The standard form

The standard form of the L.P model is used to develop the general solution method for solving any linear programming problem.

The properties of the standard L.P form are :-

1) All the constrain are equations with non-negative right hand side.
2) All the decision variable are non-negative.
3) The objective function may be maximization or minimization.

The inequality constraint of the type $\leq$ or $\geq$ can be change to equality (=) constraint by adding a slack variable to (or subtracting) a surplus variable from the left side of the constraint.

Example :- in the constraints :- $2X_1 + 5X_2 \geq 6$

$$2X_1 + 5X_2 \leq 6$$

are equivalent to :- $2X_1 + 5X_2 - S_1 = 6$

$$2X_1 + 5X_2 + S_1 = 6$$

respectively .

Example :- write the following L.P model in the standard from :-

$$\min \ Z = 4X_1 + 3X_2$$

Subject to :- $2X_1 + 4X_2 = 18$

$$X_1 - 3X_2 \leq -5$$

$$-5X_1 + 10X_2 \leq 12$$

Where X_1 unconstrained and $X_2 \geq 0$

Solution :- standard from :-

Let $X_1 = X_1' - X_1''$

Min $Z = 4X_1' - 4X_1'' + 3X_2$

Subject to :-

1) $2X_1' - 2X_1'' + 4X_2 = 8$

2) $-X_1 + 3X_2 \geq 5$

$-X_1' + X_1'' + 3X_2 - S_2 = 5$

3) $-5X_1' + 5X_1'' + 10X_2 + S_3 = 12$

$X_1', X_1'', X_2, S_2, S_3 \geq 0$

Min $Z = 4X_1' - 4X_1'' + 3X2$

Subject to :-

$2X_1' - 2X_1'' + 4X_2 = 8$

$-X_1' + X_1'' + 3X_2 - S_2 = 5$

$-5X_1' + 5X_1'' + 10X_2 + S_3 = 12$

$X_1', X_1'', X_2, S_2, S_3 \geq 0.$

Example:-

Express the following L.P model in the standard form :-

Max. $Z = 3X_1 + X_2 + 2X_3$

Subject to :- $2X_1 - 3X_2 \leq 7$

$4X_1 + X_2 + 2X_3 \geq 3$

$$X_1 + 3X_3 \leq 5$$

$$X_1 \geq 0, \; X_2 \geq 0.$$

Solution :- Standard form :-

Let $X_3 = X_3' - X_3''$

$$\text{Max. } Z = 3X_1 + X_2 + 2X_3' - 2X_3''$$

Subject to :-

$$2X_1 - 3X_2 + S_1 = 7$$

$$4X_1 + X_2 + 2X_3' - 2X_3'' - S_2 = 3$$

$$X_1 + 3X_3' - 3X_3'' + S_3 = 5$$

$$X_1, X_2, X_3', X_3'', S_1, S_2, S_3 \geq 0.$$

Example :- Max. $Z = X_1 - 2X_2 + 4X_3$

Subject to :-

$$2X_1 + X_2 + 3X_3 \leq 6$$

$$X_1 + X_3 \geq 3$$

$$3X_1 - 2X_2 - X_3 = -4$$

$$X_2, X_3 \geq 0$$

Solution :- standard from :-

Let $X_1 = X_1' - X_1''$

$$\text{Max. } Z = X_1 - X_1 - 2X_2 + 4X_3$$

Subject to :-

$$2X_1' - 2X_1'' + X_2 + 3X_3 + S_1 = 6$$

$$X_1' - X_1'' + X_3 - S_2 = 3$$

$$-3X_1' - 3X_1'' + 2X_2 + X_3 = 4$$

$$X_1' - X_1'', X_2, X_3, S_1, S_2 \geq 0$$

Example:

Maximize $Z = 120x_1 + 150x_2$

Subject to

$$2x_1 + 3x_2 \leq 30$$

$$3x_1 + 2x_2 \leq 16$$

$$16x_1 + 4x_2 \leq 64$$

$$x_1, x_2 \geq 0$$

Solution:

$Max\ or\ Min\ z = c^T x$

Subject to $A\vec{x}\ (\leq\ , \geq\ , =)\ \vec{b}$

$\vec{x} \geq 0$

$$A = \begin{pmatrix} 2 & 3 \\ 3 & 2 \\ 16 & 4 \end{pmatrix} \quad b = \begin{pmatrix} 30 \\ 16 \\ 64 \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad c = \begin{pmatrix} 120 \\ 150 \end{pmatrix}$$

Example:

Convert the linear programming problem to **standard form**:

Max $Z = 120x_1 + 150x_2$

Subject to

$$2x_1 + 3x_2 \leq 30$$

$$2x_1 + 2x_2 \leq 16$$

$$16x_1 + 4x_2 \leq 64$$

$$x_1,\ x_2 \geq 0$$

Solution

We must first convert the inequalities to equalities by the introduction of slack variable $S_1 \geq 0$, $S_2 \geq 0$ and $S_3 \geq 0$ to give:

The objective function thus:

$$\text{Max} \quad Z = 120x_1 + 150x_2 + 0s_1 + 0s_2 + 0s_3$$

S.t

$$2x_1 + 3x_2 + s_1 = 30$$
$$2x_1 + 2x_2 + s_2 = 16$$
$$16x_1 + 4x_2 + s_3 = 64$$
$$x_1, x_2, s_1, s_2, s_3 \geq 0$$
$$b_1 = 30 > 0, \quad b_2 = 16 > 0, \quad b_3 = 64 > 0$$

Thus our linear programming model in **matrix standard form** is
Where:

$$A = \begin{pmatrix} 2 & 3 & 1 & 0 & 0 \\ 2 & 2 & 0 & 1 & 0 \\ 16 & 4 & 0 & 0 & 1 \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} \geq 0, \quad \vec{c} = \begin{pmatrix} 120 \\ 150 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } \vec{b} = \begin{pmatrix} 30 \\ 16 \\ 64 \end{pmatrix} > 0$$

Example:

Convert the linear programming model, into **matrix standard form**.

Minimize $Z = 300x_1 + 500x_2$

Subject to

$$20x_1 + 40x_2 \geq 1000$$
$$25x_1 + 20x_2 \geq 800$$
$$x_1, x_2 \geq 0$$

Solution:

First we must turn the minimization model to a maximization model by simply changing sign

Maximize $Z' = -Z = -300x_1 - 500x_2$.

Next we turn the inequalities into equalities by **surplus variables**:

$S_1 \geq 0$, $S_2 \geq 0$ and to give:

The objective function:

$$\text{Max} \quad z' = -300x_1 - 500x_2 + 0s_1 + 0s_2$$

S.t

$$20x_1 + 40x_2 - S_1 = 1000$$
$$25x_1 + 20x_2 - S_2 = 800$$
$$x_1, x_2, s_1, s_2 \geq 0$$
$$b_2 = 800 > 0 . b_1 = 1000 > 0$$

Matrix standard form is:

$$A = \begin{pmatrix} 20 & 40 & -1 & 0 \\ 25 & 20 & 0 & -1 \end{pmatrix}, \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{pmatrix} \geq 0, \vec{b} = \begin{pmatrix} 1000 \\ 800 \end{pmatrix} \text{and } \vec{c} = \begin{pmatrix} -300 \\ -500 \\ 0 \\ 0 \end{pmatrix}.$$