Stepping Stone Method of Optimality Test

Once, we get the **basic feasible solution** for a transportation problem, the next duty is to test whether the solution we got is an **optimal solution** or not?

1. Steps to test **unused squares**;

2. Select an **unused square**,

3. Allocate **+** ① unit to **unused square** and locate **-** ① and **+** ① alternatively to corners of the selected closed path.

4. Calculate the **improvement index** .

5. If improvement index is **negative** allocate as **much** as you can to that unused square.

Repeat the allocation till the improvement index is ≥ **0** for all unused squares.

Factories \ Retail Agency	1	2	3	4	5
1	**50** 1 -1	9 +1	13	36	51
2	**50** 24 +1	**50** 12 -1	16	20	1
3	14	**10** 33	**50** 1	**50** 23	**40** 26

Improvement index **(1 – 2)** = 1*9-1*1+1*24-1*12= + 20 , this means if we allocate **(1 – 2)** +1 unit , then the transport cost will increase by +20.

Factories \ Retail Agency	1	2	3	4	5

1	**50** 1 -1	9	13 +1	36	51
2	**50** 24 +1	**50** 12 -1	16	20	1
3	14	**10** 33 +1	**50** 1 -1	**50** 23	**40** 26

Improvement index **(1 – 3)** = 1*13-1*1+1*24-1*12+1*33-1*1= + 56 , this means if we allocate **(1 – 3)** +1 unit , then the transport cost will increase by +56.

Retail Agency / Factories	1	2	3	4	5
1	**50** 1 -1	9	13	36 +1	51
2	**50** 24 +1	**50** 12 -1	16	20	1
3	14	**10** 33 +1	**50** 1	**50** 23 -1	**40** 26

Improvement index **(1– 4)** = 1*13-1*1+1*24-1*12+1*33-1*23= + 34 , this means if we allocate **(1 – 4)** +1 unit , then the transport cost will increase by +24.

And so on …

Improvement index **(1– 5)** = 1*51-1*1+1*24-1*12+1*33-1*26= +73, this means if we allocate **(1 – 5)** +1 unit , then the transport cost will increase by +73.

Improvemen]t index **(2– 3)** = 1*16-1*12+1*33-1*1= +36, this means if we allocate **(2 – 3)** +1 unit , then the transport cost will increase by +36.

Improvement index **(2– 4)** = 1*20-1*12+1*33-1*23=+18, this means if we allocate **(2 – 4)** +1 unit , then the transport cost will increase by +18.

Improvement index **(2– 5)** = 1*1-1*12+1*33-1*26= - 4, this means if we allocate **(2 – 5)** +1 unit , then the transport cost will **decrease by - 4**.

Since there is a decrease in the cost, we will allocate as much as we can to **(2 – 5)**. To further improve the current solution, select the "smallest" number found in the path (2-5 , 2-2 , 3-2 , 3-5) containing minus (-) signs. This number is added to all cells on the closed path with plus (+) signs, and subtracted from all cells on the path with minus (-) signs.

Retail Agency / Factories	1	2	3	4	5	**Capacity**
1	1 **50**	9	13	36	51	50
2	24 **50**	12 **10**	16	20	1 **40**	100
3	14	33 **50**	1 **50**	23 **50**	26	150
Requirement	100	60	50	50	40	

Z= 50 * 1 + 50 * 24 + 10 * 12 + 50 * 33 + 50 * 1 + 50 * 23 + 40 * 1 = 4260.

1	2	3	4	5
1 **50**	9	13	36	51

50 24 -1	**10** 12 +1	16	20	1 **40**
14 +1	**50** 33 -1	1 **50**	23 **50**	26

Improvement index **(3 – 1)** = 1*14-1*33+1*12-1*24= - 31, this means if we allocate **(3 – 1)** +1 unit , then the transport cost will **decrease by - 31**.

Since there is a decrease in the cost, so,

Retail Agency / Factories	1	2	3	4	5	**Capacity**
1	1 **50**	9	13	36	51	50
2	24	12 **60**	16	20	1 **40**	100
3	14 **50**	33	1 **50**	23 **50**	26	150
Requirement	100	60	50	50	40	

Z= 50 * 1 + 50 * 14 + 60 * 12 + 50 * 1 + 50 * 23 + 40 * 1 = 2710.

We will check the Improvement index in each cell again:

(1 – 2)	Not possible	
(1 – 3)	1*13-1*1+1*14-1*1	= +25
(1 – 4)	1*36-1*1+1*14-1*23	= +26
(1 – 5)	Not possible	

(2 – 1)	Not possible	
(2 – 3)	Not possible	
(2 – 4)	Not possible	
(3 – 2)	Not possible	
(3 – 5)	Not possible	

In the table above, no more negative improvement index, so the solution

Z = 2710 is **optimal**.

Example : According to the following table, Find the feasible solution by three methods (NWCM, LCM and VAM), then find the optimal solution.

To

From

	Albuquerque	Boston	Cleveland	**Capacity**
Des Moines	5	4	3	**100**
Evansville	8	4	3	**300**
Fort Lauderdale	9	7	5	**300**
Demand	**300**	**200**	**200**	**700**

Minimize $\mathbf{Z} = 5x_{11} + 4x_{12} + 3x_{13} + 8x_{21} + 4x_{22} + 3x_{23} + 9x_{31} + 7x_{32} + 5x_{33}$

Constraints: For **Supply** nodes;

$x_{11} + x_{12} + x_{13} \leq 100$

$x_{21} + x_{22} + x_{23} \leq 300$

$x_{31} + x_{32} + x_{33} \leq 300$

Supply **cannot be bigger** than capacity.

For **Demand** nodes;

$x_{11} + x_{21} + x_{31} \geq 300$

$x_{12} + x_{22} + x_{32} \geq 200$

$x_{13} + x_{23} + x_{33} \geq 200$

Demand cann**ot be less** than the required.

all x_{ij} and $x_{ji} \geq 0$

Solution:

1) Initial Solution with NWCM

	Albuquerque	Boston	Cleveland	Capacity

Des Moines	5	4	3	100
Evansville	8	4	3	300
Fort Lauderdale	9	7	5	300
Demand	300	200	200	700

	Albuquerque	Boston	Cleveland	Capacity
Des Moines	100 (5)	4	3	**100**
Evansville	200 (8)	100 (4)	3	**300**
Fort Lauderdale	9	100 (7)	200 (5)	**300**
Demand	**300**	**200**	**200**	**700**

Z=100×5+200×8+100×4+100×7+200×5=500+1600+400+700+1000=**4200 $**

2) Initial Solution with LCM

	Albuquerque	Boston	Cleveland	Capacity
Des Moines	5	4	100 (3)	**100**
Evansville	8	200 (4)	100 (3)	**300**
Fort Lauderdale	300 (9)	7	5	**300**
Demand	**300**	**200**	**200**	**700**

Z=300×9+200×4+100×3+100×3=2700+800+300+300=**4100 $**

3) Initial Solution with VAM

	3	0	0		
	Albuquerque	Boston	Cleveland	**Capacity**	
Des Moines	5	4	3	**100**	**1**
Evansville	8	4	3	**300**	**1**
Fort Lauderdale	9	7	5	**300**	**2**
Demand	**300**	**200**	**200**	**700**	

	~~3~~ 1	0	0		
	Albuquerque	Boston	Cleveland	**Capacity**	
Des Moines	(100) 5	4	3	**100**	**1**
Evansville	8	4	3	**300**	**1**
Fort Lauderdale	9	7	5	**300**	**2**
Demand	**300**	**200**	**200**	**700**	

	1	~~0~~ 3	~~0~~ 2		
	Albuquerque	Boston	Cleveland	**Capacity**	
Des Moines	(100) 5	4 X	3 X	**100**	~~1~~ **0**
Evansville	8	(200) 4	3	**300**	**1**
Fort Lauderdale	9	7 X	5	**300**	**2**
Demand	**300**	**200**	**200**	**700**	

	~~1~~ 0	~~3~~ 0	~~2~~ 0		
	Albuquerque	Boston	Cleveland	**Capacity**	
Des Moines	(100) [5]	X [4]	X [3]	**100**	**0**
Evansville	X [8]	(200) [4]	(100) [3]	**300**	**0** ~~5~~
Fort Lauderdale	[9]	X [7]	[5]	**300**	**4**
Demand	**300**	**200**	**200**	**700**	

	0	0	0		
	Albuquerque	Boston	Cleveland	**Capacity**	
Des Moines	(100) [5]	X [4]	X [3]	**100**	**0**
Evansville	X [8]	(200) [4]	(100) [3]	**300**	**0**
Fort Lauderdale	(200) [9]	X [7]	(100) [5]	**300**	**0**
Demand	**300**	**200**	**200**	**700**	

Z= 100×5+200×4+100×3+200×9+100×5=500+800+300+1800+500=**3900 $**

Stepping Stone Method of Optimality Test

	Albuquerque	Boston	Cleveland
Des Moines	100 [5] -1	[4] +1	[3]
Evansville	200 [8] +1	100 [4] -1	[3]
Fort Lauderdale	[9]	100 [7]	200 [5]

Improvement index=I_{DB}=+4-5+8-4=+3 , this means, if we allocate to **Des Moines-Boston +1** unit the transport cost will increase by **+3**.

	Albuquerque	Boston	Cleveland
Des Moines	100 [5] -1	[4]	[3] +1
Evansville	200 [8] +1	100 [4] -1	[3]
Fort Lauderdale	[9]	100 [7] +1	200 [5] -1

Improvement index=I_{DC}=+3-5+7-4+8-5=+4 , this means, if we allocate to **Des Moines- Cleveland +1** unit the transport cost will increase by **+4**.

	Albuquerque	Boston	Cleveland
Des Moines	100 [5]	[4]	[3]
Evansville	200 [8]	100 [4] -1	[3] +1
Fort Lauderdale	[9]	100 [7] +1	200 [5] -1

Improvement index=I_{EC}=+3-5+7-4=+1 , this means, if we allocate to **Evansville - Cleveland +1** unit the transport cost will increase by **+1**.

	Albuquerque	Boston	Cleveland
Des Moines	100 [5]	[4]	[3]
Evansville	200 [8] -1	100 [4] +1	[3]
Fort Lauderdale	[9] +1	100 [7] -1	200 [5]

Improvement index=I_{FA}=+9-8+4-7=-2 , this means, if we allocate to **Fort Lauderdale - Albuquerque +1** unit the transport cost will decrease by **-2**.

Since there is a decrease in the cost, we will allocate as much as we can to **Fort Lauderdale – Albuquerque**. The amount is the **minimum of the numbers that we are assigning -①** in the cycle.

	Albuquerque	Boston	Cleveland	**Capacity**
Des Moines	100 [5]	[4]	[3]	**100**
Evansville	**100** [8]	**200** [4]	[3]	**300**
Fort Lauderdale	**+100** [9]	**0** [7]	200 [5]	**300**
Demand	**300**	**200**	**200**	**700**

Z=100×5 + 100×8+200×4+100×9+200×5=500+800+800+900+1000=**4000 $**

The solution obtained may or may not be optimal. To check we return to first step to check the unused squares,

D to B	I_{DB}	+DB-DA+AE-EB	+4-5+8-4	+3
D to C	I_{DC}	+DC-DA+FA-FC	+3-5+9-5	+2
E to C	I_{EC}	+EC-EA+FA-FC	+3-8+9-5	**-1**
F to B	I_{FB}	+FB-EB+EA-FA	+7-4+8-9	+2

An improvement can be done by allocating to **EC**, since the minimum number in the square is 100; we allocate 100 to **EC** square.

	Albuquerque	Boston	Cleveland	**Capacity**
Des Moines	100 [5]	[4]	[3]	**100**
Evansville	**0** [8]	**200** [4]	**+100** [3]	**300**
Fort Lauderdale	**+200** [9]	**0** [7]	100 [5]	**300**
Demand	**300**	**200**	**200**	**700**

Z=100×5 + 200×9+200×4+100×3+100×5=500+1800+800+300+500=**3900 $**

We will check the Improvement index in each cell again.

D to B	I_{DB}	+DB-DA+FA-FC+EC-EB	+4-5+9-5+3-4	+2
D to C	I_{DC}	+DC-DA+FA-FC	+3-5+9-5	+2
E to A	I_{EC}	+EA-FA+FC-EC	+8-9+5-3	+1
F to B	I_{FB}	+FB-EB+EC-FC	+7-4+3-5	+1

In this table, no more negative improvement index, so the solution is **optimal**.

Exercise:

1) A company has three factories X, Y, and Z and three warehouses A, B, and C. It is required to schedule factory production and shipments from factories to warehouses in such a manner so as to minimize total cost of shipment and production. Unit variable manufacturing cost (UVMC) and factory capacities and warehouse requirements are given below:

From / To	A	B	C	**Capacity**
X	10	4	11	**70**
Y	12	5	8	**50**
Z	9	7	6	**30**
Demand	**40**	**50**	**60**	**150**

a) Load the table with **North-west method**.

b) Load the table with **least cost method**.

c) Load the table with **Vogel's approximation method**, (VAM).

d) Solve the question by **stepping stone algorithm**.

2) The demand and capacity are given

From / To	Sarajevo	Travnik	Bihac	**Capacity**
Mostar	5	7	15	**120**
Zenica	4	2	8	**200**
Tuzla	6	3	10	**150**
Demand	**210**	**160**	**100**	**470**

a) Load the table with **North-west method**.

b) Load the table with **least cost method**.

c) Load the table with **Vogel's approximation method**, (VAM).

d) Solve the question by **stepping stone algorithm**.

1	1	2	
5	7	15	**120** 2
4	2	8	**200** 2
6	3	10	**150** 3
210	**160**	**100**	**470**

1 2	**1 5**	**2**	

5 120	7 X	15 X	**120 2 10**
4 90	2 10	8 100	**200 2 4**
6 X	3 150	10 X	**150 3**
210	**160**	**100**	**470**

Z= 120*5+90*4+10*2+100*8+150*3= 2230

5 120 -1	7 +1	15	**120**
4 90 +1	2 -1 10	8 100	**200**
6	3 150	10	**150**
210	**160**	**100**	**470**

(1,2) = 1*7-1*5+1*4-1*2 = +4

(1,3) = 15-5+4-8 = +6

(3,1) = 6-3+2-4 = +1

(3,3) = 10-8+2-3 = +1

So, the optimal solution: Z= 2230 .