Helical Gears

Helical gears are very similar to spur gears. They have teeth that lie in helical paths on the cylinders instead of teeth parallel to the shaft axis.

Like spur gears, helical gears are cut from a cylindrical gear blank and have involute teeth. The difference is that their teeth are at some helix angle to the shaft axis. These gears are used for transmitting power between parallel or nonparallel shafts. The former case is shown in Figure (1-a). In nonparallel, nonintersecting shaft applications, gears with helical teeth are known as crossed helical gears, as shown in Figure (1-b). This severely reduces their load-carrying capacity. Nevertheless, crossed helical gears are frequently used for the transmission of relatively small loads, such as distributor and speedometer drives of automobiles. The helix can slope in either the upward or downward direction. The terms right-hand (RH) and left-hand (LH) helical gears are used to distinguish between the two types, as indicated in the figure.

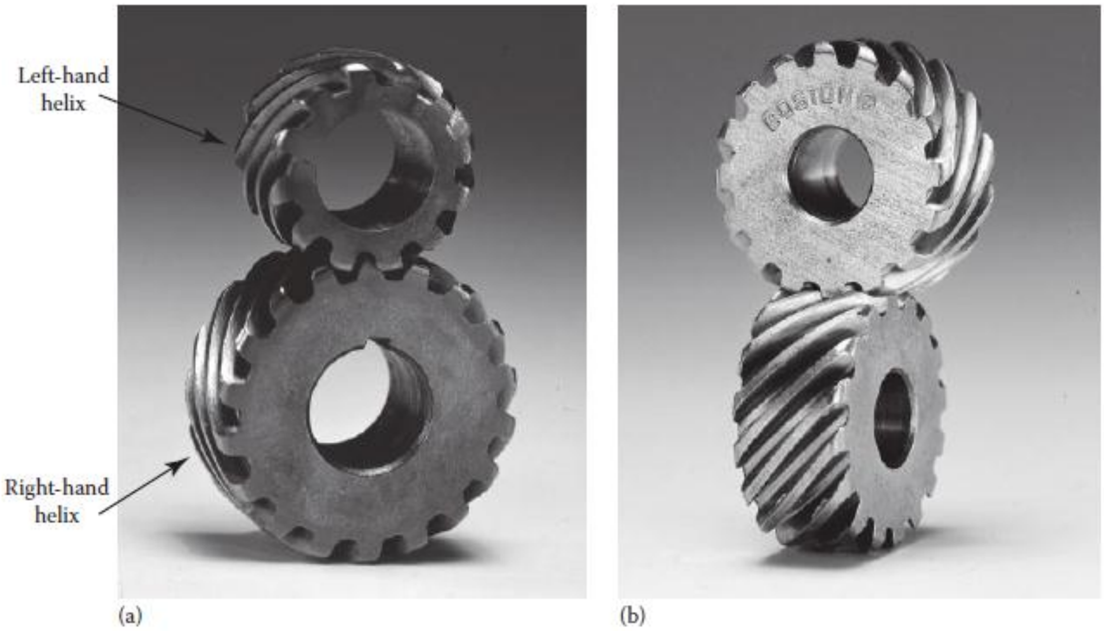


Fig.(1) Helical gears: (a) opposite-hand pair meshed on parallel axes (most common type) and (b) same-hand pair meshed on crossed axes.

Herringbone gear refers to a helical gear having half its face cut with teeth of one hand and the other half with the teeth of opposite hand (Figure 2).

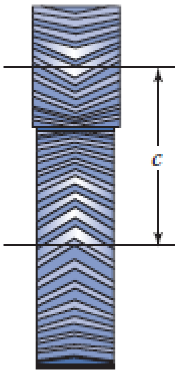


Fig.(2)Atypical herringbone gearset

Figure (3) illustrates the thrust, rotation, and hand relations for some helical gear sets with parallel shafts. Note that the direction in which the thrust load acts is determined by applying the RH or LH rule to the driver. That is, for the RH driver, if the fingers of the RH are pointed in the direction of rotation of the gears, the thumb points in the direction of the thrust. The driven gear then has a thrust load acting in the direction opposite to that of the driver, as shown in the figure.

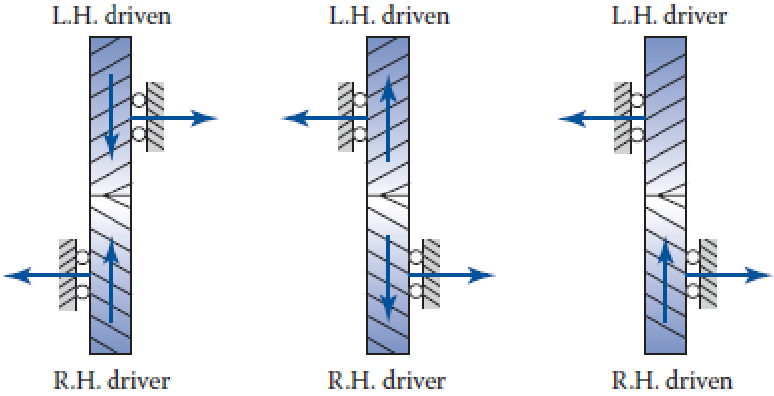


Fig.(3) Direction, rotation, and thrust load for three helical gearsets with parallel shafts.

The **advantages** of helical gears over other basic gear types include more teeth in contact simultaneously, and the load transferred gradually and uniformly as successive teeth come into engagement. Gears with helical teeth operate more smoothly and carry larger loads at higher speeds than spur gears. The line of contact extends diagonally across the face of mating gears. When employed for the same applications as spur gears, these gears have quiet operation.

The **disadvantages** of helical gears are greater cost than spur gears and the presence of an axial force component that requires thrust bearings on the shaft. Helical gear tooth proportions follow the same standards as those for spur gears. The teeth form the *helix* angle ψ with the gear axis, measured on an imaginary cylinder of pitch diameter (d). The usual range of values of the helix angle is between 15° and 30°. Various relations may readily be developed from geometry of a basic rack. Figure (4) illustrated with dimensions in transverse plane (At), normal plane (An), and axial plane (Ax). The distances between similar pitch lines from tooth to tooth are the circular pitch (p), the normal circular pitch(p_n), and the axial (circular) pitch (p_a).

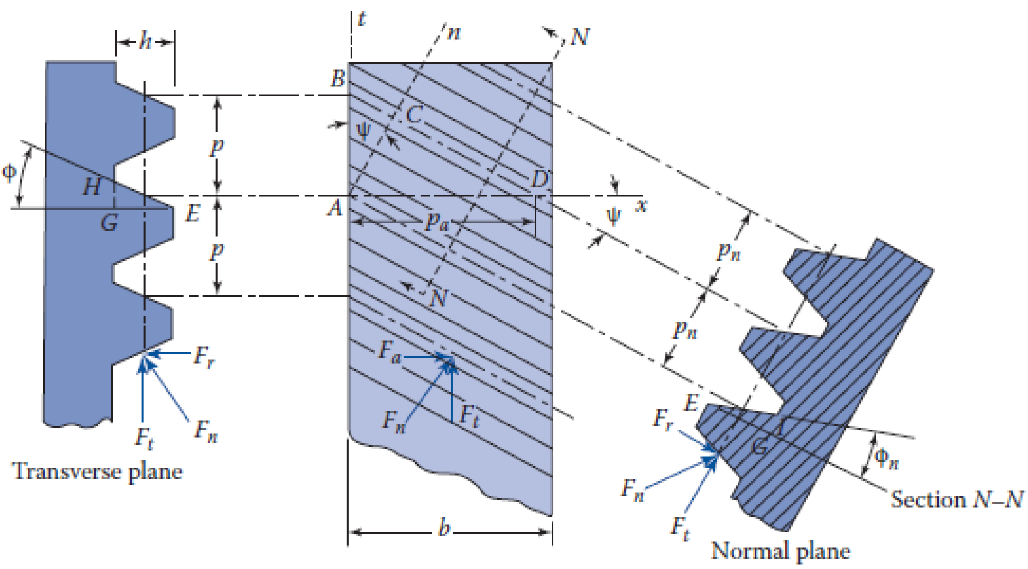


Fig.(4) Portion of a helical rack displaying transverse and normal planes and resolution of forces.

$$p_n = p\cos\psi\ , \qquad p_a = p\cot\psi = \frac{p_n}{\sin\psi}$$

Diametral pitch is more commonly employed than circular pitch to define tooth size. The product of circular and diametral pitch equals π for normal as well as transverse plane. Therefore,

$$Pp = \pi,\ P_n p_n = \pi, P_a = \frac{P}{\cos\psi}\ , P = \frac{N}{d}$$

where
P = the diametral pitch
P_n = the normal diametral pitch
N = the number of teeth

Hence, pressure angles are related by

$$\tan\Phi_n = \tan\Phi\cos\psi$$

Other geometric quantities are expressed similarly to those for spur gears:

$$d = \frac{Np}{\pi} = \frac{Np_n}{\pi\cos\psi} = \frac{N}{P_n\cos\psi}$$

$$c = \frac{d_1 + d_2}{2} = \frac{p}{2\pi}(N_1 + N_2) = \frac{N_1 + N_2}{2P_n\cos\psi}$$

where c represents the center distance of mating gears (1 and 2).

Virtual Number of Teeth

Intersection of the normal plane N–N and the pitch cylinder of diameter d is an ellipse (Figure 4). The shape of the gear teeth generated in this plane, using the radius of curvature of the ellipse, would be a nearly virtual spur gear having the same properties as the actual helical gear.

The virtual number of teeth N′ may be expressed in the following convenient form:

$$N' = \frac{N}{\cos^3\psi}$$

N is the number of actual teeth.

It is necessary to know the virtual number of teeth in design: This is considered in finding the appropriate values of the geometric factors Y and J for helical gears as discussed later.

Ex: Two helical gears have a center distance of $c = 10$ in., width $b = 1.9$ in., a pressure angle of $\varphi = 25^\circ$, a helix angle of $\psi = 30^\circ$, and a diametral pitch of $P = 6$ in^{-1} If the speed ratio is to be $r_s = 1/3$, calculate

a. The (transverse) circular, normal circular, and axial pitches.

b. The number of teeth of each gear.

c. The normal diametral pitch and normal pressure angle.

Solution:

(a) Since $pP = \pi$ then $p = \frac{\pi}{P} = \frac{\pi}{6} = 0.5236in$

Hence $p_n = pcos\psi = 0.5236cos30 = 0.453in$

$$p_a = pcot\psi = 0.5236cot30 = 0.907in$$

(b) $r_s = \frac{1}{3} = \frac{N_1}{N_2}\ or\ N_2 = 3N_1$

$$c = \frac{p}{2\pi}(N_1 + N_2) =$$

$$10 = \frac{0.5236}{2\pi}(N_1 + 3N_1)$$

$$N_1 = 30$$

$$\therefore N_2 = 3 \times N_1 = 3 \times 30 = 90\ tooth$$

$$d_1 = \frac{N_1}{P} = \frac{30}{6} = 5in = 127mm$$

$$d_2 = \frac{N_2}{P} = \frac{90}{6} = 15in = 381mm$$

(c) The normal diametral pitch can be evaluated as

$$P_n = \frac{P}{cos\psi} = \frac{6}{cos30} = 6.928$$

The normal pressure angle can be evaluated as

$$tan\emptyset_n = tan\emptyset cos\psi = tan25cos30 = 0.403$$

$$\emptyset_n = 22^o$$

Helical Gear Tooth Loads

As in the case of spur gears, the points of application of the force are in the pitch plane and in the center of the gear face. We see from Figure (5) that the normal load F_n is at a compound angle defined by the normal pressure angle φ_n and the helix angle ψ in combination.

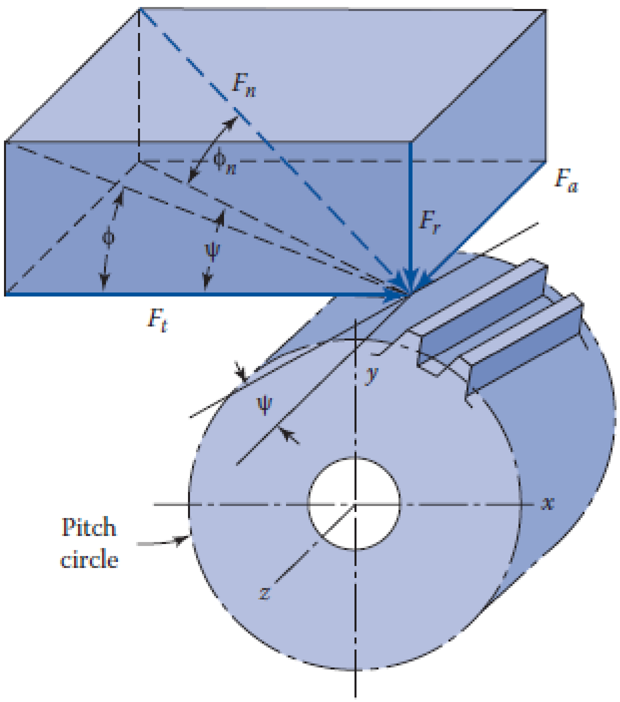


Fig.(5) Components of tooth force acting on a helical gear.

$$F_r = F_n sin\emptyset_n$$

$$F_t = F_n cos\emptyset_n cos\psi$$

$$F_a = F_n cos\emptyset_n sin\psi$$

where

F_n = the normal load or applied force

F_r = the radial component

F_t = the tangential component, also called the *transmitted load*

F_a = the axial component, also called the *thrust load*

The relation between different components of load can be presented as:

$$F_r = F_t tan\emptyset$$

$$F_a = F_t tan\psi$$

$$F_n = \frac{F_t}{cos\emptyset_n cos\psi}$$

In the foregoing, the pressure angle $\emptyset$ is related to the helix angle ψ` and normal pressure angle $\emptyset_n$by the following Equation .

$$tan\emptyset_n = tan\emptyset cos\psi$$

Helical Gear Tooth Bending and Wear Strengths

The equations for the bending and wear strengths of helical gear teeth are similar to those of spur gears. However, slight modifications must be made to take care of the effects of the helix angle ψ.

Lewis Equation

The allowable bending load of the helical gear teeth is given by the following expression:

$$F_b = \frac{\sigma_o bY}{K_f P_n}$$

The value of Y is obtained from the following table using the virtual teeth number N'

Table (1) Lewis form factor.

No. of Teeth	20° Y	25° Y	No. of Teeth	20° Y	25° Y
12	0.245	0.277	26	0.344	0.407
13	0.264	0.293	28	0.352	0.417
14	0.276	0.307	30	0.358	0.425
15	0.289	0.320	35	0.373	0.443
16	0.295	0.332	40	0.389	0.457
17	0.302	0.342	50	0.408	0.477
18	0.308	0.352	60	0.421	0.491
19	0.314	0.361	75	0.433	0.506
20	0.320	0.369	100	0.446	0.521
21	0.326	0.377	150	0.458	0.537
22	0.330	0.384	200	0.463	0.545
24	0.337	0.396	300	0.471	0.554
25	0.340	0.402	Rack	0.484	0.566

The allowable bending stress can be taken from the following table:

Table(2):

Allowable Static Bending Stresses for Use in the Lewis Equation

		σ_o		
Material	**Treatment**	**ksi**	**(MPa)**	**Average Bhn**
Cast iron				
ASTM 35		12	(82.7)	210
ASTM 50		15	(103)	220
Cast steel				
0.20% C		20	(138)	180
0.20% C	WQ&T	25	(172)	250
Forged steel				
SAE 1020	WQ&T	18	(124)	155
SAE 1030		20	(138)	180
SAE 1040		25	(172)	200
SAE 1045	WQ&T	32	(221)	205
SAE 1050	WQ&T	35	(241)	220
Alloy steels				
SAE 2345	OQ&T	50	(345)	475
SAE 4340	OQ&T	65	(448)	475
SAE 6145	OQ&T	67	(462)	475
SAE 65 (phosphor bronze)		12	(82.7)	100

Note: WQ&T, water-quenched and tempered; OQ&T, oil-quenched and tempered.

Buckingham Equation

The limit load for wear for helical gears on parallel shafts may be written in the following form:

$$F_w = \frac{d_p b Q K}{cos^2 \psi}$$

Where

$$Q = \frac{2N_g}{N_p + N_g}$$

The wear load factor K can be taken from the following table (The same for spur gear) for the normal pressure angle $\emptyset_n$.

Table(3)

Materials in Pinion and Gear	S_e ksi	S_e (MPa)	K $\phi = 20°$ psi	K $\phi = 20°$ (MPa)	K $\phi = 25°$ psi	K $\phi = 25°$ (MPa)
Both steel gears, with average Bhn of pinion and gear						
150	50	(345)	41	(0.283)	51	(0.352)
200	70	(483)	79	(0.545)	98	(0.676)
250	90	(621)	131	(0.903)	162	(1.117)
300	110	(758)	196	(1.352)	242	(1.669)
350	130	(896)	270	(1.862)	333	(2.297)
400	150	(1034)	366	(2.524)	453	(3.124)
Steel (150 Bhn) and cast iron	50	(354)	60	(0.414)	74	(0.510)
Steel (200 Bhn) and cast iron	70	(483)	119	(0.821)	147	(1.014)
Steel (250 Bhn) and cast iron	90	(621)	196	(1.352)	242	(1.669)
Steel (150 Bhn) and phosphor bronze	59	(407)	62	(0.428)	77	(0.531)
Steel (200 Bhn) and phosphor bronze	65	(448)	100	(0.690)	123	(0.848)
Steel (250 Bhn) and phosphor bronze	85	(586)	184	(1.269)	228	(1.572)
Cast iron and cast iron	90	(621)	264	(1.821)	327	(2.555)
Cast iron and phosphor bronze	83	(572)	234	(1.614)	288	(1.986)

For satisfactory helical gear performance, the usual requirement is that

$F_b \geq F_d$ and $F_w \geq F_d$.

The dynamic load F_d acting on helical gears can be estimated by the following formula:

$$F_d = \frac{78 + \sqrt{V}}{78} F_t (for\ 0 < V > 4000 fpm$$

in which the pitch-line velocity, V in fpm, is defined by the following equation :

$$\boldsymbol{V = \frac{\pi d n}{12}}$$

Note

To convert to m/s, divide the given values in this expression by 196.8.

Ex:

A motor at about n = 2400 rpm drives a machine by means of a helical gear set as shown in Figure(6). Calculate:

a. The value of the helix angle

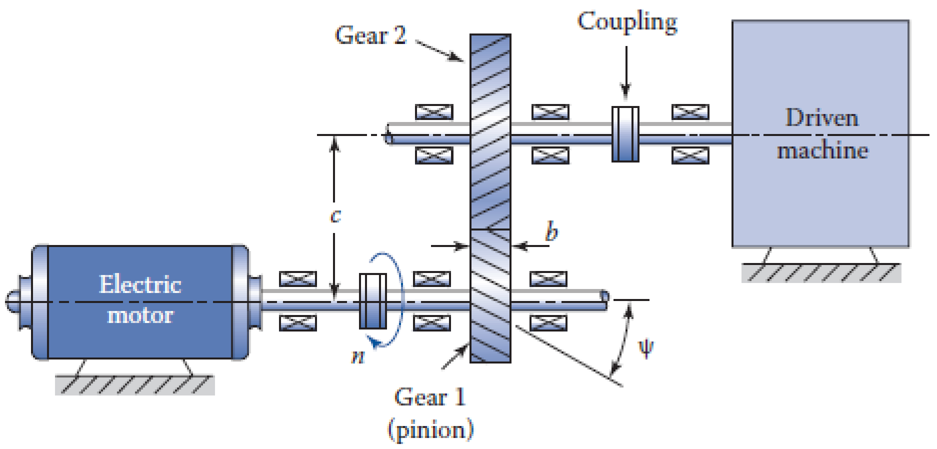
b. The allowable bending and wear loads using the Lewis and Buckingham formulas

c. The horsepower that can be transmitted by the gear set.

The gears have the following geometric quantities:

$$P_n = 5in^{-1}, \emptyset = 20^o, c = 9in, N_1 = 30, N_2 = 42, b = 2in$$

The gears are made of SAE 1045 steel, water-quenched and tempered



Solution

Since

$$d = \frac{N}{P_n cos\psi}$$

To find the diametral pitch use the following equation

Since

$$c = \frac{d_1 + d_2}{2}$$

And

$$d_1 = \frac{N_1}{P} \; and \; d_2 = \frac{N_2}{P}$$

Hence

$$P = \frac{1}{2c}(N_1 + N_2) = \frac{30 + 42}{2 \times 9} = \frac{72}{18}$$

And

$$d_1 = \frac{N_1}{P} = \frac{30 \times 18}{72} = 7.5in$$

$$d_2 = \frac{N_2}{P} = \frac{42 \times 18}{72} = 10.5in$$

Hence

$$cos\psi_1 = \frac{N_1}{P_n d_1} = \frac{30}{5(7.5)} = 0.8 \; or \; \psi_1 = \psi_2 = 36.9^o$$

b. The virtual number of teeth can be evaluated using the following equation

$$N' = \frac{N}{cos^3\psi} = \frac{30}{(0.8)^3} = 58.6$$

From table (1) an interpolation between N=50 and N=60 gives

Y=0.419, From table (2) $\sigma_o = 32ksi$. Use lewis equation with $K_f = 1$ to get

$$F_b = \frac{\sigma_o b Y}{K_f P_n}$$

$$F_b = \frac{32(2)0.419}{5} = 5.36kips$$

From Table (3), K = 79 ksi.

$$Q = \frac{2N_g}{N_p + N_g} = \frac{2(42)}{30 + 42} = \frac{84}{72} = \frac{7}{6}$$

The Buckingham equation can be used to evaluate the wear load:

$$F_w = \frac{d_p b Q K}{cos^2\psi} = \frac{7.5(2)(7)(79)}{6(0.8)^2} = 2.16kips$$

c. The horsepower capacity is based on F_w since it is smaller than F_b. The pitchline velocity equals

$$V = \frac{\pi d_1 n_1}{12} = \frac{\pi(7.5)(2400)}{12} = 4712 fpm$$

The dynamic load can be evaluated as

$$F_d = \frac{78 + \sqrt{V}}{78} F_t (for \; 0 < V > 4000fpm$$

$$F_d = \frac{78 + \sqrt{4712}}{78} F_t = 1.88F_t$$

Design based on smallest value of $F_b or \; F_w$

$$F_w \geq F_d$$
$$2.16 = 1.88F_t \; or \; F_t = 1.15kips$$

The corresponding gear power is

$$h_p = \frac{F_t V}{33000} = \frac{1150(4712)}{33000} = 164$$