

Clutches

The function of a clutch is to permit smooth, gradual connection and disconnection of two members having a common axis of rotation. The clutches considered here are of the friction type, depending on sliding friction between solid surfaces. Other types use magnetic, eddy current and hydrodynamic forces. Fluid couplings and torque converters are examples. Several types of friction clutches are considered here in. All must be designed to satisfy the following three basic requirements.

(1) The required friction torque must be produced by an acceptable actuating force. (2) The energy converted to friction heat (during braking or during clutch engagement) must be dissipated without producing destructively high temperatures. (3) The wear characteristics of the friction surfaces must be such that they give acceptable life.

Disc Clutches

Figure (1) shows a simple disk clutch with one driving and one driven surface. Driving friction between the two develops when they are forced together.

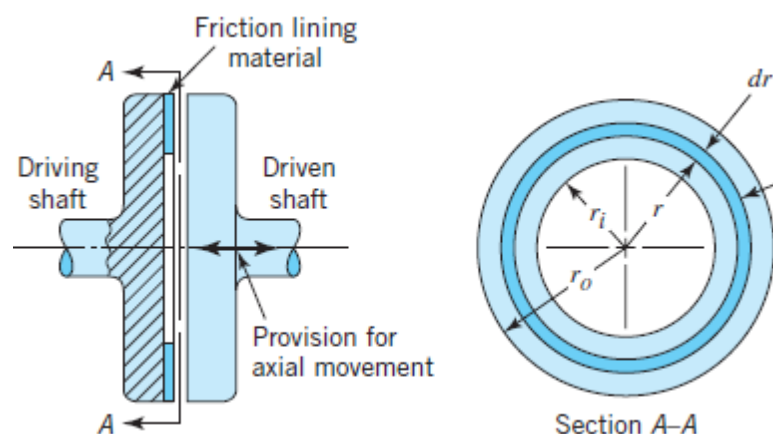


Fig.(1) Basic disk clutch

Figure (2) shows an automotive-type clutch, used with "standard" transmissions. The flywheel, clutch cover, and pressure plate rotate with the crankshaft. A series of circumferentially distributed springs force the

pressure plate toward the flywheel, clamping the clutch plate (driven disk) between them. The hub of the clutch plate is spline connected to the transmission input shaft. The clutch is disengaged by depressing the clutch pedal, which rotates the lever marked “To release.” This pushes the clutch release bearing against a series of radially oriented release levers that pull the pressure plate away from the flywheel. Note that the clutch release bearing is a thrust bearing. The right side bears against the release mechanism, which does not rotate; the left side bears against the release levers, which rotate with the crankshaft. This clutch has two driving surfaces, one on the flywheel and one on the pressure plate, and two driven surfaces, the two sides of the clutch plate.

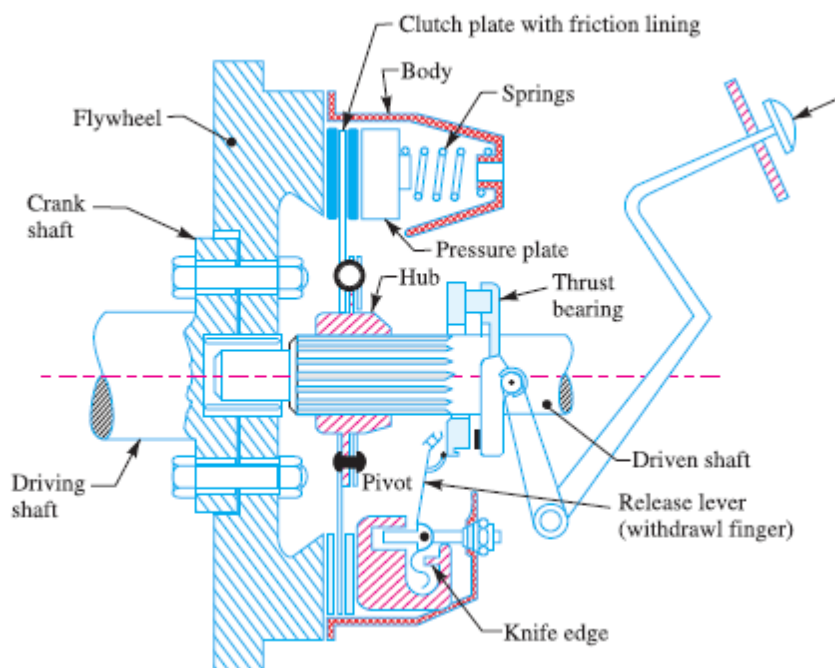


Fig.(2) Automotive type disk clutch

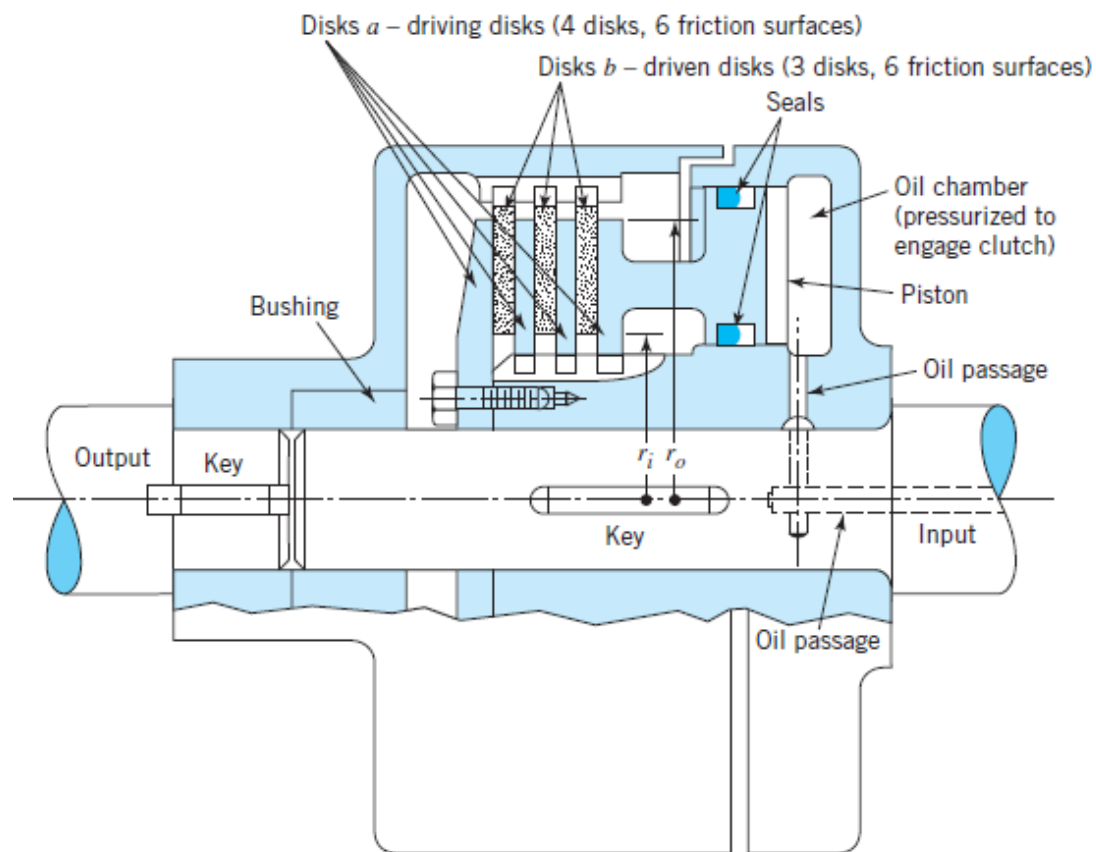


Fig.(3) Multiple-disk clutch, hydraulically operated.

Disks *a* are constrained (as with splines) to rotate with the input shaft; disks *b* are similarly constrained to rotate with the output shaft. When the clutch is disengaged, the disks are free to slide axially to separate themselves. When the clutch is engaged, they are clamped tightly together to provide (in the case illustrated) six driving and six driven surfaces. The two end disks, which have only their inner sides serving as friction surfaces, should be members of the same set in order to avoid transmitting the clamping force through a thrust bearing. Note that in Figures (2) and (3) the clutch clamping force is localized to the region of the disks, whereas in Figure (1) it would have to be transmitted through a thrust bearing.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

Mathematical Formulations:

Equations are now developed below relating clutch size, friction coefficient, torque capacity, axial clamping force, and interface pressure, using each of two basic assumptions. Throughout the development the coefficient of friction f is assumed to be constant.

Uniform Pressure Distribution Assumption:

This assumption is valid for an unworn (new), accurately manufactured clutch, with rigid outer disks. With reference to Figure (1), the normal force acting on a differential ring element of radius r is

$$dF = (2\pi r dr)p \quad (1)$$

Where p is the uniform level of interface pressure. The total normal force acting on the area of contact is

$$F = \int_{r_i}^{r_o} 2\pi p r dr = \pi p (r_o^2 - r_i^2) \quad (2)$$

where F is also the axial force clamping the driving and driven disks together. The friction torque that can be developed on a ring element is the product of the normal force, coefficient of friction, and radius,

$$dT = (2\pi r dr)pfr \quad (3)$$

and the total torque that can be developed over the entire interface is

$$T = \int_{r_i}^{r_o} 2\pi p f r^2 dr = \frac{2}{3} \pi p f (r_o^3 - r_i^3) \quad (4)$$

Equation (4) represents the torque capacity of a clutch with one friction interface (one driving disk mating with one driven disk, as in Figure (1)). Actual clutches (as shown in Figures (2) and (3)) employ N friction interfaces transmitting torque in parallel, where N is an even number (two in Figure (2) six in figure (3)). For a clutch with N friction interfaces, Eq. (4) is modified to give

$$T = \frac{2}{3} \pi p f (r_o^3 - r_i^3) N \quad (5)$$

substituting for p from equation (2) into equation (5) gives an equation for torque capacity as a function of axial clamping force::

$$T = \frac{2Ff(r_o^3 - r_i^3)}{3(r_o^2 - r_i^2)} N \quad (6)$$

Uniform Wear rate at Interface

Wear rate is generally proportional to the rate of friction work-that is, friction force times rubbing velocity. With a uniform coefficient of friction, wear rate is proportional to the product of pressure times sliding velocity on the clutch face, velocity is proportional to radius; hence, rate of work is proportional to the product of pressure and radius. The uniform wear rate is assumed to result from a uniform rate of friction work-that is, a constant product of pressure and velocity, or a constant product of pressure and radius. Thus:

$$pr = C \text{ (where } C \text{ is a constant)} \quad (7)$$

The greatest pressure, $p_{\max}$, obviously occurs at the inside radius, and this has an allowable value determined by the characteristics of the friction lining material. Hence, for a clutch of inside radius r_i and a friction lining with allowable pressure $p_{\max}$, the clutch design is based on

$$pr = C = p_{\max} r_i \quad (8)$$

$$F = \int_{r_i}^{r_o} 2\pi p_{\max} r_i dr = 2\pi p_{\max} r_i (r_o - r_i) \quad (9)$$

$$T = \int_{r_i}^{r_o} 2\pi p_{\max} r_i f r dr N = \pi p_{\max} r_i f (r_o^2 - r_i^2) N \quad (10)$$

$$\text{Since } p_{\max} = \frac{F}{2\pi r_i (r_o - r_i)}$$

Substitute $p_{\max}$ in equation (10) to get

$$T = Ff \left(\frac{r_o + r_i}{2} \right) N \quad (11)$$

Where (N) is the number of friction interfaces. **The assumption of uniform wear rate gives a lower calculated clutch capacity** than the assumption of uniform pressure.

Friction Materials Coefficient of Friction

Approximate data relative to coefficients of friction and allowable pressures for various friction lining materials are given in Tables 1 and 2.

Table (1) Representative Properties of Friction Materials, Operating Dry

Friction Material ^a	Dynamic Friction Coefficient f^b	Maximum Pressure ^c		Maximum Bulk Temperature	
		psi	kPa	°F	°C
Molded	0.25–0.45	150–300	1030–2070	400–500	204–260
Woven	0.25–0.45	50–100	345–690	400–500	204–260
Sintered metal	0.15–0.45	150–300	1030–2070	450–1250	232–677
Cork	0.30–0.50	8–14	55–95	180	82
Wood	0.20–0.30	50–90	345–620	200	93
Cast iron, hard steel	0.15–0.25	100–250	690–1720	500	260

Table (2) Representative Values of Friction Coefficient for Friction Materials Operating in Oil

Friction Material ^a	Dynamic Friction Coefficient f
Molded	0.06–0.09
Woven	0.08–0.10
Sintered metal	0.05–0.08
Paper	0.10–0.14
Graphitic	0.12 (avg.)
Polymeric	0.11 (avg.)
Cork	0.15–0.25
Wood	0.12–0.16
Cast iron, hard steel	0.03–0.06

The maximum torque for a given outside radius is obtained when

$$r_i = \sqrt{\frac{1}{3}} r_o = 0.58r_o$$

Proportions commonly used range from $r_i = 0.45r_o$ to $r_i = 0.80r_o$

Ex: A multiple-disk wet clutch is to be designed for transmitting a torque of 85 N.m. Space restrictions limit the outside disk diameter to 100 mm. Design values for the molded friction material and steel disks to be used are $f = 0.06$ (wet) and $p_{\max} = 1400$ kPa. Determine appropriate values for the disk inside diameter, the total number of disks, and the clamping force.

Solution

Use the largest allowable outside disk diameter, $d_o = 100$ mm ($r_o = 50$ mm)

Select $r_i = 29$ mm. (Since $0.45r_o < r_i < 0.80r_o$)

Using Eq. (10)

$$T = \pi p_{\max} r_i f (r_o^2 - r_i^2) N$$

Gives

$$N = \frac{T}{\pi p_{\max} r_i f (r_o^2 - r_i^2)}$$

$$= \frac{85}{\pi \times 1400000 \times 0.029 \times 0.06 (0.05^2 - 0.029^2)} = 6.69$$

Since N must be an even integer, use $N = 8$. Reference to Figure (3) shows that this requires a total of 4 + 5, or nine disks (remember that the two outer disks have friction surfaces on one side only).

Since

$$T = F f \left(\frac{r_o + r_i}{2} \right) N$$

$$85 = F (0.06) \left(\frac{0.05 + 0.029}{2} \right) (8)$$

This gives

$$F = 4483 \text{ N} \cong 4500 \text{ N}$$

Rounding up the calculated value of F to an even number, we find that the final proposed answers are (a) inside diameter = 58 mm, (b) clamping

force = 4500 N, and (c) a total of nine disks.

Cone Clutches

Figure (4-a) shows a cone clutch. It is similar to a disk clutch and can be regarded as the general case of which the disk clutch is a special case with a cone angle α of 90° . The construction of a cone clutch makes it impractical to have more than one friction interface; hence, $N = 1$. As previously noted, this requires that the shaft bearings take a thrust load equal to the clamping force. This is acceptable because the inherent wedging action of a typical cone clutch enables the clamping force to be reduced to only about one-fifth that of a corresponding disk clutch of $N=1$.

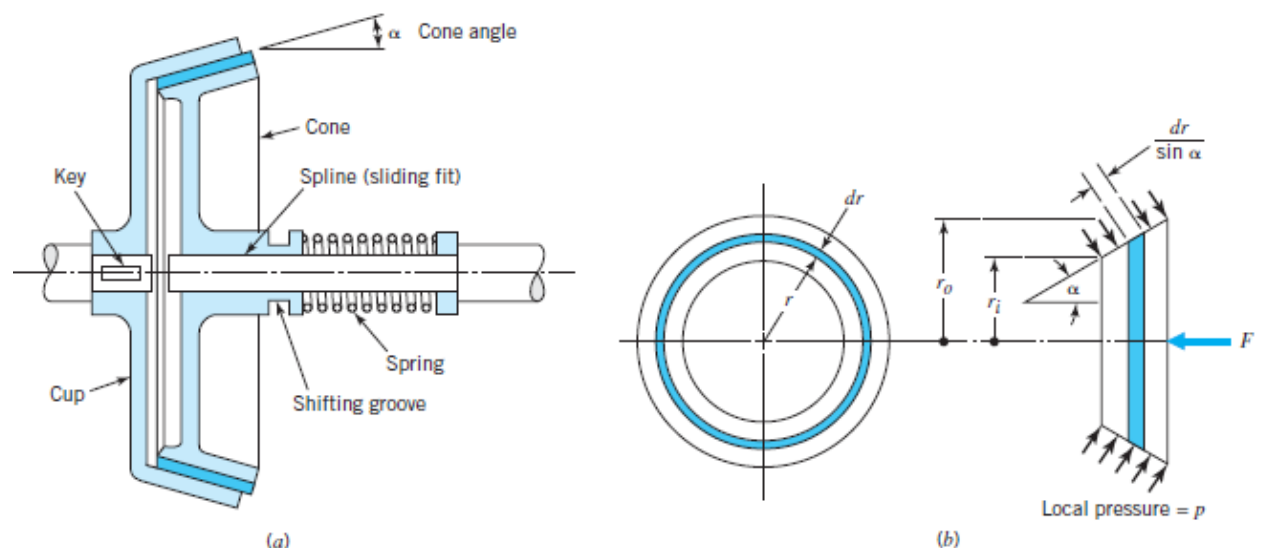


Fig.(3) Cone clutch

Figure (3-b) shows that the surface area of a ring element is

$$dA = 2\pi r dr / \sin \alpha \quad (12)$$

The normal force on the element is

$$dN = (2\pi r dr) p / \sin \alpha \quad (13)$$

The corresponding clamping force is

$$dF = dN \sin \alpha = (2\pi r dr) p \quad (14)$$

which is exactly the same as for a disk clutch ring element (Eq.1). The torque that can be transmitted by the element is

$$dT = dNfr = 2\pi pfr^2 dr / \sin\alpha \quad (15)$$

For assumed uniform pressure the torque can be calculated as that for the disk clutch divided by $\sin\alpha$ as follows:

$$T = \frac{2}{3} \pi p f (r_o^3 - r_i^3) / \sin\alpha \quad (16)$$

Or

$$T = \frac{2Ff(r_o^3 - r_i^3)}{3(r_o^2 - r_i^2)} / \sin\alpha \quad (17)$$

While for assumed uniform wear the torque transmitted can be calculated as:

$$T = \pi p_{max} r_i f (r_o^2 - r_i^2) / \sin\alpha \quad (18)$$

Or

$$T = Ff \left(\frac{r_o + r_i}{2} \right) / \sin\alpha \quad (19)$$

The smaller the cone angle α , the less theoretical clamping force required. This angle cannot be made smaller than about 8° or the clutch may tend to “grab” when engaged. In addition, for $\alpha < 8^\circ$ cone clutches tend to be difficult to disengage. An angle of 12° is usually considered about optimum, with values of α between 8° and 15° being commonly used.

Centrifugal Clutch

For the centrifugal clutch shown in the figure(4) the forces acting on the shoe can be defined as

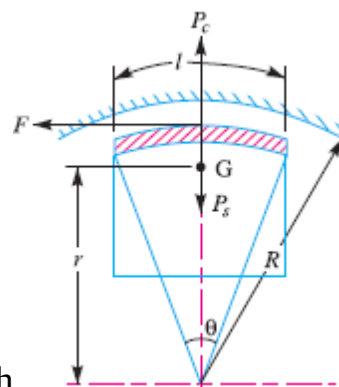


Fig.(4)Centrifugal clutch

$$P_c = \text{centrifugal force} = m \cdot \omega^2 \cdot r$$

Since the speed at which the engagement begins to take place is generally taken as 3/4th of the running speed, therefore the inward force on each shoe exerted by the spring is given by

$$P_s = m(\omega_1)^2 r = m\left(\frac{3}{4}\omega\right)^2 r = \frac{9}{16} m\omega^2 r$$

∴ Net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s = m\omega^2 r - \frac{9}{16} m\omega^2 r = \frac{7}{16} m\omega^2 r$$

and the frictional force acting tangentially on each shoe,

$$F = f(P_c - P_s)$$

Frictional torque acting on each shoe

$$T = F \times R = f(P_c - P_s)R$$

and total frictional torque transmitted,

$$T = F \times R = f(P_c - P_s)R \times N = N \times F \times R$$

Ex:

A plate clutch consists of one pair of contacting surfaces. The inner and outer diameters of the friction disk are 100 and 200mm respectively. The coefficient of friction is 0.2 and the permissible intensity of pressure is 1 N/mm². Assuming uniform wear theory ; calculate the power transmitting capacity of the clutch at 750rpm.

Solution

For uniform wear theory

$$F = 2\pi p_{max} r_i (r_o - r_i)$$

$$F = 2\pi \times 1 \times 50(100 - 50) = 15707.963N$$

$$T = Ff \left(\frac{r_o + r_i}{2} \right) N$$

$$T = 15707.963 \times 0.2 \left(\frac{0.1 + 0.05}{2} \right) \times 1 = 235.619 N.m$$

The power transmitted by the clutch can be calculated as

$$Power = T \times \omega$$

$$Power = 235.619 \times \frac{2\pi \times 750}{60} = 18505.47 W = 18.505 kW$$

Ex

An automotive plate clutch consists of two pairs of contacting surfaces with an asbestos friction lining. The torque transmitting capacity of the clutch is 550 N.m. The coefficient of friction is 0.25 and the permissible intensity of pressure is 0.5 N/mm². Due to space limitations the outer diameter of the friction disk is fixed as 250mm. Using uniform wear theory calculate

- (i) The inner diameter of the friction disk
- (ii) The spring force required to keep the clutch in an engaged position.

Solution

$$T = \pi p_{max} r_i f (r_o^2 - r_i^2) N$$

$$550 \times 1000 = \pi \times 0.5 r_i \times 0.25 (125^2 - r_i^2) \times 2$$

$$700281.5 = 15625 r_i - r_i^3$$

By trial and error the above equation may be solved

$r_i = 87 \text{ mm}$, $r_i = 56 \text{ mm}$ or 143.23 mm

chose $r_i = 87 \text{ mm}$ for minimum friction area.

$$F = 2\pi p_{max} r_i (r_o - r_i)$$

$$F = 2\pi \times 0.5 \times 87 (125 - 87) = 10386.105 N$$

Ex:

An oil immersed multi-disk clutch with cork sheet as the friction material is used on a scooter engine. The friction disk of such a clutch is shown in figure(4). The torque transmitted by the clutch is 10N.m. The coefficient of friction between the cork sheet and the steel plate in the wet condition is 0.2. The permissible pressure on the cork sheet is 0.1N/mm^2 . The inner and outer diameters of the friction lining are 65 and 95 mm respectively. There are radial slots for the circulation of the coolant, which reduces the effective friction area. To account for these slots, the number of contacting surfaces can be increased by 5%. Assuming uniform wear theory, calculate the required number of contacting surfaces.

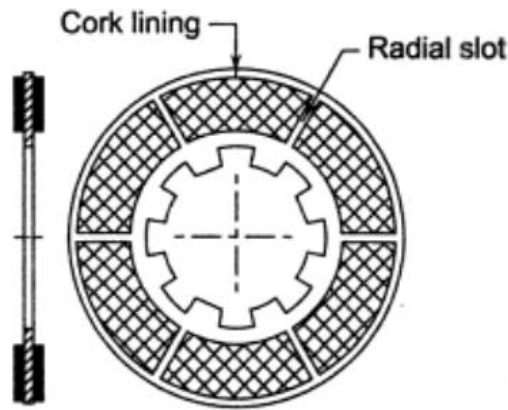


Fig.(4)

Solution

$$N = \frac{T}{\pi p_{max} r_i f (r_o^2 - r_i^2)}$$

$$N = \frac{10 \times 1000}{\pi \times 0.1 \times 32.5 \times 0.2 (47.5^2 - 32.5^2)} = 4.08 \text{ disk}$$

Since the number of contacting surfaces can be increased by 5%, hence
 $N = 1.05 \times 4.08 = 4.284$ chose number of contacting surfaces $N = 5$.

Ex:

A multi-disk clutch consists of five steel plates and four bronze plates. The inner and outer diameters of the friction disks are 75 and 150mm respectively. The coefficient of friction is 0.1 and the intensity of pressure on friction lining is limited to 0.3 N/mm^2 . Assuming uniform wear theory, calculate:

- (i) The required force to engage the clutch
- (ii) Power transmitting capacity at 750rpm.

Solution

$$F = 2\pi p_{max} r_i (r_o - r_i)$$

$$F = 2\pi \times 0.3 \times 37.5(75 - 37.5) = 2650.71 \text{ N}$$

Number of disks = N+1

Since there are five steel and four bronze plates, hence the number of disks is 9

$$N = 9 - 1 = 8$$

The torque transmitting capacity can be evaluated as:

$$T = \pi p_{max} r_i f (r_o^2 - r_i^2) N$$

$$T = \pi \times 0.3 \times 37.5 \times 0.1(75^2 - 37.5^2) \times 8 = 119282.34 \text{ N.mm}$$

$$\text{Power} = T \times \omega$$

$$\text{Power} = 119282.34 \times 10^{-3} \times \frac{2\pi \times 750}{60} = 9368.38 \text{ W}$$

Or Power in kW is 9.368.

Ex:

An oil immersed multi-disk clutch with moulded asbestos on one side and steel disks on the other, is used in an application. The torque transmitted by the clutch is 75N.m. The coefficient of friction between the asbestos lining and the steel plate in the wet condition is 0.1. The permissible intensity of pressure on the asbestos lining is 500kPa. The

outer friction lining is kept as 100mm due to space limitation. Assuming uniform wear theory, calculate the inside diameter of the

Note: Assume that the clutch transmit the maximum power.

Ex:

A cone clutch with asbestos friction lining transmits 30kW power at 500rpm. The coefficient of friction is 0.2 and the permissible intensity of pressure is 0.35N/mm^2 . The semi-cone angle α is 12.5° . The outer diameter is fixed as 300mm from space limitations. Assuming uniform wear theory calculate:

- (i) The inner diameter
- (ii) The force required to engage the clutch.

Solution

The torque transmitted by the clutch can be evaluated as:

$$T = \frac{2}{3} \pi p f (r_o^3 - r_i^3) / \sin \alpha$$

$$T = \frac{\text{power}}{\omega} = \frac{30000}{500 \times \frac{2\pi}{60}} = 572.957 \text{ N.m}$$

$$\therefore 572957 = \frac{2}{3} \pi \times 0.35 \times 0. \frac{2(150^3 - r_i^3)}{\sin 12.5}$$

Or $84567.08 = (150^3 - r_i^3)$ from which $r_i = 136.2\text{mm}$

The force required to engage the clutch can be evaluated as:

$$F = 2\pi p_{\max} r_i (r_o - r_i)$$

$$F = 2\pi \times 0.3 \times 136.2(150 - 136.2) = 3542.88 \text{ N}$$

Ex: The contact surfaces in a cone clutch have an effective diameter of 80 mm. The semi-angle of the cone is 15° and coefficient of friction is 0.3. Find the torque required to produce slipping of the clutch, if the axial force applied is 200 N. The clutch is employed to connect an

electric motor, running uniformly at 900 r.p.m. with a flywheel which is initially stationary. The flywheel has a mass of 14 kg and its radius of gyration is 160 mm. Calculate the time required for the flywheel to attain full-speed and also the energy lost in slipping of the clutch.

Solution

The torque required to produce slipping of the clutch can be expressed as

$$T = Ff \left(\frac{r_o + r_i}{2} \right) / \sin \alpha$$

$$T = 200 \times 0.3 \left(\frac{80 \times 10^{-3}}{2} \right) / \sin 15 = 9.272 \text{ N.m}$$

The time required for the flywheel to attain full speed can be determined as:

The mass moment of inertia of the flywheel can be evaluated as:

$$I = m \times k^2 = 14 \times (0.16)^2 = 0.3583 \text{ kg.m}^2$$

The torque transmitted can also expressed as

$$\begin{aligned} T &= 9.273 = I \times \text{angular acceleration} \\ &= 0.3583 \times \text{angular acceleration} \end{aligned}$$

$$\text{Angular acceleration} = \frac{9.273}{0.3584} = 25.87 \text{ rad/s}^2$$

$$\omega = 2\pi \times \frac{900}{60} = 94.26 \text{ rad/s}$$

$$\omega = 94.26 = \omega_o + \text{angular acceleration} \times t = 0 + 25.87t$$

$$t = \frac{94.26}{25.87} = 3.64 \text{ s}$$

Energy lost due to slipping can be evaluated as:

The displacement can be evaluated as

$$\begin{aligned} \theta &= \text{Average angular speed} \times \text{time} = \frac{\omega_o + \omega}{2} \times t \\ &= \frac{0 + 94.26}{2} \times 3.64 = 171.6 \text{ rad} \end{aligned}$$

$$\begin{aligned} \text{Energy lost in slipping of the clutch} &= T \times \theta = 9.273 \times 171.6 \\ &= 1591 \text{ N.m} \end{aligned}$$

Fluid Couplings

Figure (5) shows the essential parts of a fluid coupling. The rotor attached to the input shaft is called the *impeller*, which is also bolted to the case. These two units form the housing containing the hydraulic fluid (usually a low-viscosity mineral oil).

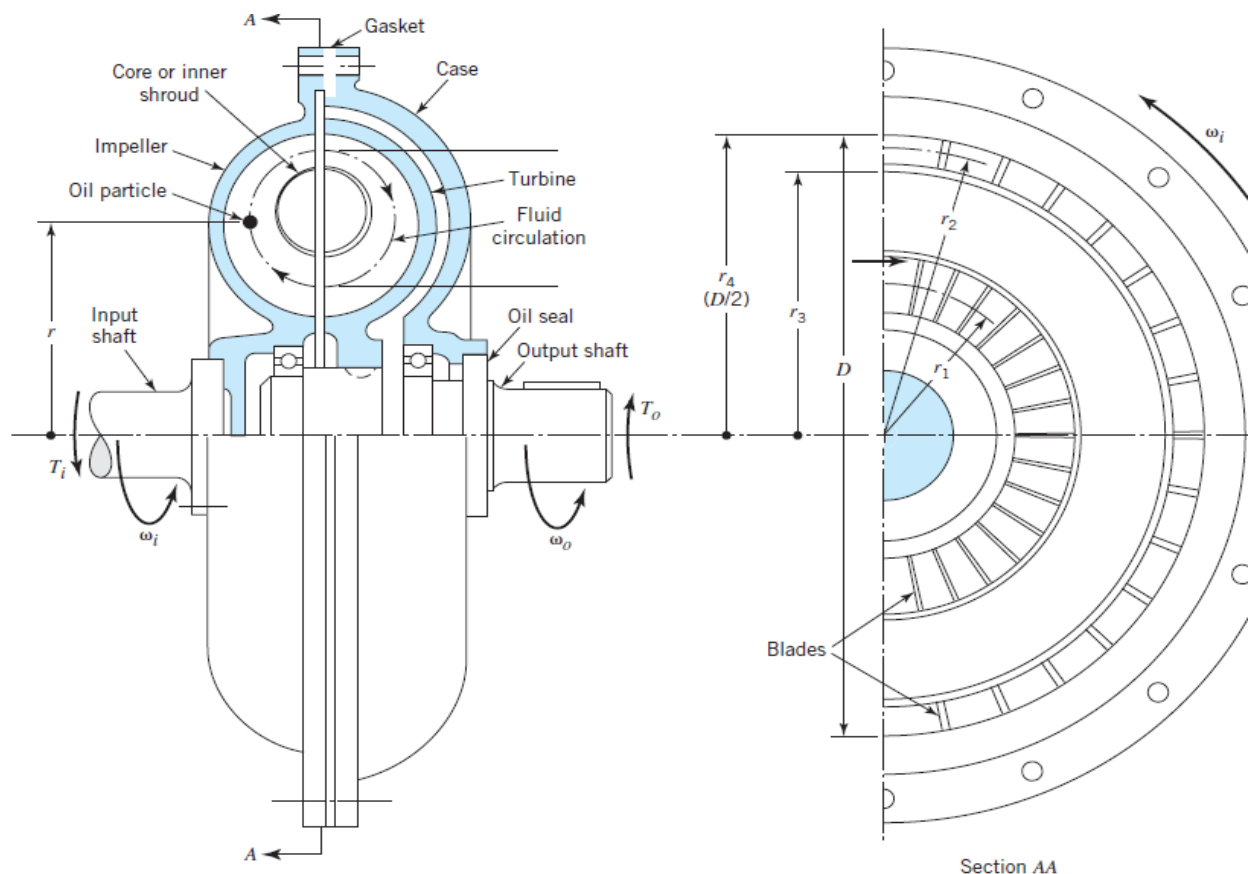


Fig.(5) Fluid coupling

The output shaft is supported in the housing by two bearings. Attached to it is the **turbine**, which is driven by oil discharged from the impeller. Rotation of the impeller creates centrifugal forces within the oil trapped between adjacent blades, causing it to flow radially outward; note the direction of arrows on the “fluid circulation” path in Figure (5). When the flow crosses over to the turbine, the oil impinges upon the turbine blades, giving up much of its kinetic energy (in the plane of impeller and turbine rotation). Pressure from the “oil behind” forces the flow to proceed radially inward through the spaces between adjacent turbine blades,

where it gives up additional kinetic energy. During normal operation the turbine rotation may be only slightly slower than that of the impeller. The centrifugal forces developed within the oil on the turbine side oppose those on the impeller side, thereby reducing the velocity of flow around the transverse “fluid circulation” paths. If the direction of the drive reverses and the output shaft runs faster than the input, the direction of fluid flow also reverses. This is what happens when a car with a fluid coupling (or torque converter) experiences “engine braking. In normal operation the output shaft rotates more slowly than the input shaft by a factor called *slip*. Representing the input and output rotating velocities by ω_i and ω_o , respectively, slip S is defined as

$$S = (\omega_i - \omega_o)/\omega_i$$

A fundamental concept is that without slip, there can be no fluid circulation and therefore no transmission of power. Centrifugal force acting on the oil particle is

$$F = mr\omega^2 = \rho r\omega^2$$

Since input and output torques are equal, the efficiency of power transmission by the coupling is

$$e = \frac{\omega_o}{\omega_i} = 1 - S$$

During normal operation, a well-designed and properly applied coupling usually operates at 95 to 98 percent efficiency.