Introduction to Ordinary Differential Equations

1-Classification of (ODE)

A ***differential equation*** is an equation involving one dependent variable and its derivatives with respective to one or more independent variables.

Examples :

1. $\frac{dy}{dx} = 5x + 3$ (ode)

2. $e^{y} \ \frac{d^2y}{dx^2} + 2\,(\frac{dy}{dx})^2 = 1$ (ode)

3. $4\,\frac{d^3y}{dx^3} + (\sin x)\ \frac{d^2y}{dx^2} + 5xy = 0$ (ode)

4. $(\frac{d^2y}{dx^2})^3 + 3y(\frac{dy}{dx})^7 + y^3(\frac{dy}{dx})^2 = 5x$ (ode)

5. $\frac{\partial^2 y}{\partial t^2} - 4\,\frac{\partial^2 y}{\partial x^2} = 0$ (pde)

6. $\frac{dy}{dx} = \frac{x^2}{y^2 cosy}$ (ode)

7. $\frac{dy}{dx} + \frac{d^2y}{dx^2} = y + x^2$ (ode)

8. $(y-1)dx + x\cos(y)\,dy = 1$ (ode)

9. $x^2y'' + x\,y' + (x^2 - n^2)y = 0$ (ode)(Bessel's eq)

10. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ (ode) (pde) (wave eq)

11. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ (ode) (pde) (heat eq)

A differential equation is an ordinary differential equation if the unknown function depends on only one independent variable .

If the unknown function depends on two or more independent variables , the diff. eq . is a partial differential equation .

If the equation contains partial derivatives of one or more dependent variables ,then the equation is called a partial differential equation.

Examples

1. u $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}$

2. $u\, u_x$ + u = u_{yy}

3. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

4. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

The first level of classification, distinguishing ordinary and partial diff. eq's . was discussed above. We extend this classification system with the following definition.

Def : The highest derivative in the differential equation is called the order of the equation .

Example : Determine the order of each of the following diff. eq's

1. $\frac{dy}{dx} = \frac{x^2}{y^2 cosy}$ (first order- first degree)

2. $u_{xx} + u_{yy} = 0$ (second order -first degree)

3. $(y-1)dx + x\cos(y)dy = 1$ (first order- first degree)
4. $(\frac{dy}{dx})^4 = y + x$ (first order- fourth degree)

5. $y^3 + \frac{dy}{dx} = 1$ (first order- first degree)

- The next level of classification is based on the following definition :
- The degree of a differential equation is the highest exponent of highest order derivative which occurs in it , after the equation is converted to the form free from radical and fractions . for example eq .

$$\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = k$$

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$$\left(1+\left(\frac{dy}{dx}\right)^2\right)^3 = k^2 \left(\frac{d^2y}{dx^2}\right)^2$$

Second order and second degree

Def : A linear ordinary diff . eq (of order n) is of the form :

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots\cdots + a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

Where the function $a_j(x)$, j = 0,1, ……, n and f(x) are given .

If the equation does not meet the requirements of this equation, then the eq is said to be nonlinear.

Example : Determined which of the following differential equation are linear

1. $\frac{dy}{dx} = x^3$ (linear)

2. $\frac{\partial^2 u}{\partial x^2} + u = e^x$ (linear)

3. $(y-1)dx + x\cos(y)\,dy = 1$ (non-linear)
4. $\frac{d^3y}{dx^3} + y\frac{dy}{dx} = x$ (non-linear)

5. $\frac{dy}{dx} + x^2 y = x$ (linear)

6. $\frac{d^2x}{dt^2} + \sin x = 0$ (non-linear), (pendulum eq)