

Pressure Measurement

Absolute pressure: It refers to the absolute value of the force per unit area exerted on the containing wall by a fluid. **Gage pressure** represents the difference between the absolute pressure and the local atmospheric pressure. The absolute pressure may not be negative.

Vacuum represents the amount by which the atmospheric pressure exceeds the absolute pressure. The vacuum may not be greater than the local atmospheric pressure.

The three terms are illustrated graphically in Figure(1).

Some Common units of Pressure

1 atmosphere (atm) = 14.696 pounds per square inch absolute

= 1.01325×10^5 newtons per square meter (Pa)

= 2116 pounds-force per square foot (lbf/ft²)

1 N/m² \equiv 1 pascal (Pa)

1 atmosphere (atm) = 760 millimeters of mercury (mmHg)

1 bar = 10^5 newtons per square meter (100 kPa)

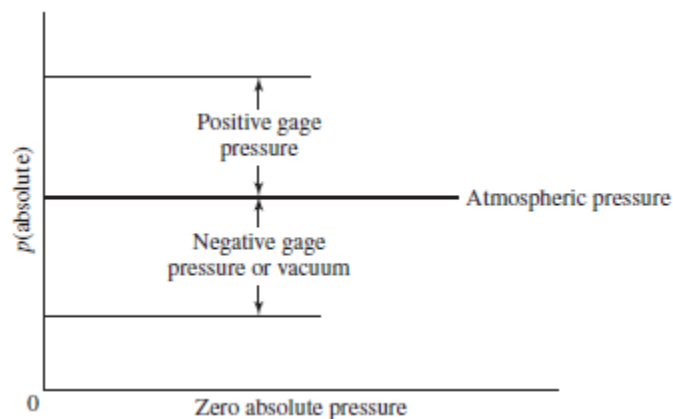


Fig.(1) Relationship between pressure terms.

1 bar = 10^5 Pa

1 microbar = 1 dyne per square centimeter

= 2.089 pounds-force per square foot

= 0.1 newton per square meter (0.1 Pa)

1 millimeter of mercury (mmHg) = 1333.22 microbar

= 133.322 newtons per square meter

(133.3 Pa)

1 micrometer = 10^{-6} meters of mercury (μm , microns)

= 10^{-3} millimeters of mercury (mmHg)

= 0.133322 newtons per square meter=(0.133 Pa)

1 torr \equiv 1 millimeter of mercury (mmHg)

1 inch of mercury = 70.73 pounds-force per square foot

1 inch of water = 5.203 pounds-force per square foot

1 pound per square inch absolute = 6894.76 newtons per square meter

(6.894 kPa)= 0.070307 kilograms-force per square

centimeter (kgf/cm^2), [kilopounds

per square centimeter (kp/cm^2)]

Mechanical Pressure-Measurement Devices

Mechanical devices offer the simplest means for pressure measurement. It consists the following measuring devices:

U-Tube Manometer:

The fluid manometer is a widely used device for measurement of fluid pressures under steady-state and laboratory conditions. Consider the u-tube manometer shown in figure 1, the difference in pressure between the unknown pressure p and the atmosphere is determined as a function of the differential height h .

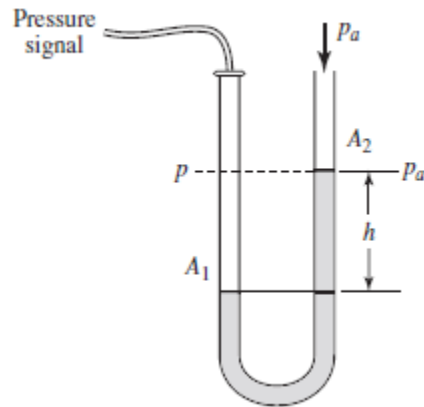


Fig.(1) U-tube manometer

The density of the fluid transmitting the pressure p is ρ_f , and the density of the manometer fluid is designated as ρ_m . A pressure balance of the two columns dictates that:-

$$p_a + \frac{g}{g_c} h \rho_m = p + \frac{g}{g_c} h \rho_f$$

$$p - p_a = \frac{g}{g_c} h (\rho_m - \rho_f) \tag{1}$$

The sensitivity of the U-tube manometer may be defined as

$$\text{Sensitivity} = h / (p - p_a) = h / \Delta p = 1 / (g / g_c) (\rho_m - \rho_f)$$

or for a manometer with $\rho_m \gg \rho_f$,

$$\text{Sensitivity} = 1 / \rho_m (g / g_c)$$

Ex:

A U-tube manometer employs special oil having a specific gravity of 0.82 for the manometer fluid. One side of the manometer is open to local atmospheric pressure of 29.3 inHg and the difference in column heights is measured as $20 \text{ cm} \pm 1.0 \text{ mm}$ when exposed to an air source at 25°C . Standard acceleration of gravity is present. Calculate the pressure of the air source in Pascals and its uncertainty.

Solution

The manometer fluid has a density of 82 percent of that of water at 25°C; so,

$$\rho_m = 0.82\rho_w = (0.82)(996 \text{ kg/m}^3) = 816.7 \text{ kg/m}^3$$

The local atmospheric pressure is

$$p_a = 29.3 \text{ inHg} = 9.922 \times 10^4 \text{ Pa}$$

The “fluid” in this problem is the air which has a density at the above pressure and 25°C (298 K) of

$$\rho_f = \rho_a = p/RT = 9.922 \times 10^4 / (287)(298) = 1.16 \text{ kg/m}^3$$

For this problem the density is negligible compared to that of the manometer fluid, but we shall include it anyway. From Eq. (1)

$$p - p_a = (\rho_m - \rho_f)gh = (816.7 - 1.16)(9.807)(0.2) = 1600 \text{ Pa}$$

$$\text{or } p = 1600 + 9.922 \times 10^4 = 1.0082 \times 10^5 \text{ Pa}$$

The uncertainty of the column height measurement is

$$1 \text{ mm} / (20 \text{ cm} \times 10 \text{ mm/cm}) = 1/200 = 0.5 \%$$

Well-type manometer:

A well-type manometer operates in the same manner as the U-tube manometer, except that the construction is as shown in Fig. 6.4. In this case the pressure balance of Eq. (1) still yields

$$p - p_a = \rho_m g h - \rho_f g h'$$

The well-type manometer is filled to a certain level at zero-pressure differential conditions. A measurement is then made of the displacement of the small column from this zero level. Designating this displacement by h' , we have

$$h' A_2 = (h - h') A_1 \tag{2}$$

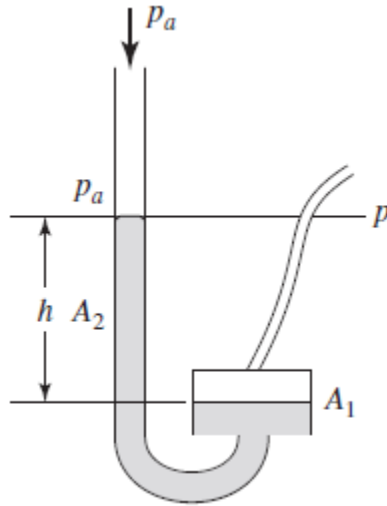


Fig.(2) Well-type manometer

since the volume displacements are the same on both sides of the manometer. Inserting Eq. (2) in (1) gives

$$\Delta P = P_1 - P_2 = \frac{g}{g_c} h' \left(\frac{A_2}{A_1} + 1 \right) (\rho_m - \rho_f) \quad (3)$$

Commercial well-type manometers have the scale for the manometer column graduated so that the user need not apply the area correction factor to the indicated displacement h' . Thus, for an area ratio of $A_2/A_1 = 0.03$ a true reading of 10.0 in for h' would be indicated as 10.3 as a result of the special scale graduation. The indicated value is then substituted for h in Eq. (1).

Inclined Manometer

Manometers may be oriented in **an inclined position to lengthen the scale and to improve readability**. For a manometer inclined at an angle θ with the horizontal the sensitivity becomes :

$$\text{Sensitivity} = L/\Delta p = L/(g/g_c) (\rho_m - \rho_f) \times L \sin \theta$$

where L is the measured fluid displacement along the incline and

$$h = L \sin \theta$$

Micro-manometer

It consists of two reservoirs of diameter D connected by a U-tube of smaller diameter d . Reservoir 1 contains a fluid with density ρ_1 , while

reservoir 2 contains a fluid having a density ρ_2 . The assembly is subjected to a pressure differential Δp . The sensitivity may be derived

$$\text{Sensitivity} = h/\Delta p = \{[(d/D)^2(\rho_2 + \rho_1) + (\rho_2 - \rho_1)](g/g_c)\}^{-1} \quad 3$$

$$h = h_{\text{initial interface}} - h_{\text{final interface}} \quad (4)$$

When $D \gg d$ and the difference in the two fluid densities is small, a very large sensitivity can result.

Dead-Weight Tester

The dead-weight tester is a device used for balancing a fluid pressure with a known weight. Typically, it is a device used for static calibration of pressure gages. Consider the schematic in Fig. (3). The apparatus is set up for calibration of the pressure gage G . The chamber and cylinder of the tester are filled with a clean oil. The gage to be tested is installed and the piston inserted in the cylinder. The pressure exerted on the fluid by the piston is now transmitted to the gage when the valve is opened. This pressure may be varied by adding weights to the piston. The viscous friction between the piston and the cylinder in the axial direction may be substantially reduced by rotating the piston-weight assembly while the measurement is taken. As the pressure is increased, it may be necessary to advance the plunger to account for the compression of the oil and any entrapped gases in the apparatus. The accuracies of dead-weight testers are limited by two factors:

- (1) The friction between the cylinder and the piston.
- (2) The uncertainty in the area of the piston.

The friction is reduced by rotation of the piston and use of long enough surfaces to ensure negligible flow of oil through the annular space between the piston and the cylinder. The area upon which the weight force acts is not the area of the piston or the area of the cylinder; it is

some effective area between these two which depends on the clearance spacing and the viscosity of the oil. The smaller the clearance, the more closely the effective area will approximate the cross-sectional area of the piston.

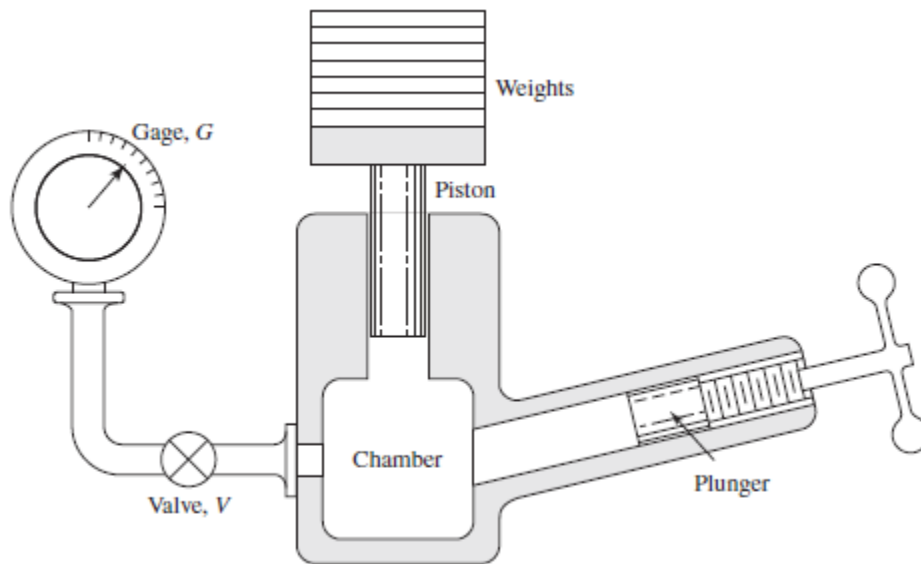


Fig.(3) Schematic of a dead-weight tester.

It can be shown that the percentage error due to the clearance varies according to

$$\text{Percent error} \sim \frac{(\rho \Delta p)^{1/2} b^3}{\mu DL} \quad (5)$$

where

ρ = density of the oil

Δp = pressure differential on the cylinder

b = clearance spacing

μ = viscosity

D = piston diameter

L = piston length

Bourdon-Tube Pressure Gage

Bourdon-tube pressure gages enjoy a wide range of application where consistent, inexpensive measurements of static pressure are desired. They are commercially **available in many sizes (1- to 16-in diameter)** and accuracies. The construction of a bourdon-tube gage is shown in Figure (4). The bourdon tube itself is usually an elliptical cross-sectional tube having a C-shape configuration. When the pressure is applied to the inside of the tube, an elastic deformation results, which, ideally, is proportional to the pressure. The degree of linearity depends on the quality of the gage. **The end of the gage is connected to a spring-loaded linkage, which amplifies the displacement and transforms it to an angular rotation of the pointer.**

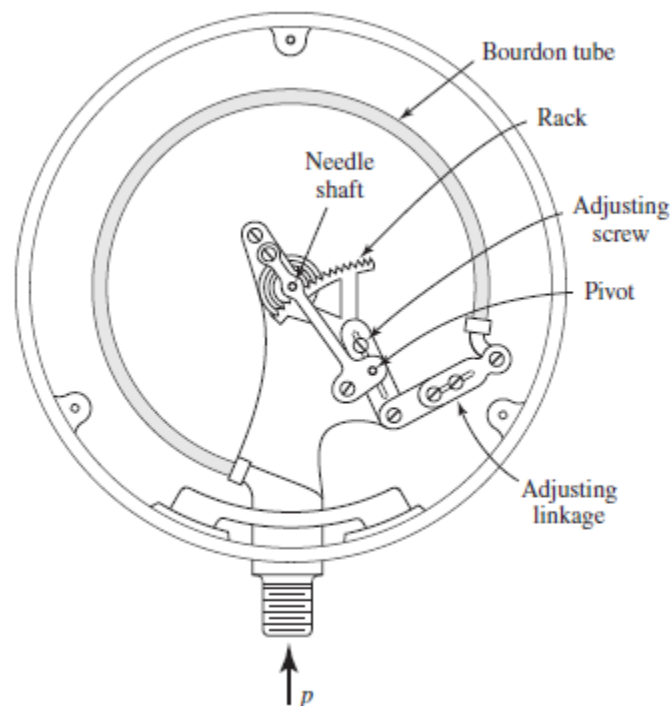


Fig.(4) Schematic of a bourdon-tube pressure gage.

Electrical-resistance strain gages may also be installed on the bourdon tube to sense the elastic deformation. A proprietary design of digital pressure transducers based upon piezo-resistance strain gage response

claims an accuracy of 0.02 percent of full scale and is available in full-scale ranges from 10 to 10,000 psia.

Diaphragm and Bellows Gages

Diaphragm and bellows gages represent similar types of elastic deformation devices useful for many pressure-measurement applications.

Diaphragm Gages

Consider first the flat diaphragm subjected to the differential pressure $p_1 - p_2$, as shown in Figure (5). The diaphragm will be deflected in accordance with this pressure differential and the deflection sensed by an appropriate displacement transducer.

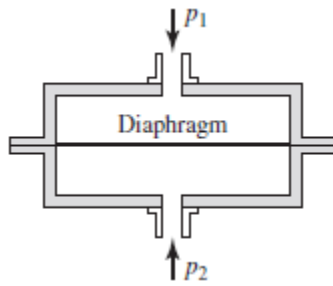


Fig.(5) Schematic of a diaphragm gage.

Electrical-resistance strain gages may also be installed on the diaphragm, as shown in Fig. (6). The output of these gages is a function of the local strain, which, in turn, may be related to the diaphragm deflection and pressure differential.

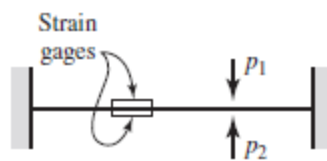
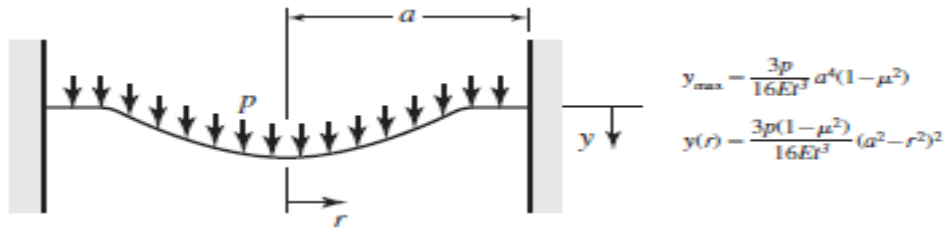


Fig.(6) Diaphragm gage using electrical-resistance strain gages

The output of these gages is a function of the local strain, which, in turn, may be related to the diaphragm deflection and pressure differential.

Accuracies of ± 0.5 percent of full scale are typical. The deflection generally follows a linear variation with Δp when the deflection is less than one-third the diaphragm thickness.



Capacitive Transducer

The deflection of a diaphragm under pressure may be sensed by a capacitance variation, as shown in Figure(7). Such pressure pickups are well suited for dynamic measurements since the natural frequency of diaphragms can be rather high. The capacitance pickup, however, involves low sensitivity, and special care must be exerted in the construction of readout circuitry.

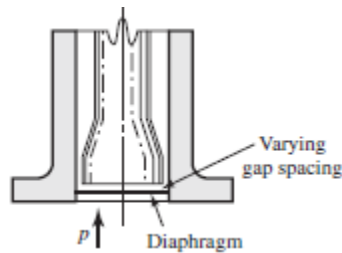


Fig.(7)Capacitance pressure gage

Bellow Gages

The bellows gage is depicted schematically in Figure(8). A differential pressure force causes a displacement of the bellows, which may be converted to an electrical signal or undergo a mechanical amplification to permit display of the output on an indicator dial. **The bellows gage is generally unsuitable for transient measurements because of the larger relative motion and mass involved. The diaphragm gage, on the other hand, which may be quite stiff, involves rather small displacements and is suitable for high-frequency pressure measurements.**

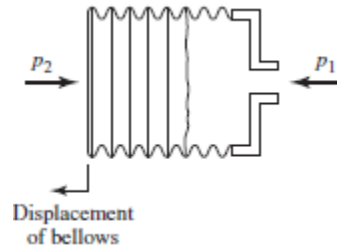


Fig.(8) Schematic of a bellows pressure gage

Differential Pressure Gage

Differential pressure gage is shown in Figure(9). Commercial models of this type of gage permit measurement of pressures as low as 0.00035 psi (0.25 Pa). The natural frequency of a circular diaphragm fixed at its perimeter is given by

$$f = \frac{10.21}{a^2} \sqrt{\frac{g_c E t^2}{12(1 - \mu^2)\rho}} \text{ Hz} \quad (6)$$

where E = modulus of elasticity, psi or Pa

t = thickness, in or m

a = radius of diaphragm, in or m

ρ = density of material, lbm/in³ or kg/m³

g_c = dimensional conversion constant = 385.9 lbm · in/lbf · s² or 1.0 kg · m/N · s²

μ = Poisson's ratio

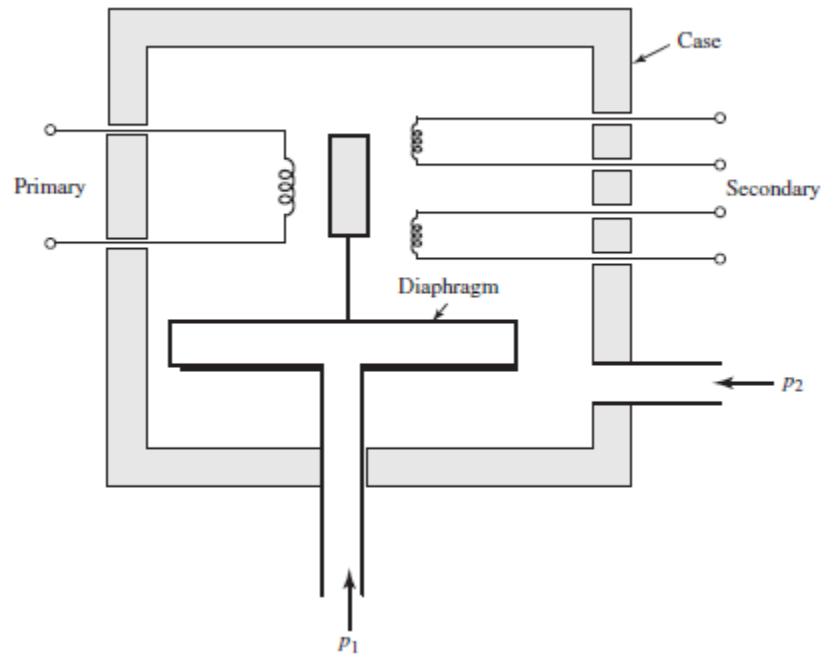


Fig.(9) Schematic of diaphragm LVDT combination used as differential pressure gage.

Equation (6) may be simplified to the following relation for steel diaphragms:

$$f = 1.934 \times 10^6 t / \pi a^2 \quad (t \text{ and } a \text{ in in}) \quad (7)$$

$$= 4.912 \times 10^4 t / \pi a^2 \quad (t \text{ and } a \text{ in m}) \quad (8)$$

Piezoresistive or semiconductor pressure transducers usually consist of a silicon diaphragm with a semiconductor strain gage bonded to the diaphragm to measure deflection.

Summary

Figure (10) gives a convenient summary of the pressure ranges for which the pressure gages are normally employed in practice.

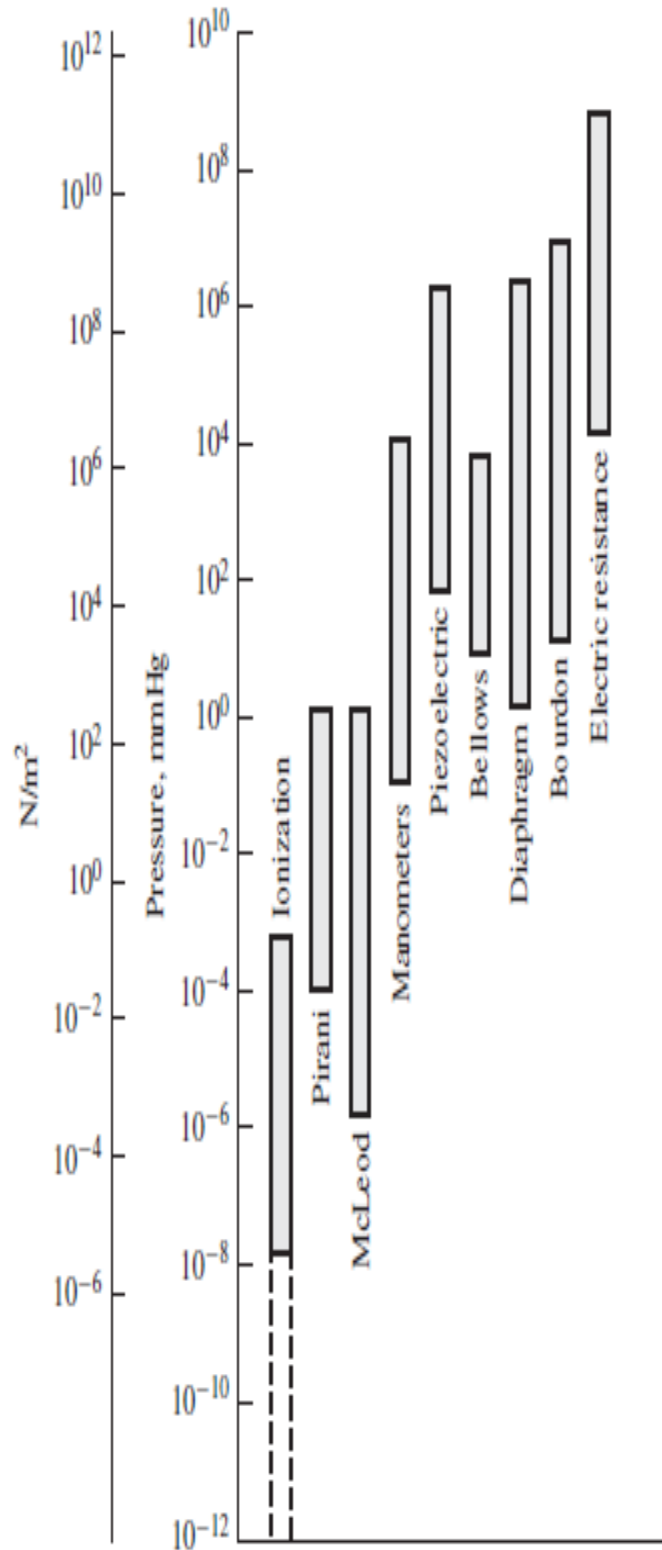


Fig.(10)Summary of applicable range of pressure gages.

Dynamic Response Considerations

The fluctuating pressure has a frequency of ω and an amplitude of p_0 and is impressed on the tube of length L and radius r . At the end of this tube is a chamber of volume V as shown in figure (11) where the connection to the pressure-sensitive transducer is made. If the conventional formula for laminar friction resistance in the tube flow is used to represent this friction, the resulting expression for the pressure-amplitude ratio is:

$$\left| \frac{P}{P_0} \right| = \frac{1}{\left\{ \left[1 - \left(\frac{\omega}{\omega_n} \right)^2 \right]^2 + 2\zeta^2 \left(\frac{\omega}{\omega_n} \right)^2 \right\}^{1/2}} \quad (9)$$

In this equation p is the amplitude of the pressure signal impressed on the transducer. The natural frequency ω_n is given by

$$\omega_n = \sqrt{\frac{3\pi r^2 c^2}{4LV}} \quad (10)$$

and the damping ratio ζ is

$$\zeta = \frac{2\mu}{\rho c r^3} \sqrt{\frac{3LV}{\pi}} \quad (11)$$

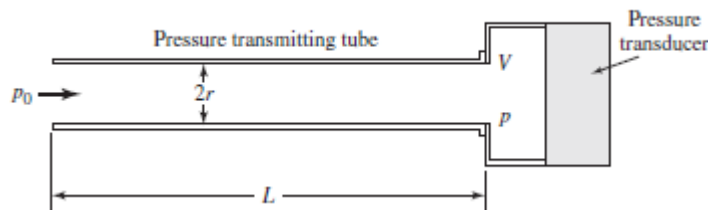


Fig.(11)

In the above formulas c represents the velocity of sound in the fluid, μ is the dynamic viscosity of the fluid, and ρ is the fluid density. The phase angle for the pressure signal is

$$\phi = \tan^{-1} \frac{-2\zeta\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2} \quad (12)$$

The velocity of sound for air may be calculated from

$$c = 49.1T^{1/2} \text{ ft/s with } T \text{ in } ^\circ\text{R}$$

$$c = 20.04T^{1/2} \text{ m/s with } T \text{ in K}$$

When the tube diameter is very small, as in a capillary, it is possible to produce a very large damping ratio so that Eq. (9) will reduce to the following for frequencies below the natural frequencies:

$$\left| \frac{P}{P_o} \right| = \frac{1}{\{1+4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\}^{1/2}} \quad (13)$$

(Q1). Distinguish among gage pressure, absolute pressure, and vacuum.

(Q2). To transmit a high-frequency pressure signal, one should select

(a) A short, small-diameter tube

(b) A short, large-diameter tube

(c) A long, small-diameter tube

(d) A long, large-diameter tube

(Q3). What are the advantages of the manometer pressure-measurement device?

(Q4). What is the advantage of a well-type manometer?

(Q5). What are some advantages of the bourdon-tube, diaphragm, and bellows gages?

(Q) A well-type manometer uses a special bromide fluid having a specific gravity of 2.95. The well has a diameter of 75mm and the tube has a diameter of 5mm. The manometer is to be used to measure a differential pressure in a water-flow system. The scale placed alongside the tube has

no correction factor for the area ratio of the manometer. Calculate the value of a factor that may be multiplied by the manometer reading in inches to find the pressure differential in pounds per square inch.

$$\Delta P = P_1 - P_2 = \frac{g}{g_c} h' \left(\frac{A_2}{A_1} + 1 \right) (\rho_m - \rho_f)$$

$$A_2 = 1.95 \times 10^{-5} \text{ m}^2, A_1 = 4.41 \times 10^{-3}, \gamma_m = 2.95$$

$$\gamma_f = 1$$

$$\Delta P = P_1 - P_2 = h' \left(\frac{1.95 \times 10^{-5}}{4.41 \times 10^{-3}} + 1 \right) (2.950 - 1)$$

$$\Delta P = h' (1.950) \text{ N/m}^2$$

Or the factor is 1.95 N/m^3

(Q) AU-tube manometer uses tubes of 0.250 and 0.500 in diameters for the two legs. When subjected to a certain pressure, the difference in height of the two fluid columns is 10.0 inHg. What would have been the reading if both tubes were the same diameter? The measurement is performed on air.

Solution:

The reading would be the same. 10 in. Hg

(Q) A manometer uses a fluid having a specific gravity of 1.85. The sensing fluid is water. What is the pressure difference when the difference in heights of the columns is 5.0 in? Assume that both legs of the manometer are filled with water.

Solution:

$$\Delta P = (\gamma_m - \gamma_f) h = (1.85 - 1) \times 5 = 4.25 \text{ in } H_2O$$

(Q) A diaphragm-pressure gage is constructed of spring steel to measure a pressure differential of 1000 psi. The diameter of the diaphragm is 0.5 in. Calculate the diaphragm thickness so that the maximum deflection is one-third the thickness. What is the natural frequency of this diaphragm? Use $E=26 \times 10^6 \text{Psi}$, $\nu = 0.3$, $a=0.25 \text{in}$,

Solution:

$$y_{max} = \frac{3\Delta p a^4}{16Et^3} (1 - \nu^2)$$
$$y_{max} = \frac{1}{3}t = \frac{3(1000)0.25^4}{16 \times 28 \times 10^6 t^3} (1 - 0.3^2)$$
$$t^4 = 6.87 \times 10^{-8}$$

Or $t = 0.0162 \text{in}$.