Chapter One
Direct Design Method
**Flat slabs:**

Concrete slabs may be carried directly by columns as shown in Fig. (1-1) without the use of beams or girders. Such slabs are described as flat plates and are commonly used where spans are not large and loads not particularly heavy.

![Fig. (1-1) Slab supported directly on column.](image)

At point (1) there is more (-M) and shearing stress, and col. try to punch the slab.

Flat slab construction shown in Fig. (1-2) is also beamless but incorporates a thickened slab region (drop panels) and column capitals.

![Fig. (1-2) Flat slab.](image)
**Column capital:** An element at the end of the column to give a wider support for the floor slab

Column capital and drop panel are used to reduce:

1- Stresses due to shear.
2- Negative bending around the columns.

Size of drop panel shall be in accordance with the following ACI-code (8.2.4 –ACI-2014). Drop panel shall be extended in each direction from center line of support a distance not less than one-sixth the span length measured from center to center of supports in that direction.

The side of the drop panel shall be at least \((L/3)\),

Where

\(L: (c.to.c)\)

\(t: \text{Thickness of slab}\)

\(t_1: \text{Thickness of slab with drop}\)

\(t_2: \text{Thickness of drop}\)

\(t_2 \geq \frac{t}{4}\)
**Bending moments in flat slab floors:**

For purposes of design, a typical panel is divided into column strips and middle strips.

ACI-2014 (8.4.1.5) column strip is a design strip with a width on each side of a column centerline equal to \((0.25l_1)\) or \((0.25l_2)\), whichever is less. Column strip includes beams, if any.

In all cases

- \(l_1\): is the span in the direction of the moment analysis (c. to c.).
- \(l_2\): is the span in the lateral direction (transvers to \(l_1\) c.to.c).
- \(l_n\): clear span in \(l_1\) direction.
In case of **flat slab** the column strip is more critical than middle strip because it work as beam carrying the middle strip load to the column therefore column strip need more reinforcement. Plus that the beam takes 85% of the slab moment in the case of two-way slab with beams.

<table>
<thead>
<tr>
<th>$l_1$ direction</th>
<th>Panels (1,2,3,7,8,9)</th>
<th>Exterior slab</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panels (4,5,6)</td>
<td>Interior slab</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$l_2$ direction</th>
<th>Panels (1,4,7,3,6,9)</th>
<th>Exterior slab</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panels (2,5,8)</td>
<td>Interior slab</td>
</tr>
</tbody>
</table>

In the diagram, the moment expressions are:

- $M_{CD} = \frac{w_u l_2 l_n^2}{8}$
- $M_{EF} = \frac{w_u l_2 l_n^2}{8}$
- $M_{AB}$
\[
\frac{1}{2}(M_{ab} + M_{cd}) + M_{ef} = \frac{1}{8}w_{u}l_{2}l_{n}^2
\]

A similar requirement exists in the perpendicular direction.

**Deflection control of two-way slab:**

**Design limits (ACI-code 2014):**

**Minimum slab thickness:**

8.3.1.1 For nonprestressed slabs without interior beams spanning between supports on all sides, having a maximum ratio of long-to-short span of 2, overall slab thickness \(h\) shall not be less than the limits in Table 8.3.1.1 and shall be at least the value in (a) or (b) unless the calculated deflection limits of 8.3.2 are satisfied:

(a) Slabs without drop panels as defined in 8.2.4..........................125 mm.

(b) Slabs with drop panels as defined in 8.2.4 ..............................100 mm.
### Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exterior panels</td>
<td>Interior panels</td>
</tr>
<tr>
<td></td>
<td>Without edge beams</td>
<td>With edge beams</td>
</tr>
<tr>
<td>280</td>
<td>$t_y/33$</td>
<td>$t_y/36$</td>
</tr>
<tr>
<td>420</td>
<td>$t_y/30$</td>
<td>$t_y/33$</td>
</tr>
<tr>
<td>520</td>
<td>$t_y/28$</td>
<td>$t_y/31$</td>
</tr>
</tbody>
</table>

<sup>[1]</sup> $t_y$ is the clear span in the long direction, measured face-to-face of supports (mm).

<sup>[2]</sup> For $f_y$ between the values given in the table, minimum thickness shall be calculated by linear interpolation.

<sup>[3]</sup> Drop panels as given in 8.2.4.

<sup>[4]</sup> Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if $\alpha_f$ is less than 0.8. The value of $\alpha_f$ for the edge beam shall be calculated in accordance with 8.10.2.7.

### 8.3.1.2

For nonprestressed slabs with beams spanning between supports on all sides, overall slab thickness $h$ shall satisfy the limits in Table 8.3.1.2 unless the calculated deflection limits of 8.3.2 are satisfied.
### Direct design method of two way slabs (8.10.2 ACI-2014):

**Limitations:**

Moments in two-way slab can be found using direct design method subject to the following restrictions:

**8.10.2.1** There shall be at least three continuous spans in each direction.

**8.10.2.2** Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span.

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**Table 8.3.1.2—Minimum thickness of nonpre-stressed two-way slabs with beams spanning between supports on all sides**

<table>
<thead>
<tr>
<th>$\alpha_{fn}$</th>
<th>Minimum $h$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{fn} \leq 0.2$</td>
<td>8.3.1.1 applies (a)</td>
</tr>
<tr>
<td>$0.2 &lt; \alpha_{fn} \leq 2.0$</td>
<td>$\frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fn} - 0.2)}$ (b)$^{[2],[3]}$</td>
</tr>
<tr>
<td>Greater of:</td>
<td>125 (c)</td>
</tr>
<tr>
<td>Greater of:</td>
<td>$\frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$ (d)$^{[2],[3]}$</td>
</tr>
<tr>
<td>$\alpha_{fn} &gt; 2.0$</td>
<td>90 (e)</td>
</tr>
</tbody>
</table>

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$^{[1]}\alpha_{fn}$ is the average value of $\alpha_f$ for all beams on edges of a panel and $\alpha_f$ shall be calculated in accordance with 8.10.2.7.

$^{[2]}\ell_n$ is the clear span in the long direction, measured face-to-face of beams (mm).

$^{[3]}\beta$ is the ratio of clear spans in long to short directions of slab.

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**8.3.1.2.1** At discontinuous edges of slabs conforming to 8.3.1.2, an edge beam with $\alpha_f \geq 0.80$ shall be provided, or the minimum thickness required by (b) or (d) of Table 8.3.1.2 shall be increased by at least 10 percent in the panel with a discontinuous edge.
8.10.2.3 Panels shall be rectangular, with a ratio of longer to shorter panel dimensions measured center-to-center of supports, not exceed 2.

8.10.2.4 Columns offset shall not exceed 10 percent of the span in direction of offset from either axis between centerlines of successive columns.

8.10.2.5 All loads shall be due to gravity only and uniformly distributed over an entire panel.

8.10.2.6 Unfactored live load shall not exceed two times the unfactored dead load.

\[
\frac{L.L}{D.D} \leq 2.0
\]

8.10.2.1 For a panel with beams between supports on all sides, Eq. (8.10.2.7a) shall be satisfied for beams in the two perpendicular directions

\[
0.2 \leq \frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2} \leq 5.0
\]

(8.10.2.7a)

Where \( \alpha_{f1} \) and \( \alpha_{f2} \) are calculated by:

\[
\alpha_f = \frac{E_{cb} l_b}{E_{cs} l_s}
\]

(8.10.2.7b)

\( \alpha_{f1} = \alpha_f \) in direction \( l_1 \)

\( \alpha_{f2} = \alpha_f \) in direction \( l_2 \)

In the case of monolithic construction (two-way slab with beams)
\[
\alpha_f = \frac{E_{cb} l_b}{E_{cs} l_s}, \quad l_s = \frac{l_2 h_f^3}{12}
\]

For internal strip \( l_2 = \frac{l_a + l_B}{2} \)

For external strip \( l_2 = \frac{l_A}{2} + \frac{c}{2} \)

Total static moment for end span

\[
M_o = \frac{w_u l_2 l_n^2}{8}
\]

\( l_1 = l_a \)

\( l_n \rightarrow l_1 \)

\( l_n \rightarrow \) clear span face to face of columns, capitals, brackets or walls.

\( l_n \geq 0.65 l_1 \)

For other direction:
For internal strip  \[ l_2 = \frac{l_a + l_b}{2} \]

For external strip  \[ l_2 = \frac{l_a}{2} + \frac{c}{2} \]

Total static moment for end span, \[ M_o = \frac{w l_2 l_n^2}{8} \]

\( l_1 = l_A \quad l_n \rightarrow l_1 \)

\( l_n \): Clear span, circular or regular polygon shaped support shall be treated as square support with the same area.
Negative and positive factored moment:

**For interior span (8.10.4.1):**

Negative $M_u = 0.65 M_o$
Positive $M_u = 0.35 M_o$

**For end span:**

Use Table 8.10.4.2 (ACI-2014)

8.10.4.2 In an end span, $M_o$ shall be distributed in accordance with Table 8.10.4.2.

Table 8.10.4.2—Distribution coefficients for end spans

<table>
<thead>
<tr>
<th></th>
<th>Exterior edge unrestrained</th>
<th>Slab with beams between all supports</th>
<th>Slab without beams between interior supports</th>
<th>Exterior edge fully restrained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior negative</td>
<td>0.75</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>Positive</td>
<td>0.63</td>
<td>0.57</td>
<td>0.52</td>
<td>0.50</td>
</tr>
<tr>
<td>Exterior negative</td>
<td>0</td>
<td>0.16</td>
<td>0.26</td>
<td>0.30</td>
</tr>
</tbody>
</table>
8.10.4.5 Negative moment $M_u$ shall be the greater of the two negative $M_u$ calculated for spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with stiffnesses of adjoining elements.
Factored moments in column strips

8.10.5.1 The column strips shall resist the portion of interior negative $M_u$ in accordance with Table 8.10.5.1.

<table>
<thead>
<tr>
<th>$\alpha_n \ell_2/\ell_1$</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$\geq 1.0$</td>
<td>0.90</td>
<td>0.75</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Note: Linear interpolations shall be made between values shown.

8.10.5.2 The column strips shall resist the portion of exterior negative $M_u$ in accordance with Table 8.10.5.2.
The relative restrained provided by the torsional resistance of the effective transverse edge beam is reflected by the parameter $\beta_t$, defined as:

$$\beta_t = \frac{E_{cb} C}{2E_{cs} I_s} \quad (8.10.5.2a)$$

The constant $C$ for T- or L-section is calculated by dividing the section into separate rectangular parts, each having smaller dimension ($x$) and larger dimension ($y$), and summing the values of $C$ for each part.

$$C = \sum \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3 y}{3} \quad (8.10.5.2b)$$

The subdivision can be done in such away as to maximize $C$.

8.10.5.5 The column strips shall resist the portion of positive $M_u$ in accordance with Table 8.10.5.5.
Factored moments in beams

8.10.5.7.1 Beams between supports shall resist the portion of column strip $M_u$ in accordance with Table 8.10.5.7.1.

Direct loads on beams: factored beam self-weight + factored wall weight

$$ (w_u)_b = (h - hf)b_w \times 24 \times 1.2 + \text{wall weight} \times 1.2 $$

$$ (M_o)_b = \frac{(w_u)_b l_n^2}{8} , \quad l_n = \text{clear span for beam} $$

Interior beam

Total negative moment $= 0.85 \text{ col. Strip moment} + 0.65 (M_o)_b$

Total pos. moment $= 0.85 \text{ col. Strip moment} + 0.35 (M_o)_b$
End beam (Use Table 8.10.4.2):

Internal negative moment = 0.85 col. Strip moment + factor \( (M_o)_b \)
pos. moment = 0.85 col. Strip moment + factor \( (M_o)_b \)
External neg. moment = 0.85 col. Strip moment + factor \( (M_o)_b \)

Design of moment reinforcement for slab

\[
\rho = \frac{1}{m} \left( 1 - \sqrt{1 - \frac{2R_m}{f_y}} \right), \quad R_u = \frac{M_u}{\phi bd^2}
\]

\[
m = \frac{f_y}{0.85 * f_{c'}} \quad A_s = \rho * bd
\]

\( A_{s_{\text{min}}} = 0.002 A_g \) for \( f_y < 420 \) N/mm\(^2\)

\( A_{s_{\text{min}}} = \frac{0.0018 * 420}{f_y} \) for \( f_y \geq 420 \) N/mm\(^2\) or \( A_{s_{\text{min}}} = 0.0014 A_g \)

\( S_{\text{main}} = \frac{A_s (\text{provided by one bar})}{\text{total } A_s \text{ (req.)}} * \text{width of strip} \)

\( S_{\text{max}} = 2t(2hf) \)
Ext. Strip

\[ l_2 = \frac{l_a}{2} + \frac{b_w}{2} \]

Int. Strip

\[ l_2 = \frac{l_a + l_b}{2} \]

\[ \beta_t = \frac{\beta_1 + \beta_2}{2} \]
8.10.6 Factored moments in middle strips

8.10.6.1 That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips.

8.10.6.2 Each middle strip shall resist the sum of the moments assigned to its two half middle strips.

8.10.6.3 A middle strip adjacent and parallel to a wall-supported edge shall resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.
A two-way reinforced concrete building floor system is composed of slab panels measuring 6*7.5 m in plan supported by column-line beams as shown in figure below. Using concrete with $f_{c'}=27.6\ \text{MPa}$ and steel having $f_{y}=414\ \text{MPa}$, design typical exterior panel to carry a service live load of 6.9 kN/m² in addition to its own weight of the floor.
Sol:

Try h=17 cm

Either x=h-hf=500-170=330 mm=33 cm
Or x=4*hf=4*170=680 mm=68cm

Chose min. value (x=33 cm)

**Beam (B₁)**

\[ I_{beam} = \frac{(36 \times 50^3)}{12} \times 1.5 = 562500 \text{ cm}^4 \]

Slab strip width = \( \frac{7.5}{2} + \frac{0.36}{2} = 3.93 \text{ m} \)

\[ I_{slab} = \frac{393 \times 100 \times 17^3}{12} = 160901 \text{ cm}^4 \]

\[ \alpha_1 = \frac{E_{cb} \times I_b}{E_{cs} \times I_s} = \frac{562500}{160901} = 3.5 \]

**Beam (B₂)**

\[ I_b = \frac{36 \times 50^3}{12} \times 2 = 750000 \text{ cm}^4 \]

Slab strip width = 6 m

\[ I_s = \frac{600 \times 17^3}{12} = 245650 \text{ cm}^4 \]

\[ \alpha_2 = \frac{750000}{245650} = 3.1 \]

**Beam (B₃)**

\[ I_b = 750000 \text{ cm}^4 \]

slab strip width = 7.5 m

\[ I_s = \frac{750 \times 17^3}{12} = 307063 \text{ cm}^4 \]

\[ \alpha_3 = \frac{750000}{307063} = 2.44 \]

\[ \alpha_{fm} = \frac{3.5 + 3.1 + 3.1 + 2.44}{4} = 3.035 \]

\[ \beta = \frac{7.5 - 0.36}{6 - 0.36} = 1.27 \]
\[ \alpha_{fm} > 2.0 \text{ slab with interior beam use eq 9.13 to find h} \]

\[
h = \frac{\ln \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 9 \times \beta} = \frac{7.14 \left( 0.8 + \frac{414}{1400} \right)}{36 + 9 \times 1.27} \times 10^3 = 165 \text{ mm} > 90 \text{ mm ok}
\]

\[
\therefore h = 17 \text{ cm is ok}
\]

self-weight of slab \[
= \frac{17}{100} \times 24 = 4.08 \text{ kN/m}^2
\]

\[
w_u = (1.2 \times 4.08) + 1.6 \times 6.9 = 16 \text{ kN/m}^2
\]

**Short span direction**

\[
M_o = \frac{w_u l_2 l_n^2}{8}
\]

*interior slab strip (7.5 m width)*

\[
M_0 = \frac{16 \times 7.5 \times (5.64)^2}{8} = 477 \text{ kN.m}
\]

sec 8.10.4.1

\[+M = 0.35 \times 477 = 167 \text{ kN.m} \]

\[-M = 0.65 \times 477 = 310 \text{ kN}.
\]

\[
l_2/l_1 = \frac{7.5}{6} = 1.25
\]

\[
\frac{\alpha_1 l_2}{l_1} = \frac{2.44 \times 7.5}{6} = 3.1
\]

sec 8.10.5.1 68% from \((-M) \rightarrow C.S\)

0.68 \times 310 = 211 \text{ kN.m} \ C.S

0.85 \times 211 = 179 \text{ kN.m} \ Beam

0.15 \times 211 = 32 \text{ kN.m} \ Slab \ C.S

310 - 211 = 99 \text{ kN.m} \ M.S

sec 8.10.5.5 68% \text{ from} \((+M) \rightarrow C. S\)

0.68 \times 167 = 114 \text{ kN.m} \ C.S

0.85 \times 114 = 47 \text{ kN.m} \ \text{beam}

0.15 \times 114 = 17 \text{ kN.m} \ Slab \ C.S

167 - 114 = 53 \text{ kN.m} \ M.S
Exterior slab strip (3.93 m width)

\[ M_0 = \frac{16 \times 3.93 \times 5.643}{8} = 250 \text{ kN.m} \]

**sec. 8.10.4.1**

\[ +M = 0.35 \times 250 = 88 \text{ kN.m} \]
\[ -M = 0.65 \times 250 = 163 \text{ kN.m} \]

\[ \frac{l_2}{l_1} = \frac{7.5}{6} = 1.25 \]
\[ \frac{\alpha l_2}{l_1} = \frac{3.5 \times 7.5}{6} = 3.1 \]

**sec. 8.10.5.1**

68% from \((-M) \rightarrow C.S\)

\[ 0.68 \times 163 = 111 \text{ kN.m} \quad \text{C.S} \]
\[ 0.85 \times 111 = 94 \text{ kN.m} \quad \text{beam} \]
\[ 0.15 \times 111 = 17 \text{ kN.m} \quad \text{Slab C.S} \]
\[ 163 - 111 = 52 \text{ kN.m} \quad \text{M.S} \]

68% from \((+M) \rightarrow C.S\)

\[ 0.68 \times 88 = 60 \text{ kN.m} \quad \text{C.S} \]

0.85*60= 51 kN.m beam

\[ 0.15 \times 60 = 9 \text{ kN.m} \quad \text{Slab C.S} \]

\[ 88 - 60 = 28 \text{ kN.m} \quad \text{M.S} \]

<table>
<thead>
<tr>
<th></th>
<th>Beam moment</th>
<th>Slab C.S moment</th>
<th>M.S moment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interior slab strip 6m span</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive</td>
<td>97</td>
<td>17</td>
<td>53</td>
</tr>
<tr>
<td>Negative</td>
<td>179</td>
<td>32</td>
<td>99</td>
</tr>
<tr>
<td><strong>Exterior slab strip 6m span</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive</td>
<td>51</td>
<td>9</td>
<td>28</td>
</tr>
<tr>
<td>Negative</td>
<td>94</td>
<td>17</td>
<td>52</td>
</tr>
</tbody>
</table>
• Long span direction (slab strip 6 m)

\[ M_0 = \frac{16 \times 6 \times 7.14^2}{8} = 612 \text{ kN.m} \]

sec 8.10.4.2

Ext. \(-M = 0.16 \times 612 = 98 \text{ kN.m} \)

\[ +M = 0.57 \times 612 = 349 \text{ kN.m} \]

Int \(-M = 0.7 \times 612 = 428 \text{ kN.m} \)

\[
c = \sum \left(1 - 0.63 \times \frac{x}{y}\right) \times \frac{x^3y}{3}
\]

case 1

\[
c = \left(1 - 0.63 \times \frac{33}{36}\right) \times \frac{33^3 \times 36}{3} + \left(1 - 0.63 \times \frac{17}{69}\right) \times \frac{17^3 \times 69}{3}
\]

\[= 277 \times 10^3 \text{ cm}^4\]

case 2

\[
c = \left(1 - 0.63 \times \frac{36}{50}\right) \times \frac{36^3 \times 50}{3} + \left(1 - 0.63 \times \frac{17}{33}\right) \times \frac{17^3 \times 33}{3}
\]

\[= 461 \times 10^3 \text{ cm}^4\]

use \(c = 461000 \text{ cm}^4\)

\[
\frac{l_2}{l_1} = \frac{6}{7.5} = 0.8 \quad \frac{\alpha_1 l_2}{l_1} = 3.1 \times 0.8 = 2.5
\]

\[
\beta_t = \frac{E_{cb} C}{2E_{cs} l_s}
\]

Sec 8.10.5

93% from Ext. \(-M \rightarrow C.S \quad Table \ 8.10.5.2\)

81% from \(+ M \rightarrow C.S \quad Table \ 8.10.5.5\)

81% from Tnt. \(-M \rightarrow C.S \quad Table \ 8.10.5.1\)
2015-2016

<table>
<thead>
<tr>
<th>Slab - beam strip (7.5m span)</th>
<th>Beam moment</th>
<th>Slab C.S moment</th>
<th>M.S moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ext. $-M$ (98)</td>
<td>77</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>+$M$ (349)</td>
<td>240</td>
<td>42</td>
<td>66</td>
</tr>
<tr>
<td>Int. $-M$ (428)</td>
<td>295</td>
<td>52</td>
<td>81</td>
</tr>
</tbody>
</table>

$A_{min} = 0.002 \times 1000 \times 170 = 340 \text{ mm}^2$

$d_{long} = 170 - 20 - 1.5 \times 12 = 132 \text{ mm}$

$d_{short} = 170 - 20 - \frac{12}{2} = 144 \text{ mm}$

In long direction $\rho_{min} = \frac{340}{132 \times 1000} = 0.0025$

In short direction $\rho_{min} = \frac{340}{144 \times 1000} = 0.0023$

$\rho_{max} = 0.0206$

$S_{max} = 2h = 2 \times 170 = 340 \text{ mm}$

Required (d) for flexure:

$$Mu = \phi \rho fy b d^2 \left(1 - 0.59 \times \rho \times \frac{fy}{f'c'}\right)$$

$$= 0.9 \times 0.0206 \times 414 \times 1000 \times d^2 \left(1 - 0.59 \times 0.0206 \times \frac{414}{276}\right)$$

In long direction

$$d = \sqrt{\frac{Mu}{6276}} = \sqrt{\frac{27 \times 10^6}{6276}} = 65.6 \text{ mm} < 132 \text{ mm o.k}$$

In short direction

$$d = \sqrt{\frac{22 \times 10^6}{6276}} = 59.2 \text{ mm} < 144 \text{ mm o.k}$$

$$\rho = \frac{1}{m} \left[1 - \sqrt{1 - \frac{2 \times Ru \times m}{fy}}\right]$$

$$m = \frac{fy}{0.85f'c'} = \frac{414}{0.85 \times 272} = 17.65$$
\[ Ru = \frac{Mu}{\Theta * b * d^2} = \frac{16 \times 10^6}{\Theta * 1000 * 132^2} = 1.0203 \]

\[ \rho = 0.0025 \]

\[ V_u \text{ from face of long beam} \]
\[ V_u = 16 \left( 3 - \frac{360}{2 \times 1000} - \frac{144}{1000} \right) = 42.8 \text{ kN} \]
\[ \phi V_c = 0.17 \times 0.75 \times \sqrt{27.6} \times 1000 \times 144 \times 10^{-3} = 96.45 \text{ kN} \rightarrow V_u < \phi V_c \text{ o.k} \]

<table>
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<tr>
<th>Location</th>
<th>M_n kN.m</th>
<th>b mm</th>
<th>d mm</th>
<th>( \frac{Mu}{b*1000} )</th>
<th>( \rho )</th>
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