1. WELL HYDRAULICS

1.1 Groundwater Flow through Porous Media

Flow along the three principal co-ordinate axes can be described as:

$$u = -K_x \frac{\partial h}{\partial x} \qquad \text{.....[1 a]}$$

$$v = -K_y \frac{\partial h}{\partial y} \qquad \text{.....[1 b]}$$

$$w = -K_z \frac{\partial h}{\partial z} \qquad \text{.....[1 c]}$$

u, *v*, and *w* are the velocity components in the *x*, *y*, and *z* directions respectively, and K_x, K_y, and K_z are the coefficients of permeability in these directions. When **Darcy's** law is substituted in the **continuity equation of motion**, one obtains the equation governing the flow of water through a porous medium. The resulting equations for confined and unconfined aquifers are, respectively, as follows:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad \text{.....[2]}$$

Above equation is known as ***Boussinesq's equation***, which represents three-dimensional unsteady groundwater flow. Here if *H* represents the hydraulic head in unconfined aquifer and *n* is the porosity of the soil is:

$$\frac{\partial^2 H^2}{\partial x^2} + \frac{\partial^2 H^2}{\partial y^2} = \frac{2n}{K}\frac{\partial H}{\partial t} \qquad \text{.....[3]}$$

This known as **Dupuit's** equation. Both these equations assume that the medium is homogeneous, isotropic, and water is incompressible.

1.2 Dupuit's Theory (Steady Radial Flow):

1.2.1 Equilibrium Equations for Unconfined Aquifer:

When a well penetrates in an extensive homogenous **Unconfined Aquifer** in which water table was initially horizontal, a circular cone of depression in the water table near the well takes place when water is pumped since no flow to the well takes place without a gradient towards the well. This depression is called **Cone of Depression**. The decrease in water level from h_e to h_w (as shown in Figure 1) is called drawdown or drop of piezometric head

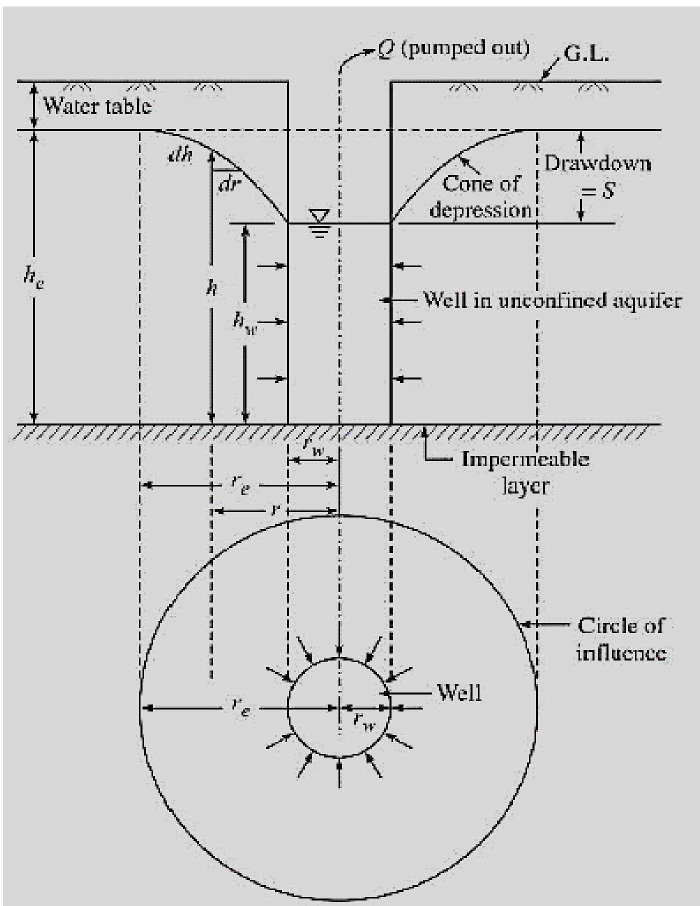

Figure 1: Well in unconfined aquifer with steady radial discharge

The analysis of such radial flow towards the well was originally proposed by Dupuit (1863) and subsequently modified by Thiem (1906). To derive an expression of the steady radial discharge Q.

At a distance r from the center of the well, let h be the height of water table. By continuity, amount of water Q flowing towards the well,

Q = area of flow at radius r x velocity at radius r

$$Q = (2\pi r h) \times \left(k \frac{dh}{dr} \right)$$

$$Q \frac{dr}{r} = 2\pi k h dh$$

By Integration:

$$Q \int \frac{dr}{r} = 2\pi k \int h \ dh$$

Boundary conditions of limits of integration are:

When $r = r_w$, $h = h_w$, $r = r_e$, $h = h_e$, where r_e is radius of influence as shown from Figure 1 above:

$$Q \int_{r_w}^{r_e} \frac{dr}{r} = 2\pi k \int_{h_w}^{h_e} h \ dh$$

$$Q \log_e \left(\frac{r_e}{r_w} \right) = \pi k [h_e^2 - h_w^2]$$

$$Q = \frac{\pi k [h_e^2 - h_w^2]}{\log_e \left[\frac{r_e}{r_w} \right]} \qquad .....[4]$$

Equation [4] is known as Dupuit's equation for steady radial flow to unconfined aquifer. It may be further be simplified as:

$$Q = \frac{\pi k (h_e + h_w)(h_e - h_w)}{\log_e \left[\frac{r_e}{r_w} \right]} \qquad .....[5]$$

For drawdown condition:

$S = h_e - h_w$

and $h_e + h_w = (h_e - h_w) + 2h_w = S + 2h_w$ sub. in Eq.[5] yields;

$$Q = \frac{\pi k S (S + 2h_w)}{\log_e \left(\frac{r_e}{r_w} \right)} \qquad .....[6]$$

<u>Note:</u> ***log_e*** is the natural logarithm.

1.2.2 Equilibrium Equations for Confined Aquifer:

The governing equation of flow, Eq.[2], can be written in polar cylindrical coordinates (r, θ, z) as:

$$\frac{1}{r}\left(\frac{\partial}{\partial r} r \frac{\partial h}{\partial r} \right) + \frac{\partial^2 h}{r^2 \partial \theta^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad .....[7]$$

If one assumes radial symmetry (*i.e.*, h is independent of θ) and the aquifer to be horizontal and of constant thickness (*i.e.*, h is independent of z), Eq.[7] reduces to:

$$\frac{d}{dr}\left(r\frac{dh}{dr}\right) = 0 \qquad \text{.....[8]}$$

For flow towards a well, penetrating the entire thickness of a horizontal confined aquifer, Eq. [9] needs to be solved for the following boundary conditions (see Figure 2):

(i) At $r = r_0$, $h = h_0$ (r_0 is the radius of influence)

(ii) At $r = r_w$, $h = h_w$ (r_w is the radius of well)

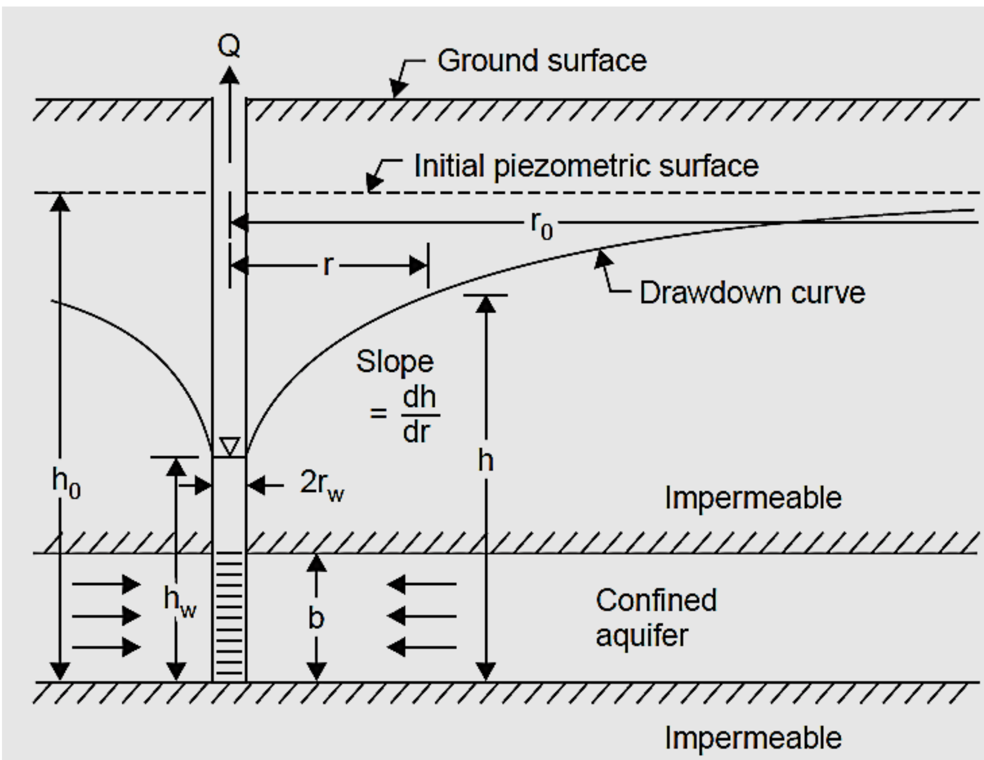

Figure 2: Radial flow to a well penetrating an extensive confined aquifer

By integrating Eq. [8] yields

$$r\frac{dh}{dr} = C_1$$

$$h = C_1 \ln r + C_2$$

In which C_1 and C_2 are constants of integration to be obtained by substituting the boundary conditions which yields

$$h_0 = C_1 \ln r_0 + C_2$$
$$h_w = C_1 \ln r_w + C_2$$

Hence,

$$C_1 = \frac{h_0 - h_w}{\ln (r_0/r_w)}$$

Also,

$$C_2 = h_w - \frac{h_0 - h_w}{\ln (r_0/r_w)} \ln r_w$$

Finally,

$$h = h_0 - \frac{h_0 - h_w}{\ln (r_0/r_w)} \ln (r_0/r) \qquad \text{.....[9 a]}$$

$$h = h_w + \frac{h_0 - h_w}{\ln (r_0/r_w)} \ln (r/r_w) \qquad \text{.....[9 b]}$$

Further, the discharge Q through any cylinder of radius r and height equal to the thickness of the aquifer B is expressed as

$$Q = -K(2\pi r B)\frac{dh}{dr}$$
$$= -2\pi T\left(r\frac{dh}{dr}\right)$$
$$= -2\pi T C_1$$

$$Q = -2\pi T \frac{h_0 - h_w}{\ln (r_0/r_w)} \qquad \text{.....[10]}$$

Thus, Eqs. [9 a] and [9 b] can be rewritten as

$$h = h_0 + \frac{Q}{2\pi T} \ln (r_0/r) \qquad .....[11]$$

$$h = h_w - \frac{Q}{2\pi T} \ln (r/r_w) \qquad .....[12]$$

It should be noted that the coordinate *r* is measured positive away from the well and that the discharge towards the well is in the negative direction of *r*. Therefore, for a discharging well, *Q* is substituted as a negative quantity in Eqs. [10] through [12]. If the drawdown at any radial distance *r* from the well is represented by *s*, then

$$s = h_0 - h = -\frac{Q}{2\pi T} \ln r_0/r \qquad .....[13]$$

and the well drawdown s_w is given as

$$s_w = h_0 - h_w = -\frac{Q}{2\pi T} \ln r_0/r_w \qquad .....[14]$$

Example [1]:

A well with a radius of 0.3 m, including gravel envelope and developed zone, completely penetrates an unconfined aquifer with K = 25 m/day and initial water table at 30 m above the bottom of the aquifer. The well is pumped so that the water level in the well remains at 22 m above the bottom of the aquifer. Assuming that pumping has essentially no effect on water table height at 300 m from the well, determine the steady-state well discharge. Neglect well losses.

Solution:

From Eq.[4]

$$Q = \frac{\pi K\left[{h_e}^2 - {h_w}^2\right]}{ln\left[\frac{r_0}{r_w}\right]}$$

$$Q = \frac{3.14 \times 25 \times [30^2 - 22^2]}{\ln\left[\frac{300}{0.3}\right]} = 4729.84 \text{ m}^3/\text{day}$$