General L.P model

Max (min) $Z = \sum_{j=1}^{n} c, xj$

Subject to $\sum aj, xj$ (≤ , = ,≥) bi   i =1,2,….., m

$x_j \geq 0$, j $=$ 1,2, … … , n.

When $c_j = (c_1, \dots, c_n)_{m \times n}$ raw vector.

$$a_{ij} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

$$X_j = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad b_j = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}.$$

There are two forms can be used before solving the L.P model which are :-

1) Canonical form.
2) Standard form.

The characteristics of the canonical form :-

1) All decision variables are non-negative.
2) All constraint are of the (≤) type.
3) Objective function is of the maximization on type.

Any L.P model can be put in the canonical form by the use of some term information:-

1) The minimization of at a function is equivalent to the maximization of the negative of it.

Example :-

Min $Z = 5X_1 + 3X_2$ is equivalent to max (-Z) = $-[5X_1 + 3X_2]$

$= -5X_1 - 3X_2$

$-1\times \{ \max(-7) = -[5X_1 + 3X_2]\}$

$[-\max(-7) = 5X_1 + 3X_2]$

$-\max(z) = \min z$.

2) The direction of an in quality is reverse when both sides are multiplied by -1 :-

Example:- $(3X_1 - 5X_2 \geq 5)\ (-1)$ equivalent to

$-3X_1 + 5X_2 \leq -5$

3) The right side of an equation can always be made nor-negative by multiplying both side by -1 :-

Example : $3X_1 + 2X_2 = -10$ is equivalent to $-3X_1 - 2X_2 = 10$

4) The variable X (unrestricted in sign) can be expressed as the different between two non-negative.

Example : if X is a negative variable. the $X = \bar{X} - \bar{\bar{X}}$ where $\bar{X} \geq 0$, $\bar{\bar{X}} \geq 0$.

Example:

Convert the following linear programming model into **matrix canonical form**.

$$\text{Maximize } Z = 30x_1 + 20x_2$$

Subject to

$$5x_1 + x_2 \geq 60$$
$$4x_1 + 3x_2 \leq 20$$
$$x_1, x_2 \geq 0$$

Solution:

We must change the inequality $\geq$ into inequality $\leq$ by multiplying it by -1, we get $-5x_1 - x_2 \leq -60$

$$A = \begin{pmatrix} -5 & -1 \\ 4 & 3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -60 \\ 20 \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad c = \begin{pmatrix} 30 \\ 20 \end{pmatrix}$$

Example :

Convert the linear programming problem to the **matrix standard form**

$$\text{Max} \quad Z = 120x_1 + 150x_2$$

Subject to

$$2x_1 + 3x_2 \leq 30$$
$$2x_1 + 2x_2 \leq 16$$
$$16x_1 + 4x_2 \leq 64$$
$$x_1, \; x_2 \geq 0$$

Solution

We must first convert the inequalities to equalities by the introduction of slack variable $S_1 \geq 0$, $S_2 \geq 0$ and $S_3 \geq 0$ to give:

The objective function thus:

$$\text{Max} \quad Z = 120x_1 + 150x_2 + 0s_1 + 0s_2 + 0s_3$$

S.t

$$2x_1 + 3x_2 + s_1 = 30$$

$$2x_1 + 2x_2 + s_2 = 16$$

$$16x_1 + 4x_2 + s_3 = 64$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$

$$b_1 = 30 > 0, \quad b_2 = 16 > 0, \quad b_3 = 64 > 0$$

Thus our linear programming model in **matrix standard form** is

$$A = \begin{pmatrix} 2 & 3 & 1 & 0 & 0 \\ 2 & 2 & 0 & 1 & 0 \\ 16 & 4 & 0 & 0 & 1 \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} \geq 0, \quad \vec{c} = \begin{pmatrix} 120 \\ 150 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } \vec{b} = \begin{pmatrix} 30 \\ 16 \\ 64 \end{pmatrix} > 0$$

Example (2) :

Convert the following linear programming model into matrix **canonical form**.

$$\text{Maximize } Z = 30x_1 + 20x_2$$

Subject to

$$5x_1 + x_2 \geq 60$$

$$4x_1 + 3x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

Solution:

We must change the inequality $\geq$ into inequality $\leq$ by multiplying it by -1, we get $-5x_1 - x_2 \leq -60$

$$A = \begin{pmatrix} -5 & -1 \\ 4 & 3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -60 \\ 20 \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad c = \begin{pmatrix} 30 \\ 20 \end{pmatrix}$$

Example (3):

Convert the following linear programming model, into matrix **canonical form**

$$\text{Minimize} \quad Z = 300x_1 + 500x_2$$

Subject to

$$20x_1 + 40x_2 \geq 1000$$
$$25x_1 + 20x_2 \leq 800$$
$$x_1, x_2 \geq 0$$

Solution

we must change the inequality $\leq$ to $\geq$ by multiplying them by -1 to get

$$-25\,x_1 - 20x_2 \geq -800$$

$$A = \begin{pmatrix} 20 & 40 \\ -25 & -20 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1000 \\ -800 \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad c = \begin{pmatrix} 300 \\ 500 \end{pmatrix}$$