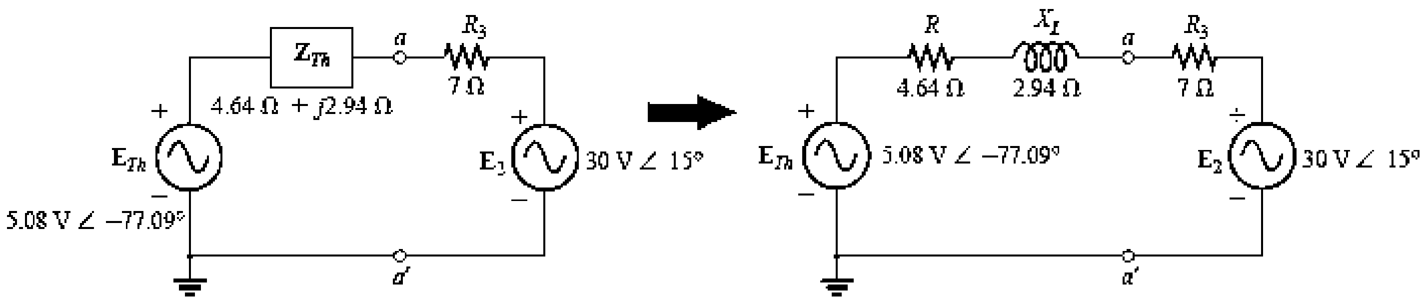
$$Z_{Th} = Z_3 + \left(\frac{Z_1 Z_2}{Z_1 + Z_2}\right) = (4.64 + j2.94)\Omega = 5.49\ \Omega\angle 32.36^0$$



Voltage divider rule to find E_{TH}

$$V_{Th} = V\frac{Z_2}{Z_1 + Z_2} = 5.08\ V\angle -77.09^0$$

The Thévenin equivalent circuit is shown in Figure below



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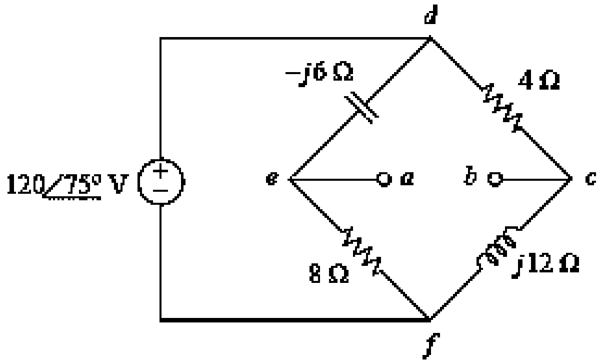
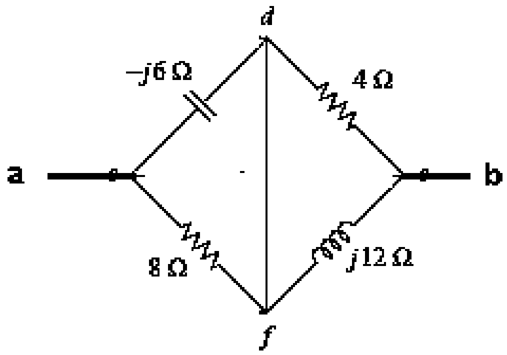
Example: **Obtain the Thevenin equivalent at terminals *a*-*b* of the circuit in Figure below.**



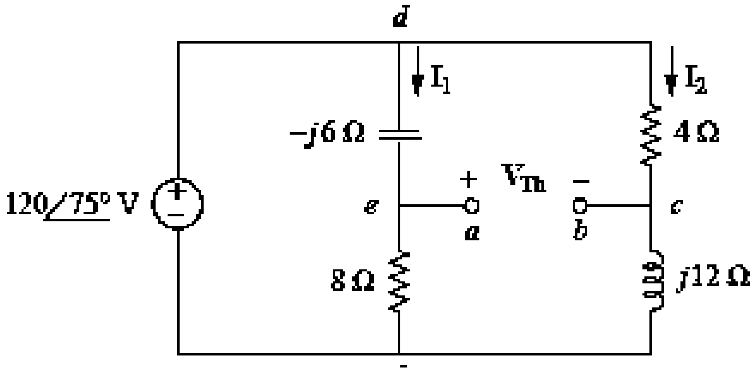
Sol/

The Thevenin impedance is



$$Z_{Th} = (-6j \parallel 8) + (4 \parallel j12) = (6.48 - j2.64)\Omega$$

To find V_{Th} consider the circuit in Figure below. Currents I_1 and I_2 are obtained as:



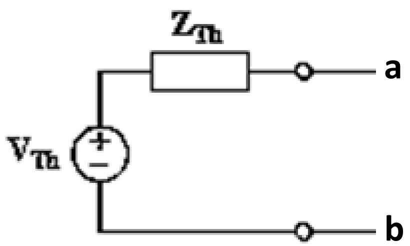
$$I_1 = \frac{120\angle 75^0}{8-j6} = 12\ A\angle 111.86^0$$

$$I_2 = \frac{120\angle 75^0}{4+j12} = 9.486A\angle 3.434^0$$

Applying KVL around loop *(aedcba)*

$$V_{Th} - 4I_2 + (-j6I_1) = 0$$

$$V_{Th} = 37.95\ V\angle 220.31^0$$



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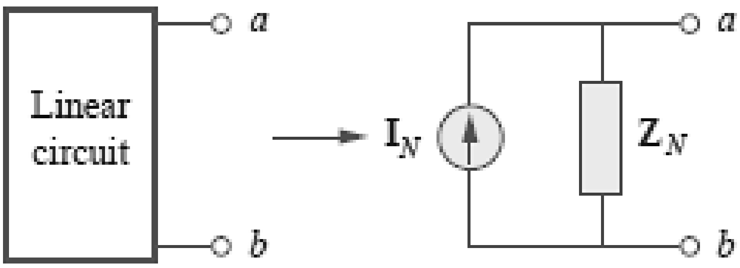
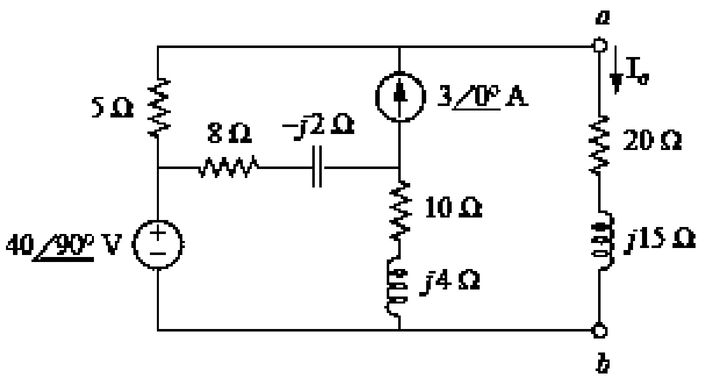
Norton's Theorem-Independent sources:

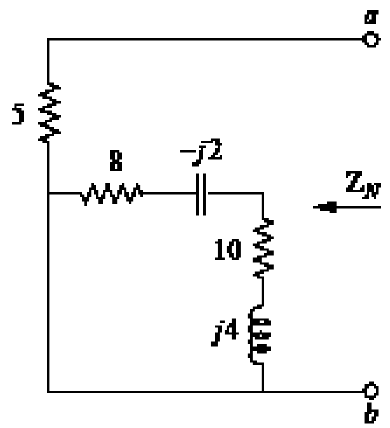
Norton's theorem allows us to replace any two-terminal linear bilateral AC network with an equivalent circuit consisting of a current source and an impedance, as in Figure.

The Norton equivalent circuit, like the Thévenin equivalent circuit, is applicable at only one frequency since the reactances are frequency dependent.



Example: Obtain current I*o* in Figure. using Norton's theorem



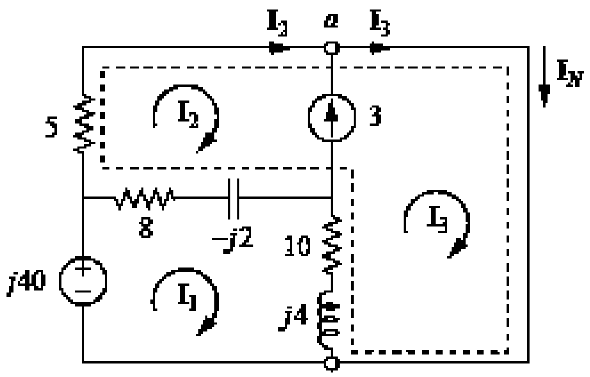


Sol/

Z_N is found in the same way as Z_{Th}

$$Z_N = 5\Omega$$

To get we short-circuit terminals *a-b* as in Figure below and apply mesh analysis. Notice that meshes 2 and 3 form a supermesh because of the current source linking them.



For mesh 1,

$$(18 + j2)I_1 - (8 - j2)I_2 - (10 + 4j)I_3 = j40 \qquad \text{..........(1)}$$

For supermesh 2-3:

$$-(18 + j2)I_1 + (13 - j2)I_2 + (10 + j4)I_3 = 0 \text{ ....(2)}$$

$$I_3 = I_2 + 3 \text{ ....(3)}$$

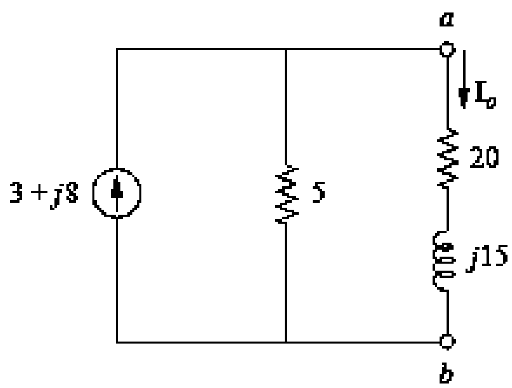
Adding Eqs. (1) and (2) gives

$$5I_2 = j40 \qquad , \qquad I_2 = j8 = 8\angle 90^0$$

From eq.(3)

$$I_3 = I_N = j8 + 3 = 8.544\ A\angle 69.443^0$$

The Norton equivalent circuit along with the impedance at terminals *a-b.* By current division



$$I_0 = (8.544\angle 69.443^0)\left(\frac{5}{25+j15}\right) = 1.465\ A\angle 38.48^0$$

****

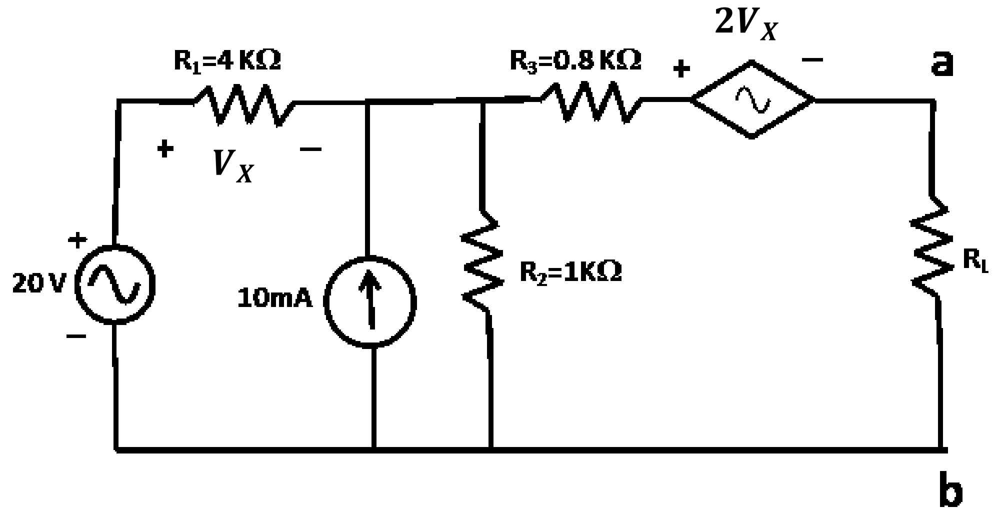
THEVENIN'S AND NORTON'S THEOREM FOR DEPENDENT SOURCE:

If a circuit contains one or more dependent source which are controlled by an element in the circuit being analyzed , all previous methods fail to provide equivalent circuits. In order to determine the thevenin or Norton equivalent circuit of a circuit having a dependent source controlled by a local voltage or current, the following steps must be taken:

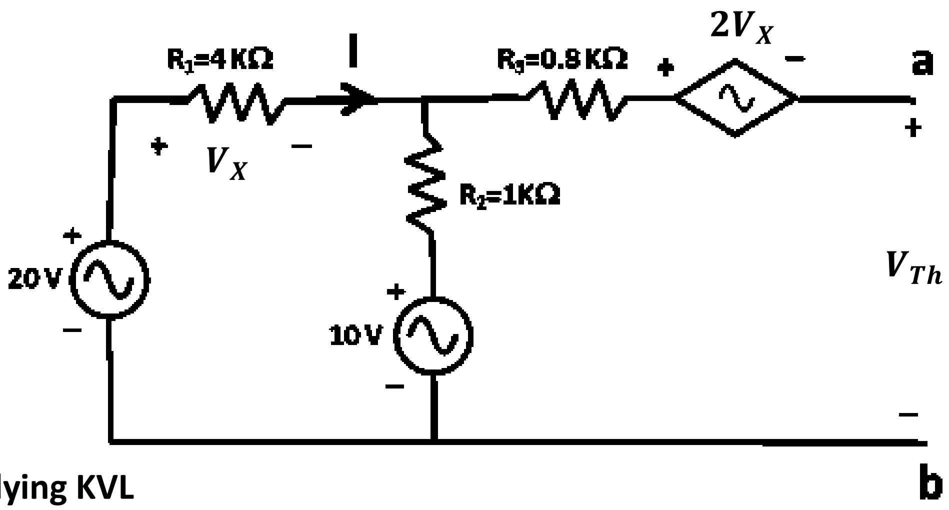
1) **Remove the branch a cross two terminals a and b.**
2) **Calculate the thevenin voltage a cross the two terminal a and b.**
3) **Determine the Norton current that would occur between the terminals.**
4) **Determine the thevenin or Norton impedance by applying equations,**

$$Z_N = Z_{Th} = \frac{V_{Th}}{I_N}$$

Example: **Find the thevenin and Norton equivalent circuits external to the load resistance in the circuit shown.**



removed the load resistance , and converting the current source into an equivalent voltage source.



Applying KVL

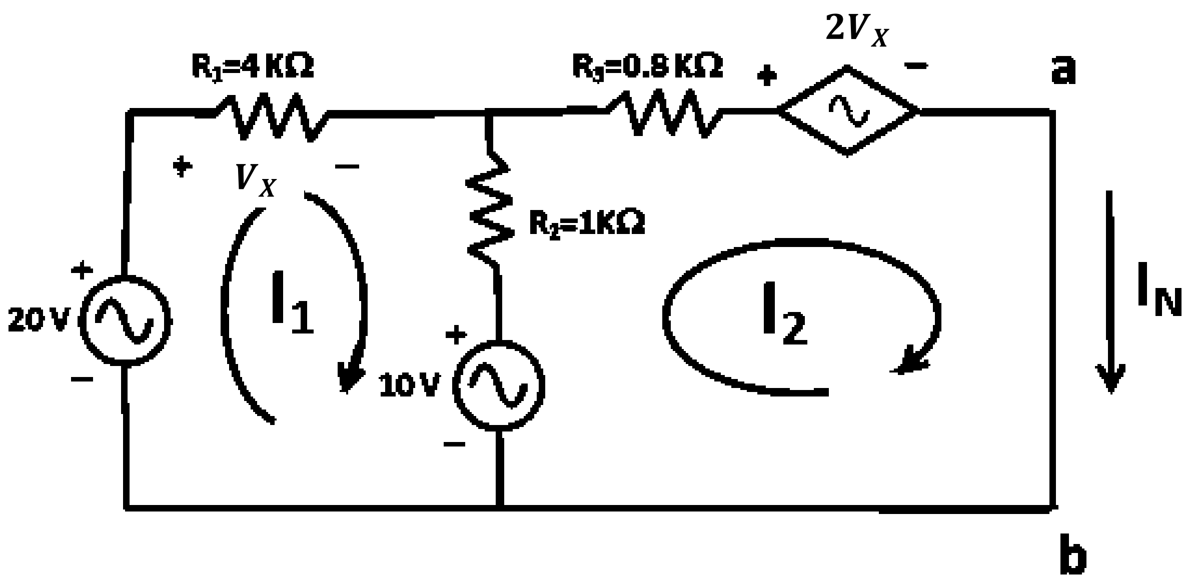
$$(5K\Omega \times I) + 20 - 10 = 0$$

I=2mA, V=8 Volt

$$V_{Th} + 16 + 8 - 20 = 0$$

$$V_{Th} = -4\,Volt$$

The Norton current is determine by mesh method



mesh (1)

$5K\Omega I_1 - 1K\Omega I_2 = 10$ ………….(1)

mesh (2)

$-1K\Omega I_1 + 1.8K\Omega I_2 = 10 - 2V \ldots\ldots\ldots\ldots\ldots\ldots (2)$

V=4KΩI$_1$, sub. In eq. (2) we gets,

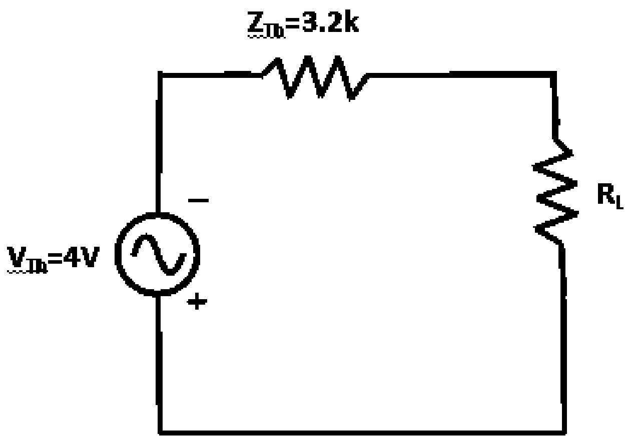
$7K\Omega I_1 + 1.8K\Omega I_2 = 10$ ………………(3)

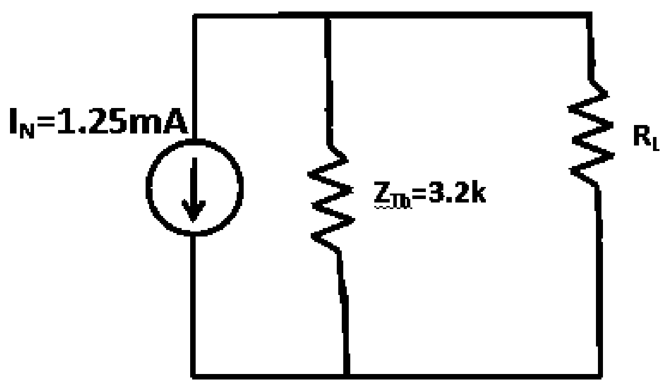
$$\begin{bmatrix} 5K & -1K \\ 7K & 1.8K \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

$I_1 = 1.75mA,$
$I_2 = I_N = -1.25mA$

The Thevenin (or Norton) impedance is determine using ohm's law

$$Z_N = Z_{Th} = \frac{-4}{-1.25\ mA} = 3.2K\Omega$$





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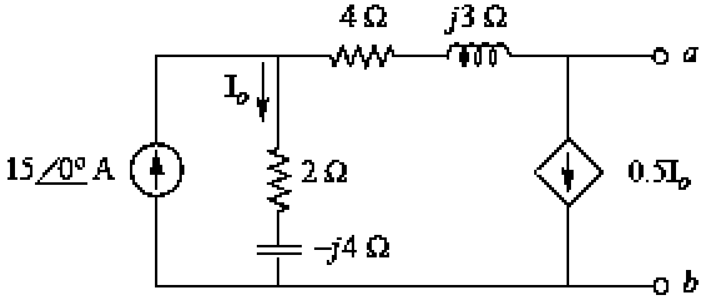
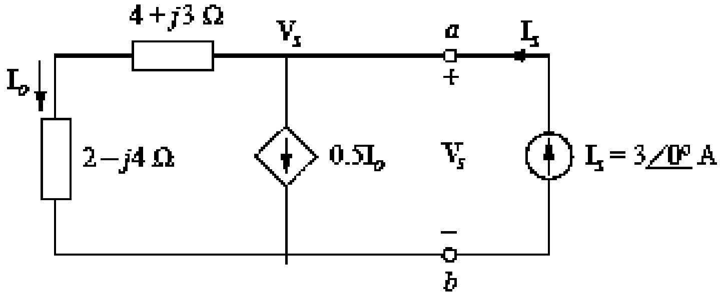
<u>Example:</u> Find the Thevenin equivalent of the circuit in Figure as seen from terminals *a-b.*

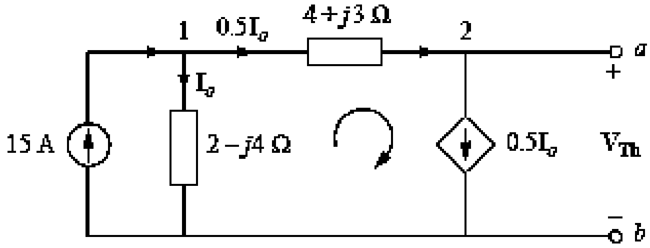
sol/



Finding Z_{Th}.

To obtain we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node).





At the node, KCL gives

$$I_0 + 0.5I_0 = 3 \ ,$$

$$I_0 = 2A$$

Applying KVL to the outer loop

$$V_S = (4 + j3 + 2 - j4)I_0 = 12 - j2 = 12.16\ V\angle{-9.462^0}$$

$$Z_{Th} = \frac{V_S}{I_0} = (4 - j0.6667)\Omega$$

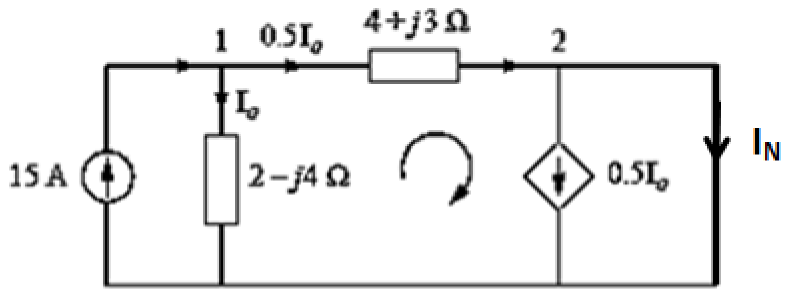
To find V_{Th} we apply KCL at node 1 ,

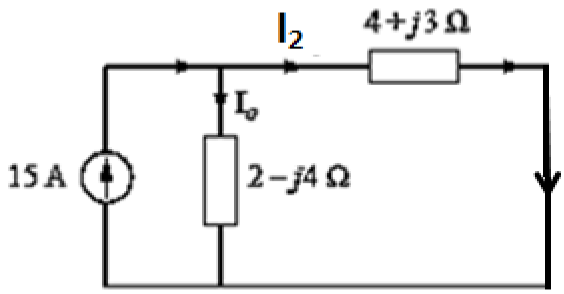
$$-V_{Th} - (4 + j3)0.5I_0 + (2 - j4)I_0 = 0$$

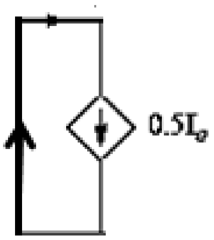
At node 1 taken KCL

$$0.5\, I_0 + I_0 = 15 \ \Rightarrow \ I_0 = 10\ A \qquad , \qquad V_{Th} = -j55 = 55\ V\angle{-90^0}$$

2nd methods find I_N







$$I_2 = 15\frac{(2-j4)}{(6-j)} = 11.028\ A\angle{-53.97^0}$$

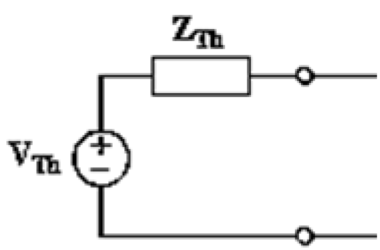
$$I_0 = 15\frac{(4+j3)}{(6-j)} = 12.329\ A\angle 46.33^0$$

Current source (0.5 I_0)= $6.1645\ A\angle 46.33^0$

$$I_N = \left(11.028\angle{-53.97^0}\right) - \left(6.1645\angle 46.33^0\right)$$

$$I_N = 13.562\ A\angle{-80.536^0}$$

$$Z_{Th} = \frac{V_{Th}}{I_N} = (4 - j0.6667)\Omega$$



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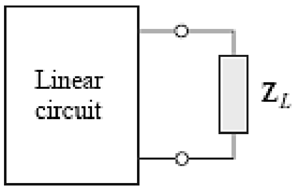
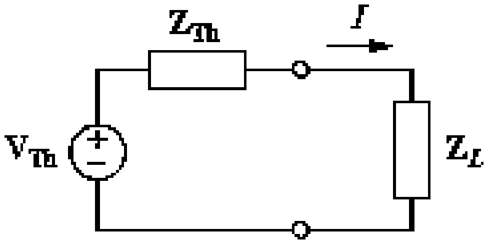
Maximum Power Transfer

The load is usually represented by an impedance, which may model an electric motor, an antenna, a TV, and so forth. In rectangular form, the Thevenin impedance Z_{Th} and the load impedance Z_L are,





$Z_{Th}=R_{Th}+jX_{Th}$, $Z_L=R_L+jX_L$

$$I = \frac{V_{Th}}{Z_{Th}+Z_L} = \frac{V_{Th}}{(R_{Th}+R_L)+j(X_{Th}+X_L)}$$

$$P = |I^2|R_L = \frac{V_{Th}^2 R_L}{(R_{Th}+R_L)^2+(X_{Th}+X_L)^2}$$

Where $|I| = \frac{V_{th}}{\sqrt{(R_{th}+R_L)^2+(X_{th}+X_L)^2}}$

$$\frac{dP}{dX_L} = [V_{Th}^2 R_L]\left[\frac{-2(X_{Th}+X_L)}{[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2]^2}\right]$$

$$\frac{dP}{dX_L} = 0 \quad \rightarrow \quad X_L = -X_{Th}$$

$$\frac{dP}{dR_L} = [V_{Th}^2]\left[\frac{[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2]-2R_L(R_{Th}+R_L)}{[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2]^2}\right]$$

$$\frac{dP}{dR_L} = 0$$

$$R_L = \sqrt{(R_{Th})^2+(X_{Th}+X_L)^2}$$

For maximum power transfer Z_L, must be selected so that X_L= - X_{Th}

$$Z_L = R_L + jX_L = R_{Th} - jX_{Th} = Z_{Th}^*$$

For maximum average power transfer, the load impedance Z_L must be equal to the complex conjugate of the Thevenin impedance Z_{Th}.

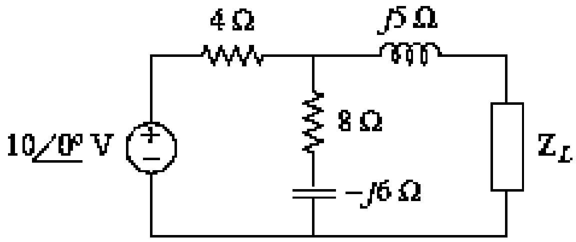
The *maximum power transfer theorem* for the sinusoidal steady state. Setting $R_L=R_{Th}$ and $X_L= -X_{Th}$ in Eq. of power gives us the maximum power as

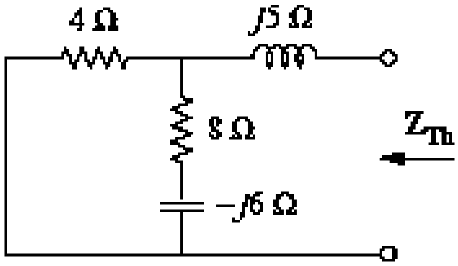
$$P_{Max.} = \frac{|V_{Th}^2|}{8R_{Th}}$$

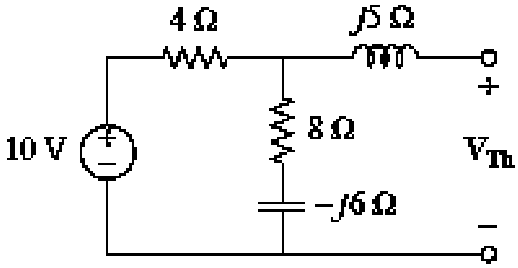
This means that for maximum power transfer to a purely resistive load, the load impedance (or resistance) is equal to the magnitude of the Thevenin impedance.

Example: Determine the load impedance that maximizes the power drawn from the circuit of Figure. What is the maximum average power?

Sol/







$$Z_{Th} = ((4) \parallel (8-j6)) + (j5) = (2.933 + j4.467)\Omega$$

$$V_{Th} = 7.45\, V\angle{-10.3^0}$$

The load impedance draws the maximum power from the circuit when

$$Z_L = Z_{Th}^* = (2.933 - j4.467)\Omega$$

The maximum average power is

$$P_{Max.} = 2.368\, Watt$$

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