Chapter three

Pre-stress concrete

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Prestressed concrete

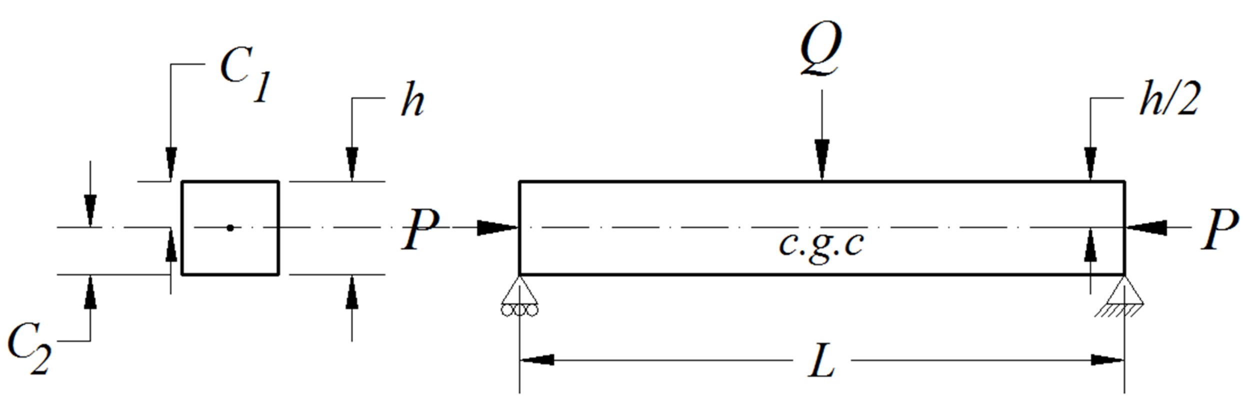
The prestressing of a structure member may be define as the creation of initial stress of opposite sign to the stress produce by the working load without increasing the actual max stresses in the member.

ACI-code define the pre-stressed concrete as follows; concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from the given external loading are counteracted to desired degree.

Prestressing applies a pre-compression to the member that reduces or eliminates undesirable tensile stresses. Cracking under service loads can be minimized or even avoided entirely. Deflection may be limited to an acceptable value.

Concrete stress control by pre-stressing

A simply supported beam with a rectangular cross section shown in (Fig.3-1) in which a longitudinal axial force P is introduced prior to the vertical loading. The longitudinal prestressing force will produce a uniform axial compression $f_c = \frac{P}{A_c}$. Where A_c is the cross-section area of the concrete. The force can be adjusted in magnitude so that when the transvers load Q is applied, the superposition of stresses due to P and Q will result in zero tensile stress at bottom of the beam as shown.



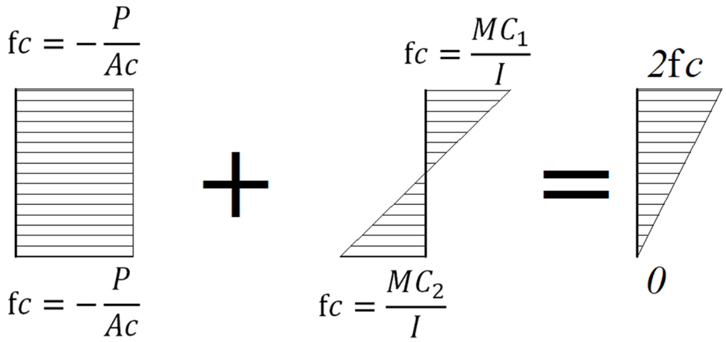


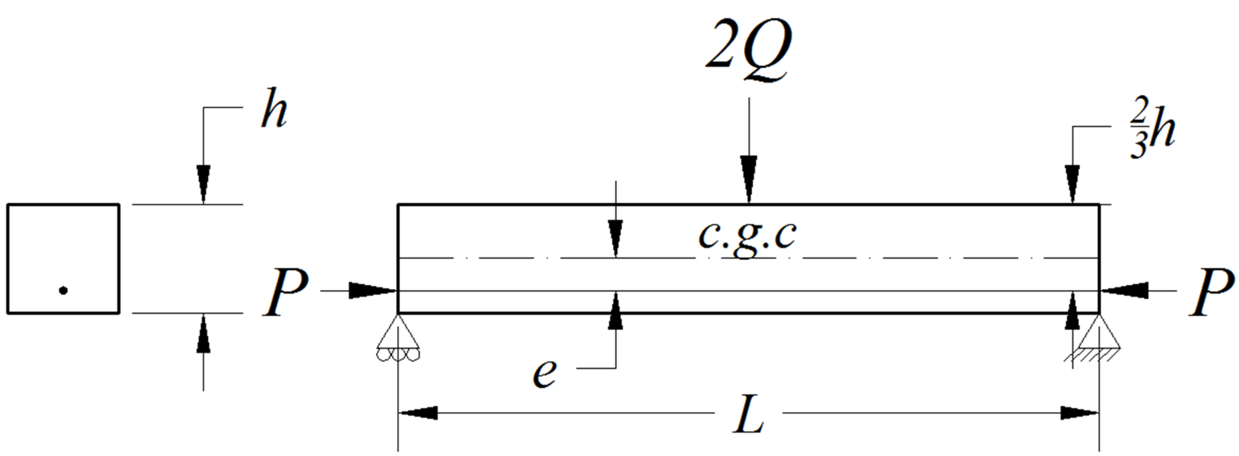
Fig (3-1) Axially prestressed beam

Where

Ic is moment of inertia of the cross-section

M is bending moment $= \frac{QL}{4}$

It would be more logical to apply the prestressing force near the bottom of the beam, to compensate more effectively for the load-induced tension. The force P, with the same value as before, but applied with eccentricity $e = h/6$ relative to the concrete centroid, will produce a longitudinal compressive stress distribution varying linearly from zero at the top surface to maximum of $2f_c = \frac{P}{A_c} + \frac{P_e C_2}{I_c}$ at the bottom



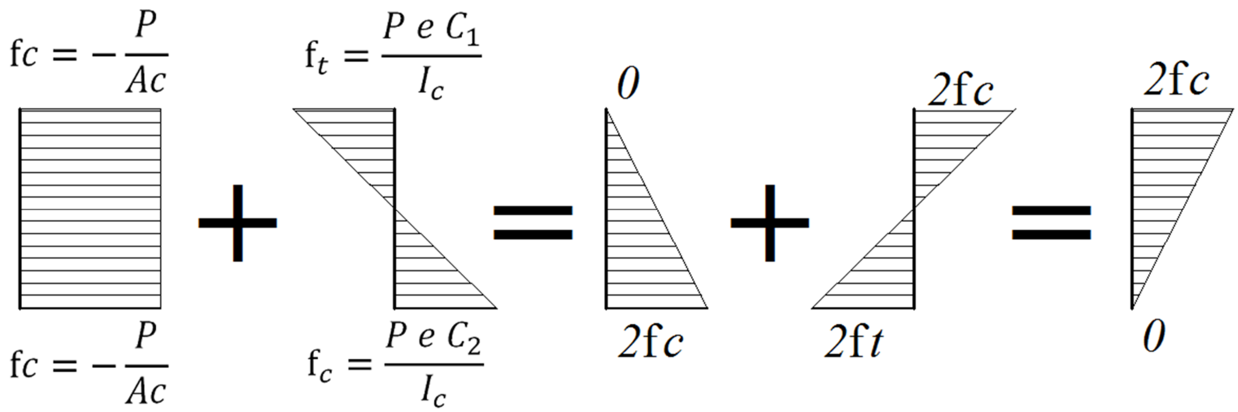
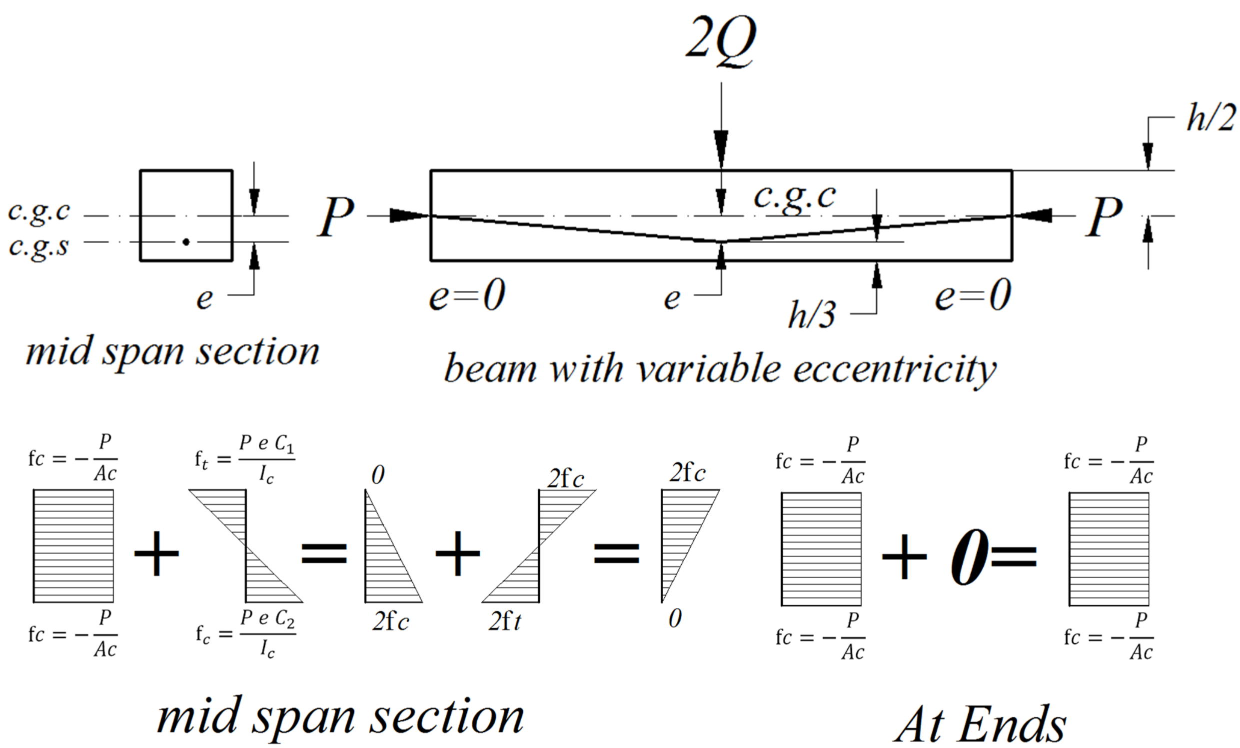


Fig (3-2) Eccentrically prestressed beam

A significant improvement can be made, by using tendon with variable eccentricity with respect to concrete center as shown in Fig(3-3). The best arrangement of prestressing would produce countermoment that act in the opposite sense to the load-induced moment. This would be achieved by giving the tendon an eccentricity that varies linearly from zero at the supports to maximum at mid span.



Final stress at mid span

$$f_{top} = -\frac{P}{Ac} + \frac{Pe\,C_1}{Ic} - \frac{MC_1}{Ic}$$

$$= -\begin{bmatrix} Comp. \\ direct \\ effect \end{bmatrix} + \begin{bmatrix} Ten. \\ eccentric \\ effect \end{bmatrix} - \begin{bmatrix} comp. \\ service \\ load \end{bmatrix}$$

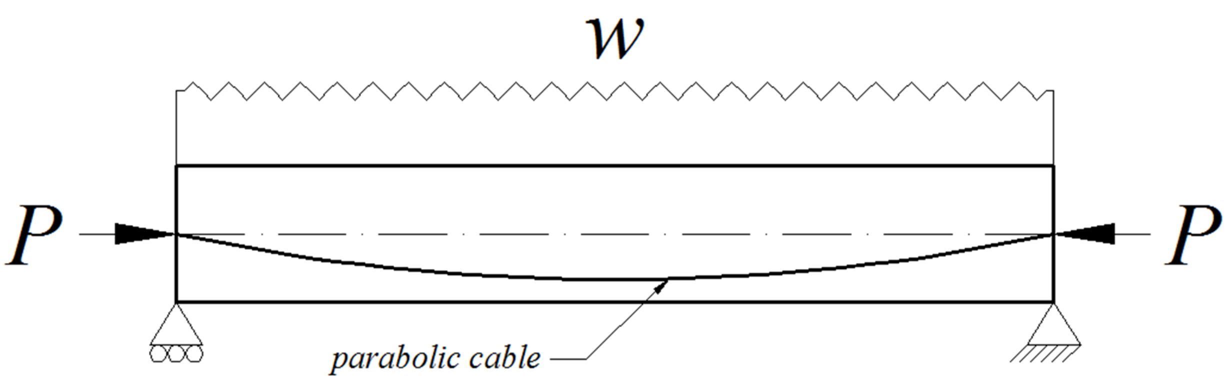
$$f_{bot} = -\frac{P}{Ac} - \frac{Pe\,C_2}{Ic} + \frac{MC_2}{Ic}$$

$$M = bending\ moment = \frac{2QL}{4}$$

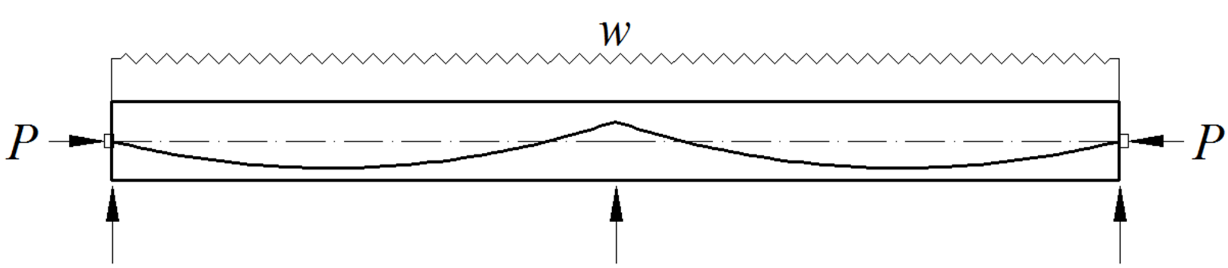
final stress at end of the beam

$$f = -\frac{P}{Ac}$$

For each characteristic load distribution, there is a best tendon profile that produces a prestress moment diagram that corresponds to that of the applied load



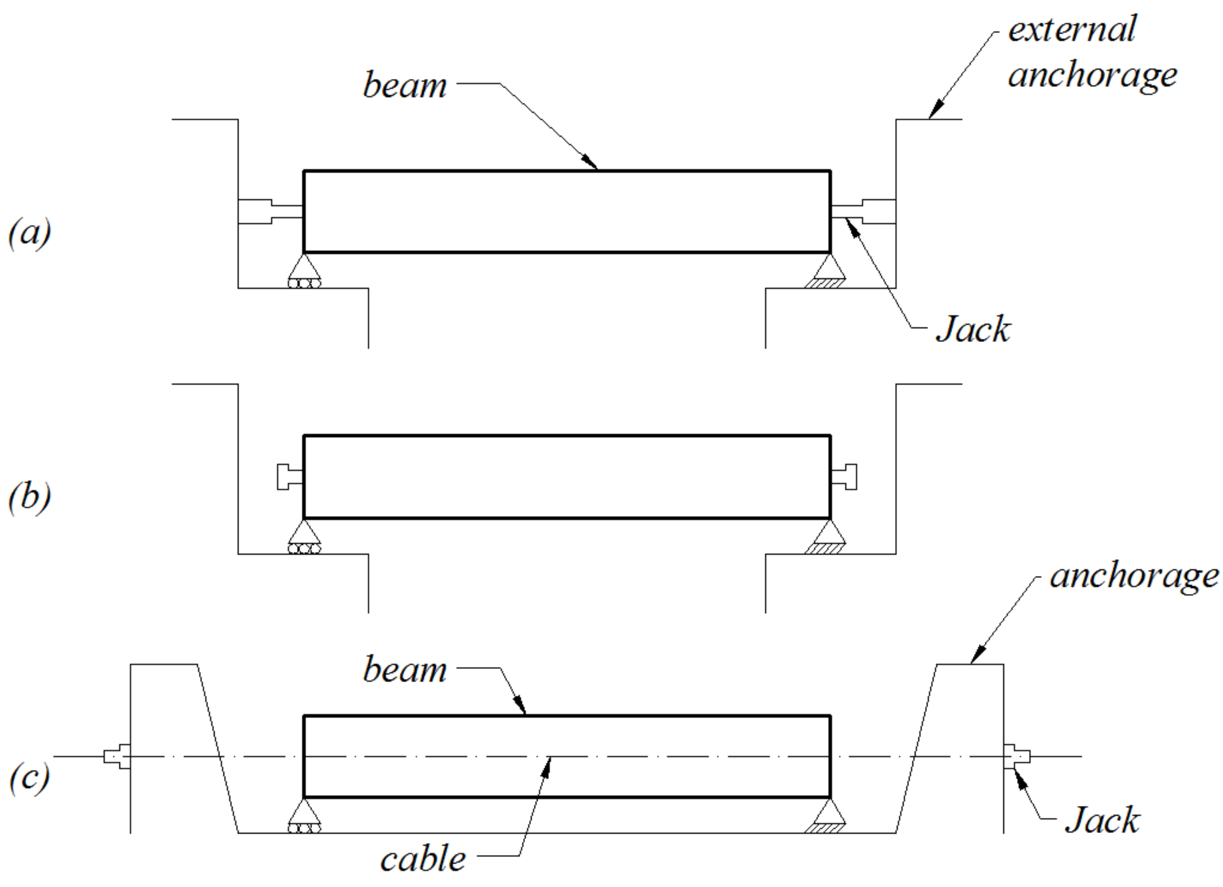
For uniformly loaded simply supported beam with parabolic tendon



Continuous beam with parabolic tendon

Sources of prestress force

Prestress can be applied to a concrete member in many ways. Perhaps the most obvious method of precompressing is to use jacks reacting against abutments as shown in Fig (3-4). Such a scheme has been employed for larger projects. Many variations are possible, including replacing the jacks with compression struts after the desired stress in the concrete is obtained or using inexpensive jacks that remain in place in the structure, in some cases with a cement grout used as the hydraulic fluid. The principal difficulty associated with such a system is that even a slight movement of the abutments will drastically reduce the prestress force.



a- Post-tensioning by jacking against abutments
b- Post-tensioning with jacks reacting against beam
c- Pretensioning with tendon stressed between external anchorages

In most cases, the same result is more conveniently obtained by tying the jack bases together with wires or cables, as shown in Fig (3-4b). These wires or cables may be external, located on each side of the beam; more usually they are passed through a hollow conduit embedded in the concrete beam. Usually, one end of the prestressing tendon is anchored, and all of the force is applied at the other end. After reaching the desired prestress force, the tendon is wedged against the concrete and the jacking equipment is removed for reuse. In this type of prestressing, the entire system is self-contained and is independent of relative displacement of the supports.

Another method of prestressing that is widely used is illustrated by Fig. (3-4c).The pre-stressing strands are tensioned between massive abutments in a casting yard prior to placing the concrete in the beam forms. The concrete is placed around the tensioned strands, and after the concrete has attained sufficient strength, the jacking pressure is released. This transfers the prestressing force to the concrete by bond and friction along the strands, chiefly at the outer ends.

It is essential, in all three cases shown in Fig (3-4), that the beam be supported in such a way as to permit the member to shorten axially without restraint so that the prestressing force can be transferred to the concrete.

Other means for introducing the desired prestressing force have been attempted on an experimental basis. Thermal pre-stressing can be achieved by preheating the steel by electrical or other means. Anchored against the ends of the concrete beam while in the extended state, the steel cools and tends to contract. The prestress force is developed through the restrained contraction. The use of expanding cement in concrete members has been tried with varying success. The volumetric expansion, restrained by steel strands or by fixed abutments, produces the pre-stress force.

Prestressing steel

Prestressing steel is most commonly used in the form of:

1- Individual wires
2- Stranded cable (stands) made up of seven wires (7-wires strand)
3- Alloy steel beam (high strength)

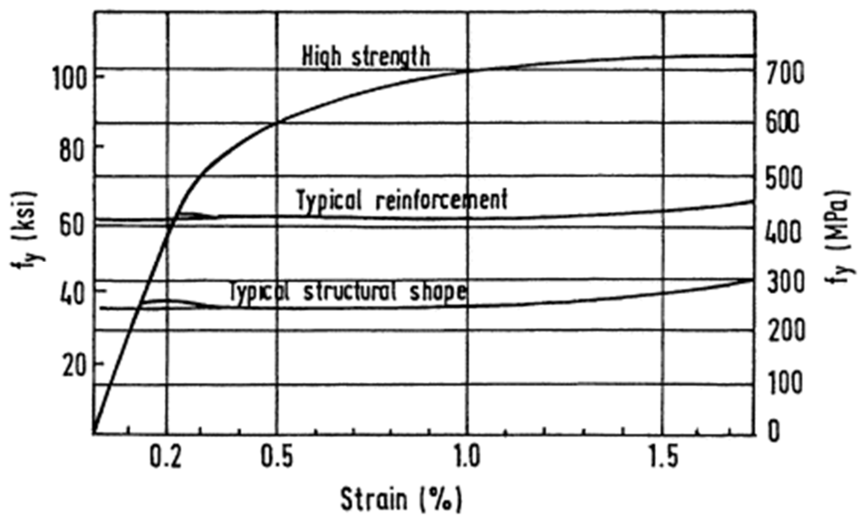


Fig (3-5) Typical stress-strain curve for steel

The tensile stress permitted by ACI-code in prestressing wires, strands or bars is dependent upon the stage of loading (ACI-code 20.3.2.5.1) permissible stresses in prestressing steel

Concrete for prestressed construction:

Ordinary concrete of substantially higher compressive strength is used for prestressed structure for those constructed of ordinary reinforced concrete $fc' > 35\ MPa\ (5000\ psi)$

As for prestressing steel, the allowable stresses in the concrete depend upon the stage of loading. ACI code defines three classification of behavior, depending on the extreme fiber stress f_t at service load in the precompressed tensile zone. The three classifications are U.T and C.

ACI-code 24.5.3.1 Serviceability requirements-Flexural members

fci' is the compressive strength of the concrete at the time of initial prestress fc' is the specified compressive strength of the concrete.

Elastic flexural analysis

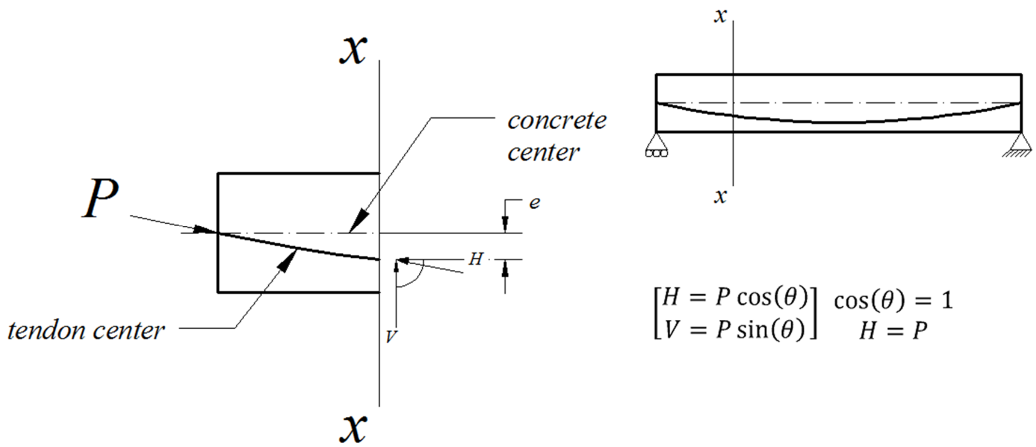


Fig (3-6). Prestressing force acting on concrete

Fig (3-6) shows a simple-span prestressed beam with curved tendons. The force N acting on the concrete from the tendon, due to tendon curvature.

It is convenient to divide the prestressing force (P) into its components

$$H = P\cos(\theta)$$
$$V = H\tan(\theta) = P\sin(\theta)$$

Where (θ) is small. $\cos(\theta) \cong 1.0$ and it is sufficient for most calculation to take $H = P$

The jack force P_j immediately reduced to P_i because of

a- Elastic shorting of concrete
b- Slip of the tendon as the force is transferred from the jack to beam ends
c- Loss due to friction

P_i: initial prestress force (immediately after transfer)

$P_i \rightarrow P_e$ because of:

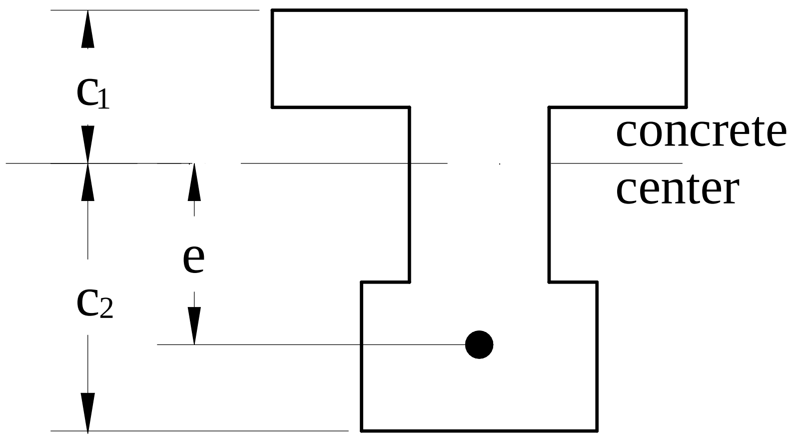
a- Shrinkage and creep in concrete
b- Relaxation in steel

P_e: effective pre-stress force

$P_e = R * P_i$ $\qquad$ R = effectiveness ratio = $(1 - losses)$

In developing elastic equations for flexural stress, the effect of prestress force, self-weight moment, and dead and live load moments are calculated separately, and the separate stresses are superimposed.

$\boldsymbol{P_i}$, When the initial prestress force is applied with an eccentricity (e) below the centroid of the cross section with area (A_c) and top and bottom fiber distance C_1 and C_2.



Compressive stresses are design as negative (-)

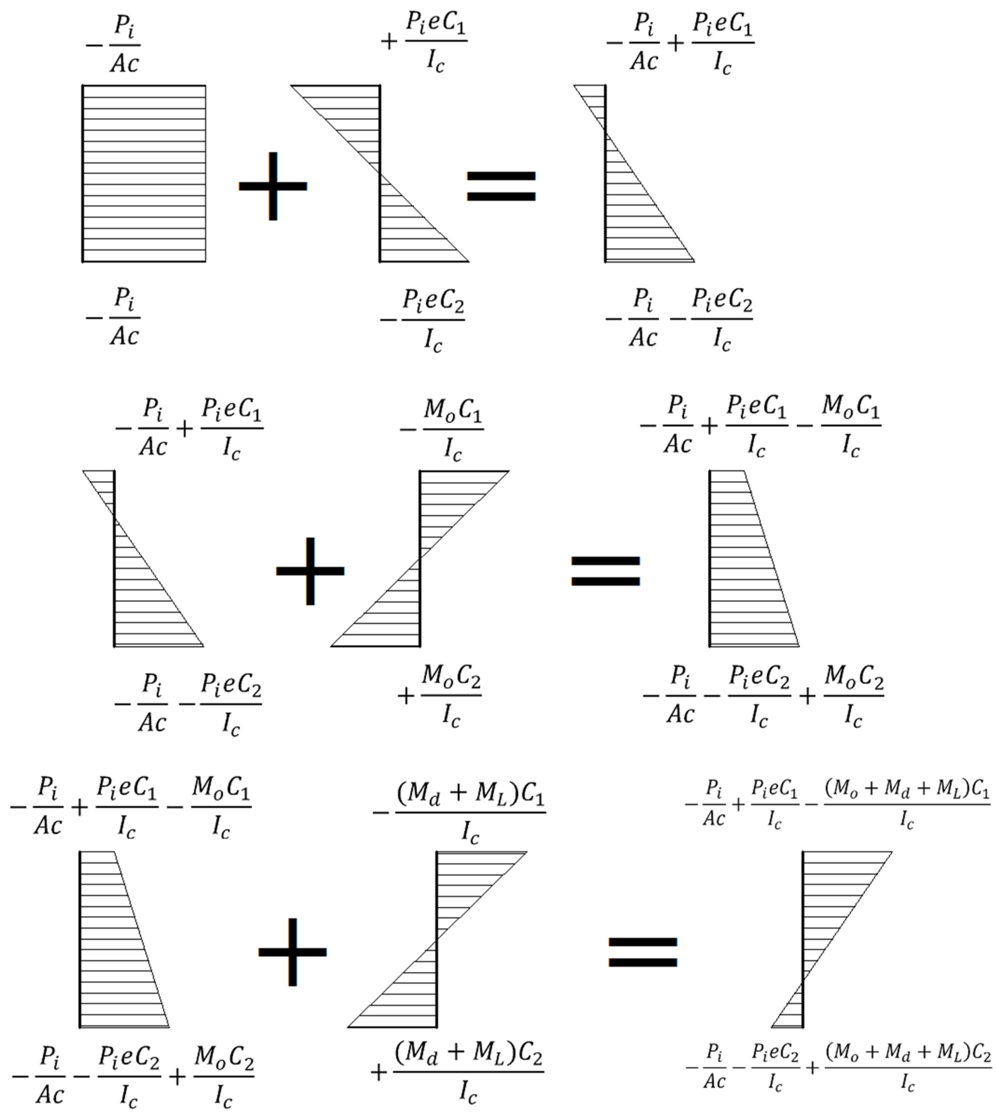
Tensile stresses as positive (+)

The flexural stress (at mid span tensile) at the top fiber

$$f_{top} = -\frac{P_i}{Ac} + \frac{P_i e\, C_1}{I_c} = -\frac{P_i}{A_c}\left(1 - \frac{eC_1}{r^2}\right)$$

r: radius of gyration of concrete section

$$r^2 = \frac{I_c}{A_c}$$



And of the bottom fiber

$$f_{bot} = -\frac{P_i}{Ac} - \frac{P_i e\, C_2}{I_c} = -\frac{P_i}{A_c}\left(1 + \frac{eC_2}{r^2}\right)$$

Where(r) is the radius of gyration of the concrete section. Normally as the eccentric pre-stress force is applied, the beam deflects upward. The beam self-weight $w_0$then causes additional moment $M_0$to act, and the net top and bottom fiber stresses become:

$$f_{top} = -\frac{P_i}{A_c}\left(1 - \frac{eC_1}{r^2}\right) - \frac{M_0 C_1}{I_c}$$
$$f_{bot} = -\frac{P_i}{A_c}\left(1 + \frac{eC_2}{r^2}\right) + \frac{M_0 C_1}{I_c}$$

As shown in Fig. 19.9b. At this stage, time-dependent losses due to shrinkage creep and relaxation commence, and the prestressing force gradually decreases from P_i to P_e. It is usually acceptable to assume that all such losses occur prior to the application of service loads, since the concrete stresses at service loads will be critical after losses not before. Accordingly, the stresses in the top and bottom fiber, with P and beam load acting, become

- $P_i + M_o$

$$f_{top} = -\frac{P_i}{Ac} + \frac{P_i e\, C_1}{I_c} - \frac{M_o C_1}{I_c}$$

$$f_{bot} = -\frac{P_i}{Ac} - \frac{P_i e\, C_2}{I_c} + \frac{M_o C_2}{I_c}$$

- $P_e + M_o$

$$f_{top} = -\frac{P_e}{Ac} + \frac{P_e e\, C_1}{I_c} - \frac{M_o C_1}{I_c}$$

$$f_{bot} = -\frac{P_e}{Ac} - \frac{P_e e\, C_2}{I_c} + \frac{M_o C_2}{I_c}$$

- $P_e + M_t$(M_o+M_d+M_L)

$$f_{top} = -\frac{P_e}{Ac} + \frac{P_e e\, C_1}{I_c} - \frac{(M_o + M_d + M_L) C_1}{I_c}$$

$$f_{bot} = -\frac{P_e}{Ac} - \frac{P_e e\, C_2}{I_c} + \frac{(M_o + M_d + M_L) C_2}{I_c}$$

Or

$$f_{top} = -\frac{P_e}{Ac} + \frac{P_e e\, C_1}{I_c} - \frac{M_o C_1}{I_c} - \frac{(M_d + M_L) C_1}{I_c}$$

$$f_{bot} = -\frac{P_e}{Ac} - \frac{P_e e\, C_2}{I_c} + \frac{M_o C_2}{I_c} + \frac{(M_d + M_L) C_2}{I_c}$$

The stress at the section of max moment, must stay within the limit states define by the distribution shown in Fig (3-8).

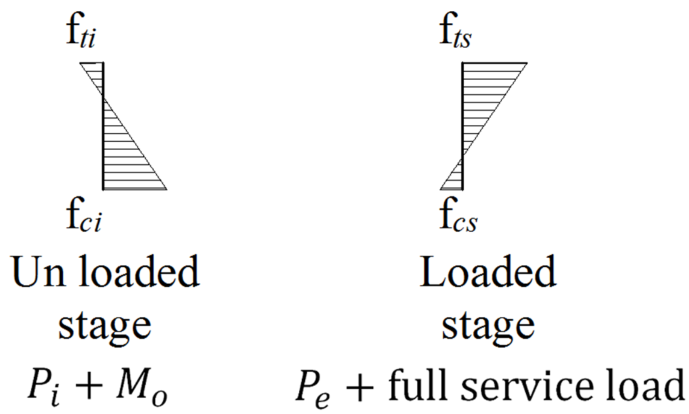


Fig (3-8) Stress limits

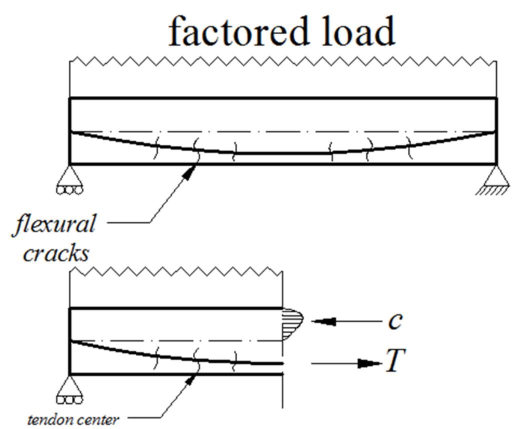
f_{ci} and f_{ti} are the permissible compression and tensile stress respectively in the concrete immediately after transfer.

f_{cs} and f_{ts} are the permissible compression and tensile stress respectively in the concrete at service load.

Flexural strength

The condition of incipient failure is shown in Fig (3-9), which shows beam carrying a factored load.

The maximum moment section, only the concrete in compression is effective, and all of the tension taken by the steel.

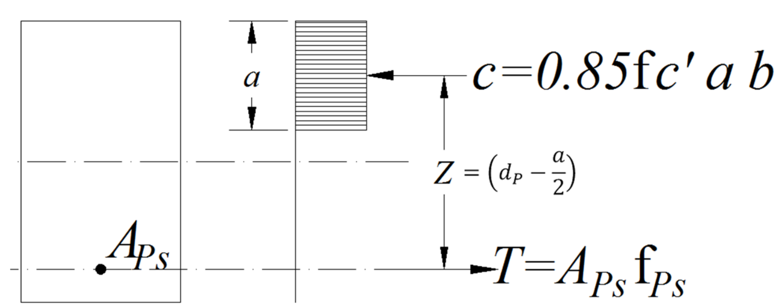


The external moment from the applied loads is resisted by the internal force couple

$$CZ = TZ$$

The strength of a pre-stressed beam can be predicted by the same methods developed for ordinary reinforced concrete beams

$a = \beta_1 c$
$\beta_1 = 0.85$ for $fc' \leq 28\ MPa$
for $fc' > 28$



$$\beta_1 = 0.05\left(\frac{fc' - 28}{7}\right) \geq 0.65$$

- For rectangular cross section or flanged section such as (I or T) beams in which the stress block depth is equal to or less than the average flange thickness, the nominal flexural strength is

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

$$a = \frac{A_{ps} f_{ps}}{0.85 fc' b}$$

or

$$M_n = \rho_p f_{ps} b d_p^2\left(1 - 0.588 * \rho_p * \frac{f_{ps}}{fc'}\right)$$

Flexural design strength$= \emptyset M_n$ where $\emptyset$: strength reduction factor $= 0.9$

If the stress block depth exceeds the average flange thickness the total pre-stressed tensile steel area is divided into two parts for computational purposes. The first part A_{pf} acting on the stress f_{ps} provides a tensile force to balance the compression in the overhanging parts of the flange thus

$$A_{Pf} = 0.85\frac{fc'}{f_{Ps}}(b - b_w)h_f$$

$$A_{Pw} = A_{Ps} - A_{Pf}$$

A_{Pw} Provided tension to balance the compression in the web

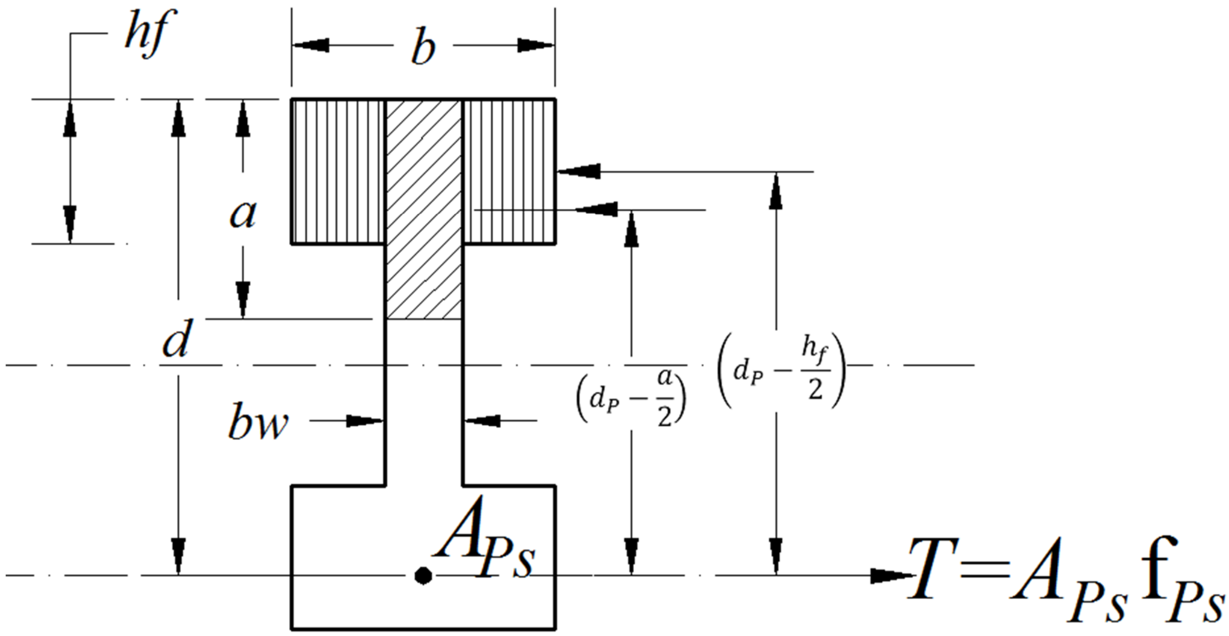
$$M_n = A_{Pw} f_{Ps}\left(d_P - \frac{a}{2}\right) + A_{pf} f_{ps}\left(d_p - \frac{h_f}{2}\right)$$

or

$$M_n = A_{Pw} f_{Ps}\left(d_P - \frac{a}{2}\right) + 0.85(b - b_w)h_f\left(d_P - \frac{h_f}{2}\right)$$

$$where \quad a = \frac{A_{Pw} f_{Ps}}{0.85 fc' b_w}$$

The design strength $= \emptyset M_n$ where $\emptyset$ is typically 0.9



(f_{Ps}) The stress in the steel at failure may be taken equal to the following according to the ACI-code (2014) ch.24

If effective pre-stress in the steel $f_{se} \geq 0.5\, f_{Pu}$

$$f_{se} = \frac{P_e}{A_{Ps}}$$

a- For member with bonded tendons

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{fc'} + \frac{d}{d_p}(\omega - \omega')\right]\right]$$

Where

$$\omega = \rho \frac{fy}{fc'}\ \omega' = \rho' \frac{fy}{fc'}\ \rho_p = \frac{A_{ps}}{bd_p}$$

b: width of compression face

β_1:the familiar relations between stress block depth and depth to the neutral axis

γ_p: is a factor that depends on the type of pre-stressing steel

$$\gamma_p = \begin{bmatrix} 0.55\ \text{for} f_{py}/f_{pu} \geq 0.80\ \text{high strength bars} \\ 0.40 \text{for} f_{py}/f_{pu} \geq 0.85\ \text{ordinary strand} \\ 0.28\ \text{for} f_{py}/f_{pu} \geq 0.90\ \text{low-relaxation strand} \end{bmatrix}$$

b- For members with unbounded tendons with

1- Span/depth ≤ 35

$$f_{ps} = f_{se} + 70 + \frac{fc'}{100\rho_p}$$

$$f_{ps} \le f_{py} \quad , \quad f_{ps} \le (f_{se} + 420)$$

2- span/depth > 35

$$f_{ps} = f_{se} + 70 + \frac{fc'}{300\rho_p}$$

$$f_{ps} \le f_{py} \quad , \quad f_{ps} \le (f_{se} + 210)$$

ACI-code requires the total tensile reinforcement must be adequate to support a factored load of at least 1.2*cracking load of beam

$$\emptyset M_n \ge 1.2 M_{cr}$$

To find (M_{cr}) the stress in the bottom fiber $= f_r$

$$-\frac{P_e}{Ac} - \frac{P_e e C_2}{I_c} + \frac{M_{cr} C_2}{I_c} = f_r$$

$$\therefore M_{cr} = f_r \frac{I_c}{C_2} + P_e\, e + \frac{P_e}{Ac}\left(\frac{I_c}{C_2}\right) = \frac{I_c}{C_2}\left(f_r + \frac{P_e e C_2}{I_c} + \frac{P_e}{Ac}\right)$$

f_r: Modulus of rapture of concrete$= 0.62\sqrt{fc'}$

To control cracking in prestressed concrete member with unbounded reinforcement some bonded reinforcement must be added in the form of non-pre-stressed reinforcement bars uniformly distributed over area of bonded reinforcement

$$As = 0.004\, A_{ct}$$

Where A_{ct}area of that part of cross section between the flexural tensile face and the center of the gross concrete cross-section.

Flexural design based on concrete stress limit

As in reinforced concrete problems in pre-stressed concrete can be separated generally as analysis problems or design problems

If the dimensions of a concrete section, the steel area and centroid location, and the amount of prestress area to be found given the load, limiting stresses and required strength the problem is complicated by the many interrelated variables.

Design problem

Notation is established pertaining to the allowable concrete stresses at limiting stage as follows:

f_{ci}: allowable compression stress immediately after transfer.

f_{ti}: allowable tensile stress immediately after transfer.

f_{cs}: allowable compression stress at service load after losses.

f_{ts}: allowable tensile stress at service load after losses.

a- Beam with variable eccentricity

For a typical class U or T beam in which tendon eccentricity permitted to vary along span flexural stress distribution in the concrete at the maximum moment section are shown in Fig (3-10).

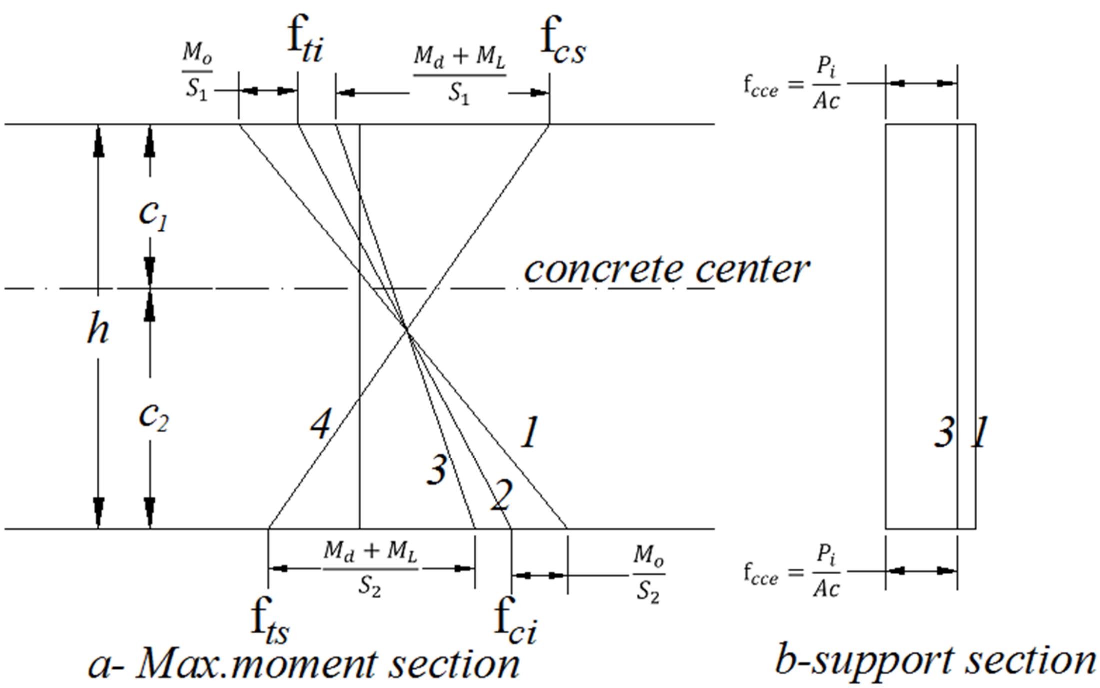


Fig (3-10) Flexural stress distribution for beam with variable eccentricity (a) maximum moment section (b) support section

Summarizing the design process to determine the best cross section and the required prestress force and eccentricity based on stress limitations.

The requirement for the section moduli S_1 and S_2 with respect to the top and bottom surfaces respectively is

$$S_1 \geq \frac{(1-R)M_o + M_d + M_L}{R\,f_{ti} - f_{cs}}$$

$$S_2 \geq \frac{(1-R)M_o + M_d + M_L}{f_{ts} - R\,f_{ci}}$$

$$R = \text{effective ratio} = \frac{P_e}{P_i}$$

$$P_e = P_i * R, \quad R = 1 - \text{losses}$$

$$I_c = S_1C_1 = S_2C_2$$

$$\frac{C_1}{C_2} = \frac{S_2}{S_1}$$

$$h = C_1 + C_2$$

$$\frac{C_1}{h} = \frac{S_2}{S_1 + S_2}$$

The concrete centroid stress under initial condition f_{cci} is given

$$f_{cci} = f_{ti} - \frac{C_1}{h}(f_{ti} - f_{ci})$$

$$P_i = Ac|f_{cci}|$$

$$e_{max} = e = (f_{ti} - f_{cci})\frac{S_1}{P_i} + \frac{M_o}{P_i}$$

The required area of prestressing steel A_{Ps}

$$A_{Ps} = \frac{P_i}{\text{permissible stress in the steel}}$$

Permissible stress in the strand immediately after transfer must not exceed $0.82\,f_{Py}$ or $0.74\,f_{Pu}$

b- Beam with constant eccentricity

The design method presented in the previous section was based on stress conditions at the maximum moment section of a beam with the maximum value of moment(M_o). If P_i and e were to held constant along the span, the eccentricity is controlled by conditions at the supports, where $M_o = Zero$

The requirements on the section moduli are that

$$S_1 \geq \frac{M_o + M_d + M_L}{R\,f_{ti} - f_{cs}}$$

$$S_2 \geq \frac{M_o + M_d + M_L}{f_{ts} - R\, f_{cs}}$$

The required eccentricity is

$$e = (f_{ti} - f_{cci})\frac{S_1}{P_i}$$

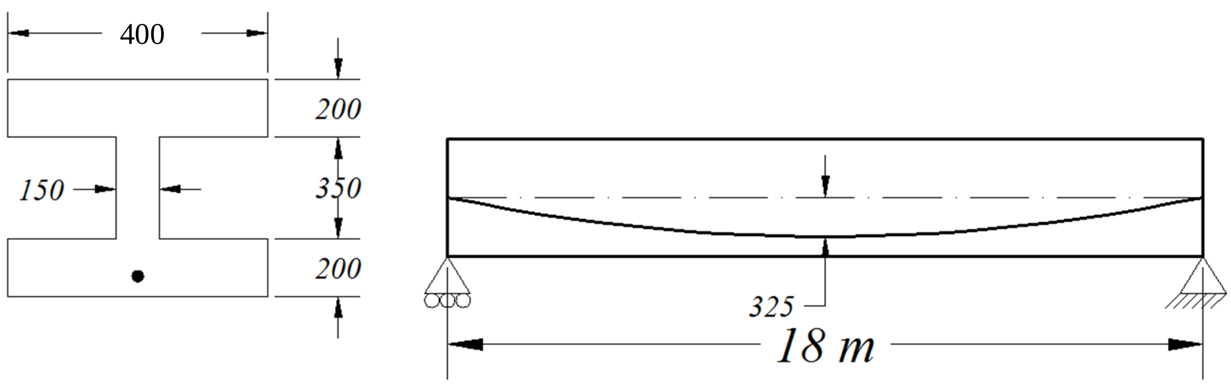
The concrete centroid stress and the initial pre-stress force may be found as before in previous section.

Step of design procedure

a- Assume a concrete section
b- Calculate the required prestresses force and eccentricity for that will be the controlling load stage
c- Check the stresses at all stage
d- Check the flexural strength

The trial section is then evicted if necessary.

Ex: Check the simply supported beam shown in Fig at mid span with respect to the permissible concrete stresses at initial and service load stage. The following data is given $fc' = 36$ MPa, $fci' = 30$ MPa, $P_i = 1200$ kN , $R = 0.8$, $e = 325$, S.D.L = 4 kN/m , L.L = 4 kN/m ssume that 75% of L.L are sustained load and class U flexural member.



Solution

$$Ag = 212.5 * 10^3 mm^2$$
$$I_g = 13169 * 10^6\ mm^4$$
$$S_{top} = \frac{I_g}{C_1} = S_{bot} = \frac{I_g}{C_2} = 35.12 * 10^6 mm^3$$

$$w_g = 212.5 * 10^3 * \frac{24}{10^6} = 5.1\ kN/m$$

$$M_g = \frac{5.1 * 18^2}{8} = 206.5\ kN.m$$

$$M_s = \frac{8 * 18^2}{8} = 324\ kN.m$$

$$M_t = M_g + M_s \rightarrow M_t = 206.5 + 324 = 530.5\ kN.m$$

Or

$$M_s = \frac{(4 + 0.75 * 4) * 18^2}{8} = \frac{7 * 18^2}{8} = 283.5\ kN.m$$

$$M_t = 206.5 + 283.5 = 490\ kN.m$$

Allowable stress at mid span

1- Initial stage (transfer

$$f_{ti} = \frac{1}{4}\sqrt{fci'} = \frac{1}{4}\sqrt{30} = 1.37\ N/mm^2$$

$$f_{ci} = 0.6 * fci' = 0.6 * 30 = 18\ N/mm^2$$

2- Service load stage

$$f_{ts} = 0.62\sqrt{fc'} = 0.62\sqrt{36} = 3.72\ N/mm^2$$

For $P_e + sustained\ load$

$$f_{cs} = 0.45 * fc' = 0.45 * 36 = 16.2\ N/mm^2$$

For $P_e + total\ load$

$$f_{cs} = 0.6 * fc' = 0.6 * 36 = 21.6\ N/mm^2$$

a- **Stage of transfer stress at mid span**

$$f_{top} = -\frac{P_i}{Ac} + \frac{P_i e}{S_{top}} - \frac{M_g}{S_{top}}$$

$$= \frac{-1200 * 10^3}{212.5 * 10^3} + \frac{1200 * 10^3 * 325}{35.12 * 10^6} - \frac{206.5 * 10^6}{35.12 * 10^6}$$

$$= -0.423\ N/mm^2 (Comp\ limit\ is\ 1.37\ N/mm^2\ tension)$$

$$\therefore o.k$$

$$f_{bot} = -\frac{P_i}{Ac} - \frac{P_i e}{S_{bot}} + \frac{M_g}{S_{bot}}$$

$$= \frac{-1200 * 10^3}{212.5 * 10^3} - \frac{1200 * 10^3 * 325}{35.12 * 10^6} + \frac{206.5 * 10^6}{35.12 * 10^6}$$

$$= -10.87\ N/mm^2\ Comp.\ (limit\ is\ 18\ N/mm^2 Comp)\ \therefore o.k$$

b- **Service load stage**

$$P_e = 0.8 * 1200 = 960\ kN$$

$$f_{top} = -\frac{P_e}{Ac} + \frac{P_e\, e}{S_{top}} - \frac{M_t}{S_{top}} = \frac{-960 * 10^3}{212.5 * 10^3} + \frac{960 * 10^3 * 325}{35.12 * 10^6} - \frac{530.5 * 10^6}{35.12 * 10^6}$$

$$= -10.739\ N/mm^2\ Comp (limit\ is\ 21.6\ N/mm^2 Comp)\ \therefore o.k$$

Or

$$f_{top} = \frac{-960*10^3}{212.5*10^3} + \frac{960*10^3*325}{35.12*10^6} - \frac{490*10^6}{35.12*10^6} =$$

$$-9.59\ N/mm^2\ Comp\ (limit\ is\ 16.2\ N/mm^2\ Comp)\ \therefore o.k$$

$$f_{bot} = -\frac{P_e}{Ac} - \frac{P_e\, e}{S_{bot}} + \frac{M_t}{S_{bot}}$$

$$= \frac{-960 * 10^3}{212.5 * 10^3} - \frac{960 * 10^3 * 325}{35.12 * 10^6} + \frac{530.5 * 10^6}{35.12 * 10^6}$$

$$= 1.704\ N/mm^2\ tens.\ (limit\ is\ 3.72\ N/mm^2 tens.)$$

$\therefore o.k$

Ex\\Check stress at the ends of simply supported beam in the previous example

Allowable stress

Initial stage

$$f_{ti} = 0.5 * \sqrt{fci'} = 2.74\ N/mm^2$$

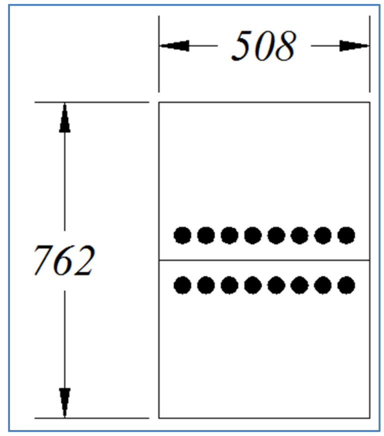
$$f_{ci} = 0.7 * fci' = 21\ N/mm^2$$

Concrete stresses at the ends

$$-\frac{P_i}{Ac} = \frac{-1200 * 10^3}{212.5 * 10^3} = -5.64\ N/mm^2\ Comp.$$

allowable is 21 N/mm^2 Comp. $\therefore o.k$

Ex\\A simply supported prestressed beam with sixteen 21.7mm (7-wire) strand ($f_{Pu} = 1723\ N/mm^2$) at initial tensile stress ($1206\ N/mm^2$) use $fc' = 34.4\ N/mm^2$, $fci' = 27.6\ N/mm^2$, $e = zero$, $span = 12m$, 20% loss of prestress, class U flexural member



1- Find concrete stresses at mid span after transfer and compare with ACI-code limit
2- Find live load moment capacity based on code permissible stresses (all live load are sustained)

Solution

Allowable stress

Initial stage

$$f_{ti} = \frac{1}{4} * \sqrt{27.4} = 1.31\ MPa$$
$$f_{ci} = 0.6 * 27.6 = 16.56\ MPa$$

Service load stage

$$f_{ts} = 0.62 * \sqrt{34.4} = 3.63\ MPa$$
$$f_{cs} = 0.45 * 34.4 = 15.48\ MPa$$

$$A_{Ps} = 92.9 * 16 = 1486.4\ mm^2$$
$$P_i = \frac{1486.4 * 1206}{1000} = 1792.6\ kN$$
$$w_g = \frac{508 * 762}{10^6} * 24 = 9.3\ kN/m$$
$$M_g = \frac{9.3 * 12^2}{8} = 167.22\ kN.m$$
$$I_g = \frac{508 * 762^2}{12} = 18.7 * 10^9\ mm^4$$

Concrete stress at transfer

$$f_{top} = -\frac{P_i}{Ac} - \frac{M_g C_1}{I_g} = \frac{-1792.6 * 10^3}{508 * 762} - \frac{167.22 * 10^6 * 381}{18.7 * 10^6}$$

$$= -4.63 - 3.4 = -8.03 \text{ MPa (allowable 1.31 MPa ten.} \therefore \text{o.k)}$$

$$f_{bot} = -4.63 + 3.4 = -1.23 \text{ MPa(allowable 16.56 MPa Comp.} \therefore \text{o.k)}$$

Service load stage

$$P_e = R * P_i = 0.8 * 1792.6 = 1434 \text{ kN}$$

$$f_{top} = -\frac{P_e}{Ac} - \frac{M_g C_1}{I_g} = f_{cs}$$

$$-\frac{1434 * 10^3}{508 * 762} - \frac{M_t * 10^6 * 381}{18.7 * 10^9} = -15.48$$

$$M_t = 578 \text{ kN.m}$$

$$f_{bot} = -\frac{P_e}{Ac} + \frac{M_g C_2}{I_g} = f_{ts}$$

$$\rightarrow \frac{-1434 * 10^3}{508 * 762} + \frac{M * 10^6 * 381}{18.7 * 10^9} = 3.63$$

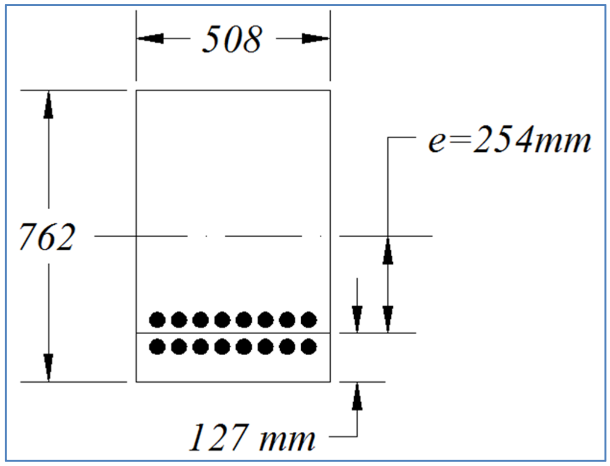
$$M_t = 360 \text{ kN.m} \quad \text{(control)}$$

$$M_t = M_g + M_d + M_L$$

$$360 = 1677 + 0 + M_L$$

$$M_L = 192.78 \text{ kN.m}$$

Ex\\The same previous example use $e = 254\ mm$



Solution

Stage of transfer (stress at mid span)

$$f_{top} = -\frac{P_i}{Ac} + \frac{P_i e\, C_1}{I_c} - \frac{M_g C_1}{I_c}$$

$$= \frac{-1792.6 * 10^3}{508 * 762} + \frac{1792.6 * 10^3 * 254 * 381}{18.7 * 10^9} - \frac{167.22 * 10^6 * 381}{18.7 * 10^9}$$

$$= -4.63 + 9.27 - 3.4 = 1.24\ N/mm^2 < 1.31\ N/mm^2 \therefore o.k$$

$$f_{bot} = -4.63 - 9.27 + 3.4 = -10.5\ N/mm^2 < 16.56\ N/mm^2 \therefore o.k$$

Service load stage

$$P_e = 1434\ kN$$

$$f_{top} = -\frac{P_e}{Ac} + \frac{P_e e\, C_1}{I_g} - \frac{M_t C_1}{I_g}$$

$$= \frac{-1434 * 10^3}{508 * 762} + \frac{1434 * 10^3 * 254 * 381}{18.7 * 10^9} - \frac{M_t * 10^6 * 381}{18.7 * 10^9}$$

$$= -15.48$$

$$M_t = 942\ kN.m$$

$$f_{bot} = -\frac{P_e}{Ac} + \frac{P_e e\, C_2}{I_g} - \frac{M_t C_2}{I_g}$$

$$= \frac{-1434 * 10^3}{508 * 762} - \frac{1434 * 10^3 * 254 * 381}{18.7 * 10^9} + \frac{M_t * 10^6 * 381}{18.7 * 10^9}$$

$$= 3.63$$

$$M_t = 724\ kN.m \quad \text{(control)}$$

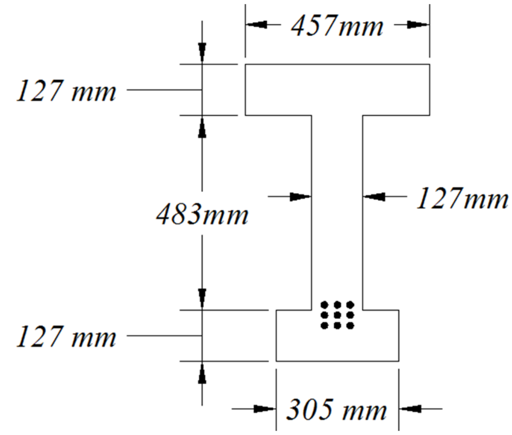
$$\therefore M_t = 724\ kN.m$$

$$M_t = M_g + M_d + M_L$$

$$724 = 1677.22 + 0 + M_L$$

$$M_L = 556.78\ kN.m$$

Ex\\ The post-tensioned bounded prestressed beam shown in cross section in Fig below is stressed using wires of $f_{Pu} = 1900\,MPa$. If concrete strength $fc' = 34.5\,MPa$ what is the ultimate strength of the member. $f_{Py}/f_{Pu} > 0.85$,$A_{Ps} = 1129\text{ mm}^2$, $C_1 = 333\text{ mm}$, $C_2 = 404\text{ mm}$, $\gamma_P = 0.4$



Solution

$$\rho_P = \frac{A_{Ps}}{bd} = \frac{1129}{457 * 622} = 0.00397$$

$$\beta_1 = 0.85 - 0.05 * \left(\frac{34.5 - 28}{7}\right) = 0.803$$

$$f_{Ps} = f_{Pu} * \left(1 - \frac{\gamma_P}{\beta_1} * \rho_P * \frac{f_{Pu}}{fc'}\right) = 1900 * \left(1 - \frac{0.4}{0.803} * 0.00397 * \frac{1900}{34.5}\right)$$
$$= 1693\text{ kN/mm}^2$$

Let $a \le h_f$

$$a = \frac{A_{Pw} f_{Ps}}{0.85 fc' b} = \frac{1129 * 1693}{0.85 * 34.5 * 457} = 142.74\text{ mm} > 127\text{mm}$$

$\therefore T - beam\ analysis$

$$A_{Pf} = \frac{0.85 * fc'(b - b_w) * hf}{f_{Ps}} = \frac{0.85 * 34.5(457 - 127) * 127}{1693}$$
$$= 726\text{ mm}^2$$

$$A_{Pw} = A_{Ps} - A_{Pf} = 1129 - 726 = 403 mm^2$$

$$a = \frac{A_{Pw} f_{Ps}}{0.85 fc' b_w} = \frac{403 * 1693}{0.85 * 34.5 * 127} = 183.2\ mm$$

Check tension failure

$$c = \frac{a}{\beta_1} = \frac{183.2}{0.803} = 228.14$$

$$\frac{c}{dt} = \frac{228.14}{622} < 0.366 < 0.375 \therefore o.k\ Tension\ failure$$

$$M_u = \emptyset M_n = 0.9 * \left[A_{Pw} f_{Ps}\left(d_P - \frac{a}{2}\right) + 0.85(b - b_w)h_f\left(d_P - \frac{h_f}{2}\right)\right]$$

$$= 0.9\left[403 * 1693\left(622 - \frac{183.2}{2}\right) + 0.85 * 34.5 * (457 - 127) * 127 * \left(622 - \frac{127}{2}\right)\right] * 10^6 = 943.5\ kN.m$$

Ex\\ A prestressed simply supported uniformly loaded beam has the following mid span moment $M_g = 172.8kN.m$ $M_s = (M_d + M_L) = 515.6\ kN.m$. the following design data is given $span = 12.19m$, $fci = 27.6N/mm^2$, $fc' = 34.5N/mm^2$. Class U flexural member all live load are sustained, variable eccentricity initial permissible stress in the steel= $1207\ N/mm^2$, use $12.7mm\ (7 - wire)$ strandes losses = 20%.

Solution

Allowable stress

$$f_{ti} = \frac{1}{4}\sqrt{27.6} = 1.31\ N/mm^2$$
$$f_{ci} = 0.6 * 27.6 = 16.56\ N/mm^2$$
$$f_{ts} = 0.62 * \sqrt{34.5} = 3.64\ N/mm^2$$
$$f_{cs} = 0.45 * 34.5 = 15.53\ N/mm^2$$

$$S_1 \geq \frac{(1 - R)M_o + M_d + M_L}{R\ f_{ti} - f_{cs}} = \frac{(1 - 0.8)172.8 * 10^6 + (515 * 10^6)}{0.8 * 1.31 - (-15.53)}$$

$$= 33.19 * 10^6 mm^3 (control)$$

$$S_2 \geq \frac{(1 - R)M_o + M_d + M_L}{f_{ts} - R\ f_{ci}} = \frac{(1 - 0.8)172.8 * 10^6 + (515 * 10^6)}{3.64 - 0.8 * (-16.56)}$$

$$= 32.6 * 10^6$$

Choose rectangular section (let $b = 510mm$)

$$S = \frac{bh^2}{6} \rightarrow 33.19 * 10^6 = \frac{510 * h^2}{6} \rightarrow h = 625mm$$

$$f_{cent} = f_{ti} - \frac{C_1}{h}(f_{ti} - f_{ci}) = 1.31 - \frac{1}{2}\left(1.31 - (-16.56)\right) = 7.625\ N/mm^2$$

P_i=A*|f_{cent}|=510*625/1000*7.625=2430kN

$$A_{Ps_{req}} = \frac{P_i}{allowable\ stress} = \frac{2430}{1207 * 10^{-3}} = 2013\ mm^2$$

A_{Ps} for 12.7 mm seven-wire strands Grade 1725= 92.9 mm^2

$$\text{No. of strands required} = \frac{2013}{92.9} = 21.66$$

Use 22 strands12.7 mm seven-wire strands

$$e = e_{max} = (f_{ti} - f_{cent})\frac{S_1}{P_i} + \frac{M_g}{P_i}$$

$$= \left(1.31 - (-7.625)\right) * \frac{33.19 * 10^6}{2430 * 10^3} + \frac{172.8 * 10^6}{2430 * 10^3}$$

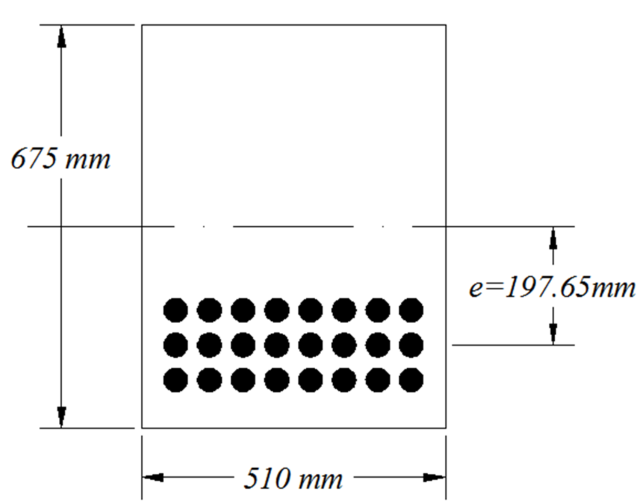
$$= 193.1\ mm$$

Or

$$e = e_{max} = (f_{cent} - f_{ci})\frac{S_2}{P_i} + \frac{M_g}{P_i}$$

$$= \left(-7.625 - (-16.56)\right) * \frac{33.19 * 10^6}{2430 * 10^6} + \frac{172.8 * 10^6}{2430 * 10^6}$$

$$= 193.1\ mm$$



Ex\\The beam in the previous example is to be redesign using straight tendons with constant eccentricity.

Solution

$$f_{ti} = 0.5 * \sqrt{27.6} = 2.62\ N/mm^2$$
$$f_{Ci} = 0.7 * 27.6 = 19.32\ N/mm^2$$

$$S_1 \geq \frac{M_o + M_d + M_L}{R\,f_{ti} - f_{cs}} = \frac{(172.8 + 515.0) * 10^6}{0.8 * 2.62 - (-15.53)} = 30.056 * 10^6$$
$$S_2 \geq \frac{M_o + M_d + M_L}{f_{ts} - R\,f_{cs}} = \frac{(172.8 + 515.0) * 10^6}{3.64 - 0.8 * (-19.32)}$$
$$= 36.09 * 10^6\ (control)$$

Let $b = 510mm$

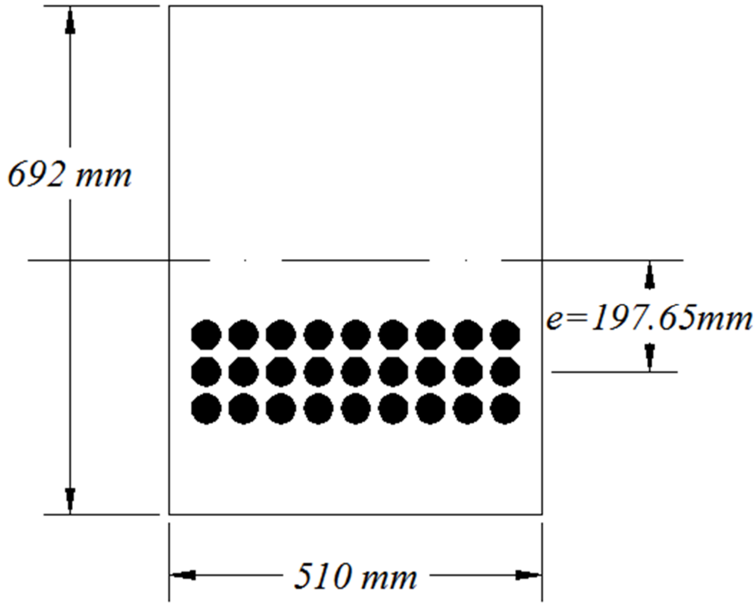
$$36.09 * 10^6 = \frac{510 * h^2}{6} \rightarrow h = 652\ mm$$
$$f_{cent} = 2.62 - \frac{1}{2}(2.62 + 19.32) = -8.35\ N/mm^2$$
$$P_i = A|f_{cent}| = 510 * 652 * |-8.35| = 2777\ kN$$
$$A_{Ps_{req}} = \frac{2777 * 1000}{1207} = 2300\ mm^2$$
$$\text{No. of strands req} = \frac{2300}{92.9} = 24.7$$

Use $25 - 12.7\ mm\ 7 - wire\ strands$

$$e = (f_{ti} - f_{cent})\frac{S_1}{P_i} = \left(2.62 - (-8.35)\right) * \frac{36.09 * 10^6}{2.947 * 10^3} = 142.5\ mm$$

Or

$$e = (f_{cent} - f_{ci})\frac{S_1}{P_i} = \left(-8.35 - (-19.32)\right) * \frac{36.09 * 10^6}{2.777 * 10^3} = 142.5mm$$


692 mm
e=197.65mm
510 mm

Shear

At loads near failure, a prestressed beam is usually extensively cracked and behaves much like an ordinary concrete beam.

$V_u \leq \emptyset V_n$

$V_n = V_c + V_s \qquad \emptyset = 0.75$

The first critical section is assumed to be at a distance $(h/2)$ from the face of support

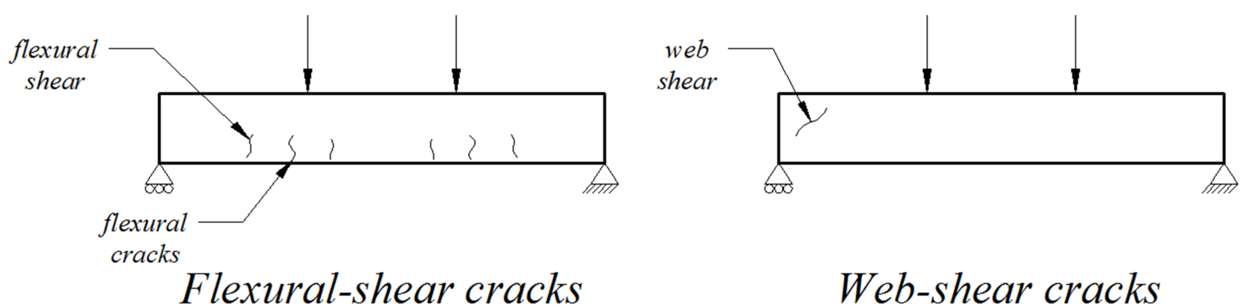
d = The distance from extreme compression fiber to centroid of prestressed and non-prestressed longitudinal reinforcement if any but need not to be taken less than $0.8h$

$$d \geq 0.8h$$

$\rho_v = \frac{Av}{b_w S}$ web reinforcement ratio

Av: Area of two legs

V_c: The smaller of V_{ci} and V_{cw} determined by flexure-shear cracking and web-shear cracking



Flexural-shear cracks start as nearly vertical flexural cracks at the tension face of the beam, and then spread diagonally upward toward the compression face.

$$V_{ci} = 0.05\lambda\sqrt{fc'}b_w d_p + V_d + \frac{V_i M_{cr}}{M_{max}}$$

V_{ci} = need not to be taken less than $0.14\lambda\sqrt{fc'}b_w d$

Where

$$M_{cr} = \frac{I}{C_2}\left(0.5\lambda\sqrt{fc'} + f_{Pe} - f_d\right)$$

And values of M_{max} and V_i shall be computed from the load combination maximum moment to occur at the section

V_d: shear force at section due to un factored dead load

f_d: stress due to unfactored dead load at tension face of the section

f_{Pe}:compressive stress at section face resulting from effective prestress force alone.

V_i: factored shear force at section due to externally applied loads occurring simultaneously with M_{max}

Web-shear cracks, start in the web due to high diagonal tension, and then spread diagonally both upward and downward.

$$V_{cw} = \left(0.29\sqrt{fc'} + 0.3f_{Pc}\right)b_w d_P + V_P$$

f_{Pc}: the compression stress after losses at the centroid of concrete section

$$V_P = P_e \sin(\theta)$$

Approximate equation for presttressed member with $A_{ps}f_{pe} \geq 0.4 (A_{ps}f_{pu}+A_sf_y)$

$$V_c = \left(0.05\sqrt{fc'} + 4.8\frac{V_u d_P}{M_u}\right) b_w d_P$$

$$0.17\sqrt{fc'}b_w d \leq V_c \leq 0.42\sqrt{fc'}b_w d$$

$$\frac{V_u d_P}{M_u} \leq 1.0$$

V_u& M_u: factored shear and moment at section considered resulting from total factored loads

M_d: moment due to unfactored dead load (moment corresponding to f_d)

Required area of web reinforcement

$$V_s = \frac{(V_u - \emptyset V_C)}{\emptyset} \qquad V_s = \frac{Av\, \text{f}y\, d}{S}$$

$$\frac{(V_u - \emptyset V_C)}{\emptyset} = \frac{Av\, \text{f}y\, d}{S} \rightarrow Av = \frac{(V_u - \emptyset V_C)S}{\emptyset \text{f}y\, d} \qquad , S = \emptyset \frac{\text{f}y\ d\ Av}{V_u - \emptyset V_C}$$

<u>Minimum web reinforcement</u>: minimum area of shear reinforcement must be provided

When $V_u > \frac{1}{2}\emptyset V_c$ minimum area is to be taken equal to the smaller of

$$\text{Av}_{\text{min}} = 0.062\sqrt{\text{fc}'}\frac{\text{b}_\text{w}\text{S}}{\text{f}_\text{yt}} \geq \frac{0.35\text{b}_\text{w}\text{S}}{\text{f}_\text{yt}}$$

Or

$$\text{Av}_{\text{min}} = \frac{\text{A}_{\text{Ps}}\text{f}_{\text{Pu}}\text{S}}{\text{S}_\text{v}\text{f}_{\text{yt}}\text{d}}\sqrt{\frac{\text{d}}{\text{b}_\text{w}}}$$

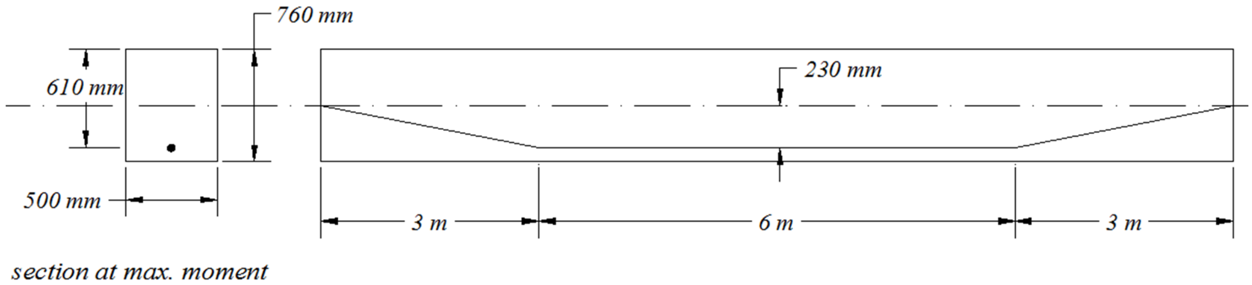
f_{yt}: specified yield strength ($\text{f}y$) of the transvers reinforcement (N/mm^2)

Max spacing

$$S \leq 0.75h \quad or\ 600mm \quad V_s \leq 0.33\sqrt{\text{f}c'}b_w d$$

$$S \leq \frac{0.75h}{2} \quad or\ 300mm \quad V_s > 0.33\sqrt{\text{f}c'}b_w d$$

Ex\\ The beam shown in figure carries an effective pre-stress force of $1450\ kN$ and supports a super imposed dead load of $7.0\ kN/m$ and service live load of $15\ kN/m$ in addition to its own weight of $9.1\ kN/m$. the wires are deflected upward $3.0m$ from support and eccentricity is reduced linearly to zero at support. What is the required stirrup spacing at point of $1.2\ m$ from the support use $7 - wire$ strands $A_{Ps} = 2036\ mm^2, f_{Pu} = 1720\ N/mm^2, fy = 400\ kN/mm^2, fc' = 35\ N/mm^2$



Solution

$$C_1 = C_2 = 380mm\ , I_g = 1.829 * 10^{10} mm^4$$

$$at\ max.moment\ section = 610 - 380 = 230mm$$

$$at\ 1.2 \rightarrow e = 230 * \frac{1.2}{3} = 92mm$$

$$d = 92 + 380 = 472mm$$

$$0.8 * h = 608mm$$

$$use\ d = 608$$

$$w_u = 1.2 * (9.1 + 7) + 1.6 * (15) = 43.32\ kN/m$$

$$V_u\ at\ 1.2\ from\ support = 208\ kN$$

$$f_{Pe} = -\frac{P_e}{Ac} - \frac{P_e\ e\ C_2}{I_g} = \frac{-1450 * 10^3}{500 * 760} - \frac{1450 * 10^3 * 92 * 380}{1.829 * 10^{10}}$$

$$= -6.59\ N/mm^2$$

$$V_d = (7 + 9.1) * \left(\frac{12}{2} - 1.2\right) = 77.28\ kN$$

$$M_d = \frac{16.1 * 1.2}{2} * (12 - 1.2) = 104.32\ kN.m$$

$$f_d = \frac{104.32 * 10^6 * 380}{1.829 * 10^{10}} = 2.16\ N/mm^2$$

$$M_{cr} = \frac{I}{C_2}\left(0.5 * \sqrt{fc'} + f_{Pe} - f_d\right)$$

$$= \frac{1.829 * 10^{10}}{380} * \left(0.5\sqrt{35} + 6.59 - 2.16\right) * 10^{-6}$$

$$= 355.6 kN.m$$

$$\frac{V_i}{M_{max}} = \frac{V_{d+L}}{M_{d+L}} = \frac{l-2x}{x(l-x)} = \frac{9.6}{12.96} = 0.74\ m^{-1}$$

$$V_{Ci} = \left(0.05\sqrt{35}\right) * \frac{500 * 608}{1000} + 77.28 + 0.74 * 355.6 = 430.3\ kN$$

$$> 0.14\sqrt{fc'} * b_w d = 251.82\ kN$$

$$V_P = P_e \sin(\theta)$$

$$\sin(\theta) = \tan(\theta) = \frac{230}{3000}$$

$$V_P = 1450 * \frac{230}{3000} = 111\ kN$$

$$f_{Pc} = \frac{1450000}{500 * 760} = 3.82\ N/mm^2$$

$$V_{cw} = \left[0.29 * \sqrt{35} + 0.3(3.82)\right] * \frac{500 * 608}{1000} + 111 = 981\ kN$$

$$\therefore use\ V_c = 430.3\ kN$$

$$V_u > \emptyset\frac{V_c}{2} \rightarrow use\ 8mm\ stirrups$$

$$S \leq \frac{80 * Av * fy * d}{A_{Ps} f_{Pu}} * \sqrt{\frac{b_w}{d}} \rightarrow S = 504\ mm$$

$$use\ 8\ mm\ stirrup@500\ mm$$

$$V_{d+L} = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

$$M_{d+L} = \frac{wL}{2}x - \frac{wx^2}{2} = \frac{w}{2}x(L-x)$$

Losses of pre-stress

$P_i \rightarrow P_i$
1- Slip in the anchorages
2- Elastic shortening of the concrete
3- Frictional losses

$P_i \rightarrow P_e$
4- Creep of concrete
5- Shrinkage of concrete
6- Relaxation of steel

1- Slip at the anchorages (only in post-tensioning tendons)

$$\Delta fs_{slip} = \frac{\Delta L}{L} E_s$$

ΔL: amount of slip

L:tendon length

E_s:Elastic modulus of pre-stressing steel

2- Elastic shortening of the concrete(in pre tension member)

$$\Delta q_{S_{elastic}} = E_s \frac{fc}{E_c} = nfc$$

fc: concrete stress at the level of steel centroid immediately after pre-stress is applied

$$fc = -\frac{P_i}{Ac}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_o e}{I_c} P_i = 0.9P_j$$

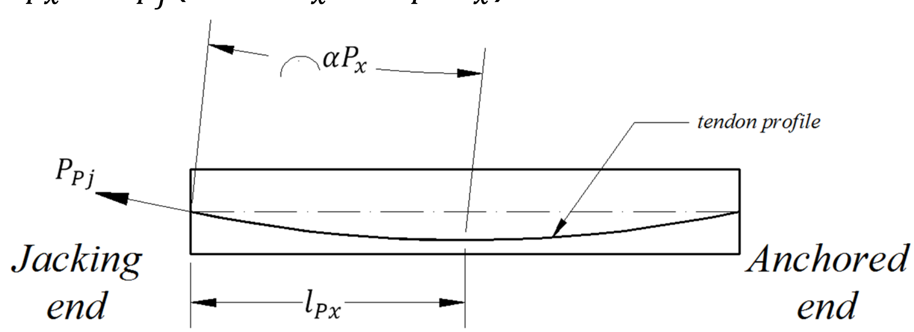
3- Frictional losses (for post tension member)

The force at the jacking end of the tendon P_{pj} required to produce the force P_{px} at any point (x) along the tendon can be found from the expression (ACI-code)

$P_{Px} = P_{PJ} * e^{-(klP_x + M_P \alpha P_x)}$

If $(klP_x + M_P \alpha P_x)$ is not greater than 0.3 P_{Px}

$P_{Px} = P_{Pj}(1 + klP_x + M_P \alpha P_x)^{-1}$



The loss of the force

$$\Delta P_{fr} = P_{Pj} - P_{Px} = P_{PJ}\left[1 - e^{-(klP_x + M_P \alpha P_x)}\right]$$

Dividing by the tendon area A_{Ps}

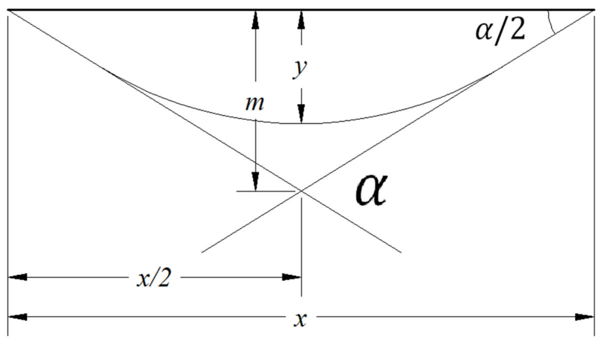
$$\Delta f_{fr} = P_{PJ}\left[1 - e^{-(klP_x + M_P \alpha P_x)}\right]$$

$$\Delta f_{fr} = f_{PJ}(klP_x + M_P \alpha P_x)$$

k:wabble friction coefficient

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\alpha}{2} = \frac{m}{x/2} \quad , \quad m \cong 2y$$

$$\frac{\alpha}{2} = \frac{4y}{x} \quad , \quad \alpha = \frac{8y}{x}\ radiaus$$



4- Creep of concrete

$$\Delta fs_{creep} = C_c n fc$$

C_c:creep coefficient

$$n = \frac{E_s}{E_c}$$

fc : concrete stress at level of steel centroid

$$fc = -\frac{P_i}{Ac}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_{sus}\, e}{I_c}$$

5- Shrinkage of concrete

$$\Delta fs_{shrinkage} = \varepsilon_{sh} E_s$$

ε_{sh}:shrinkage strain (0.0004-0.0008)

E_s:modulus of elasticity of pre-stressing steel

6- Relaxation of steel

$$\Delta f_{rel} = f_{Pi}\frac{\log t}{10}\left(\frac{f_{Pi}}{f_{Py}} - 0.55\right)$$

t:time in hours after stressing

$$f_{Pi} = 0.9 * f_{Pi}$$