

Forced-Convection Heat Transfer

Reference (J. P. Holman, **Heat Transfer, Tenth Edition**, McGraw-Hill Companies, Inc. 2010).

Ex//1) Air at 2 atm and 200°C is heated as it flows through a tube with a diameter of 1 in (2.54 cm) at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant-heat-flux condition is maintained at the wall and the wall temperature is 20°C above the air temperature, all along the length of the tube. How much would the bulk temperature increase over a 3-m length of the tube?

$$\rho = \frac{p}{RT} = \frac{(2)(1.0132 \times 10^5)}{(287)(473)} = 1.493 \text{ kg/m}^3 \quad [0.0932 \text{ lb}_m/\text{ft}^3]$$

$$\text{Pr} = 0.681$$

$$\mu = 2.57 \times 10^{-5} \text{ kg/m} \cdot \text{s} \quad [0.0622 \text{ lb}_m/\text{h} \cdot \text{ft}]$$

$$k = 0.0386 \text{ W/m} \cdot ^\circ\text{C} \quad [0.0223 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}]$$

$$c_p = 1.025 \text{ kJ/kg} \cdot ^\circ\text{C}$$

$$\text{Re}_d = \frac{\rho u_m d}{\mu} = \frac{(1.493)(10)(0.0254)}{2.57 \times 10^{-5}} = 14,756$$

so that the flow is turbulent. We therefore use Equation (6-4a) to calculate the heat-transfer coefficient.

$$\text{Nu}_d = \frac{hd}{k} = 0.023 \text{Re}_d^{0.8} \text{Pr}^{0.4} = (0.023)(14,756)^{0.8}(0.681)^{0.4} = 42.67$$

$$h = \frac{k}{d} \text{Nu}_d = \frac{(0.0386)(42.67)}{0.0254} = 64.85 \text{ W/m}^2 \cdot ^\circ\text{C} \quad [11.42 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}]$$

The heat flow per unit length is then

$$\frac{q}{L} = h\pi d(T_w - T_b) = (64.85)\pi(0.0254)(20) = 103.5 \text{ W/m} \quad [107.7 \text{ Btu/ft}]$$

$$q = \dot{m}c_p\Delta T_b = L\left(\frac{q}{L}\right)$$

We also have

$$\dot{m} = \rho u_m \frac{\pi d^2}{4} = (1.493)(10)\pi \frac{(0.0254)^2}{4}$$

$$= 7.565 \times 10^{-3} \text{ kg/s} \quad [0.0167 \text{ lb}_m/\text{s}]$$

so that we insert the numerical values in the energy balance to obtain

$$(7.565 \times 10^{-3})(1025)\Delta T_b = (3.0)(103.5)$$

and

$$\Delta T_b = 40.04^\circ\text{C} \quad [104.07^\circ\text{F}]$$

Ex//2) Water at 60°C enters a tube of 1-in (2.54-cm) diameter at a mean flow velocity of 2 cm/s. Calculate the exit water temperature if the tube is 3.0 m long and the wall temperature is constant at 80°C.

$$\begin{aligned}\rho &= 985 \text{ kg/m}^3 & c_p &= 4.18 \text{ kJ/kg} \cdot ^\circ\text{C} \\ \mu &= 4.71 \times 10^{-4} \text{ kg/m} \cdot \text{s} & & [1.139 \text{ lb}_m/\text{h} \cdot \text{ft}] \\ k &= 0.651 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 3.02 \\ \text{Re}_d &= \frac{\rho u_m d}{\mu} = \frac{(985)(0.02)(0.0254)}{4.71 \times 10^{-4}} = 1062\end{aligned}$$

so the flow is laminar. Calculating the additional parameter, we have

$$\text{Re}_d \text{Pr} \frac{d}{L} = \frac{(1062)(3.02)(0.0254)}{3} = 27.15 > 10$$

so Equation (6-10) is applicable. We do not yet know the mean bulk temperature to evaluate properties so we first make the calculation on the basis of 60°C, determine an exit bulk temperature, and then make a second iteration to obtain a more precise value. When inlet and outlet conditions are designated with the subscripts 1 and 2, respectively, the energy balance becomes

$$q = h\pi dL \left(T_w - \frac{T_{b1} + T_{b2}}{2} \right) = \dot{m} c_p (T_{b2} - T_{b1}) \quad [a]$$

At the wall temperature of 80°C we have

$$\mu_w = 3.55 \times 10^{-4} \text{ kg/m} \cdot \text{s}$$

From Equation (6-10)

$$\begin{aligned}\text{Nu}_d &= (1.86) \left[\frac{(1062)(3.02)(0.0254)}{3} \right]^{1/3} \left(\frac{4.71}{3.55} \right)^{0.14} = 5.816 \\ h &= \frac{k \text{Nu}_d}{d} = \frac{(0.651)(5.816)}{0.0254} = 149.1 \text{ W/m}^2 \cdot ^\circ\text{C} \quad [26.26 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}]\end{aligned}$$

The mass flow rate is

$$\dot{m} = \rho \frac{\pi d^2}{4} u_m = \frac{(985)\pi(0.0254)^2(0.02)}{4} = 9.982 \times 10^{-3} \text{ kg/s}$$

Inserting the value for h into Equation (a) along with \dot{m} and $T_{b1} = 60^\circ\text{C}$ and $T_w = 80^\circ\text{C}$ gives

$$(149.1)\pi(0.0254)(3.0) \left(80 - \frac{T_{b2} + 60}{2} \right) = (9.982 \times 10^{-3})(4180)(T_{b2} - 60) \quad [b]$$

This equation can be solved to give

$$T_{b2} = 71.98^\circ\text{C}$$

Thus, we should go back and evaluate properties at

$$T_{b,\text{mean}} = \frac{71.98 + 60}{2} = 66^\circ\text{C}$$

We obtain

$$\begin{aligned}\rho &= 982 \text{ kg/m}^3 & c_p &= 4185 \text{ J/kg} \cdot ^\circ\text{C} & \mu &= 4.36 \times 10^{-4} \text{ kg/m} \cdot \text{s} \\ k &= 0.656 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 2.78 \\ \text{Re}_d &= \frac{(1062)(4.71)}{4.36} = 1147 \\ \text{Re Pr} \frac{d}{L} &= \frac{(1147)(2.78)(0.0254)}{3} = 27.00 \\ \text{Nu}_d &= (1.86)(27.00)^{1/3} \left(\frac{4.36}{3.55} \right)^{0.14} = 5.743 \\ h &= \frac{(0.656)(5.743)}{0.0254} = 148.3 \text{ W/m}^2 \cdot ^\circ\text{C}\end{aligned}$$

We insert this value of h back into Equation (a) to obtain

$$T_{b2} = 71.88^\circ\text{C} \quad [161.4^\circ\text{F}]$$

H.W.

Q1//Air at 1 atm and 27°C enters a 5.0-mm-diameter smooth tube with a velocity of 3.0 m/s. The length of the tube is 10 cm. A constant heat flux is imposed on the tube wall. Calculate the heat transfer if the exit bulk temperature is 77°C. Also calculate the exit wall temperature and the value of h at exit.

Q2//A 2.0-cm-diameter tube having a relative roughness of 0.001 is maintained at a constant wall temperature of 90°C. Water enters the tube at 40°C and leaves at 60°C. If the entering velocity is 3 m/s, calculate the length of tube necessary to accomplish the heating.

Q3//Air at 300 K and 1 atm enters a smooth tube having a diameter of 2 cm and length of 10 cm. The air velocity is 40 m/s. What constant heat flux must be applied at the tube surface to result in an air temperature rise of 5°C? What average wall temperature would be necessary for this case?