LINEAR MODULATION

1. AMPLTUDE MODULATION

The transmission of an information-bearing signal (or the message signal) over a bandpass communication channel, such as a telephone line or a satellite channel, usually requires a shift of range of frequencies contained in the signal to another frequency range suitable for transmission. A shift in the signal frequency range is accomplished by modulation. *Modulation* is defined as the process by which some characteristic of a carrier signal is varied in accordance with a modulating signal. Here the message signal is referred to as the *modulating signal,* and the result of modulation is referred to as the *modulated signal*.

In continuous-wave modulation, a sinusoidal signal $A_c\ cos\ (\omega_c t + \phi)$ is used as a *carrier signal.* Then a general modulated carrier signal can be represented mathematically as

$$x_c(t)=A(t)\ cos\ [\omega_c t + \phi(t)] \quad \omega_c=2\pi f_c \tag{1.1}$$

In Eq. (1.1), ω_c is known as the *carrier frequency.* And $A(t)$ and $\phi(t)$ are called the *instantaneous amplitude* and *phase angle* of the carrier, respectively. When $A(t)$ is linearly related to the message signal $m(t)$, the result is *amplitude modulation.* If $\phi(t)$ or its derivative is linearly related to $m(t)$, then we have *phase* or *frequency modulation.*

2. DOUBLE-SlDEBAND MODULATION

DSB modulation results when $A(t)$ is proportional to the message signal $m(t)$, that is,

$$x_{DSB}(t) = m(t)\ cos(\omega_c t) \tag{2.1}$$

where we assumed that the constant of proportionality is 1. Equation (2.1) indicates that *DSB* modulation is simply the multiplication of a carrier, $cos(\omega_c t)$, by the message signal $m(t)$. By application of the modulation theorem, the spectrum of a *DSB* signal is given by

$$X_{DSB}(\omega) = \tfrac{1}{2}M(\omega - \omega_c) + \tfrac{1}{2}M(\omega + \omega_c) \tag{2.2}$$

2.1. Generation of *DSB* Signals

The process of *DSB* modulation is illustrated in Fig. (2.1a).The time-domain waveforms are shown in Fig.(2.l b and c) for an assumed message signal. The frequency-domain representations of $m(t)$ and $x_{DSB}(t)$ are shown in Fig.(2.ld) and (e) for an assumed $M(\omega)$ having bandwidth ω_M. The spectra $M(\omega-\omega_c)$ and $M(\omega-\omega_c)$ are the message spectrum translated to $\omega=\omega_c$ and $\omega= -\omega_c$ respectively. The part of the spectrum that lies above ω_c is called the *upper sideband,* and the part below ω_c is called the *lower sideband.* The spectral range occupied by the message signal is called the *baseband,* and thus the message signal is often referred to as the *baseband signal.* As seen Fig.(2.1e), the spectrum of $x_{DSB}(t)$ has no identifiable carrier in it. Thus, this type of modulation is also known as *double-sideband suppressed-carrier* (*DSB.SC*) modulation. The carrier frequency ω_c is normally much higher than the bandwidth ω_M, of the message signal $m(t)$; that is, $\omega_c >> \omega_M$.

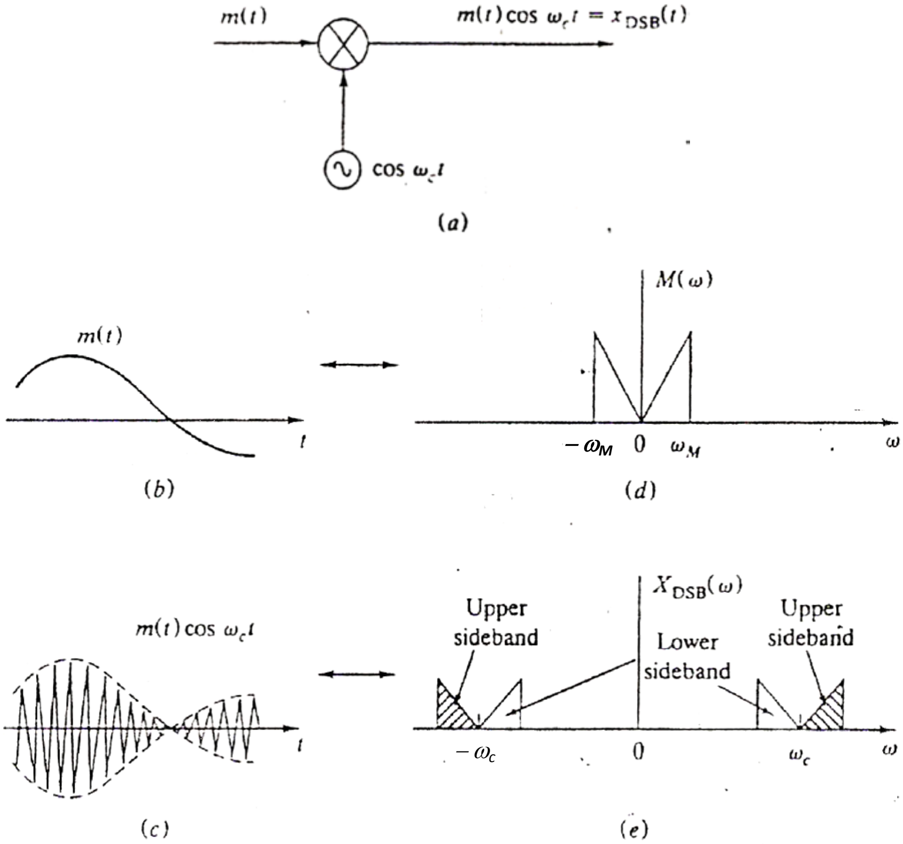


Fig.(2.1)

<u>2.2 Demodulation of *DSB* Signals</u>

Recovery of the message signal from the modulated signal is called demodulation, or detection. The message signal $m(t)$ can be recovered from the modulated signal $x_{DSB}(t)$ by multiplying $x_{DSB}(t)$ by a local carrier and using a low-pass filter (*LPF*) on the product signal, as shown in Fig.(2.2)

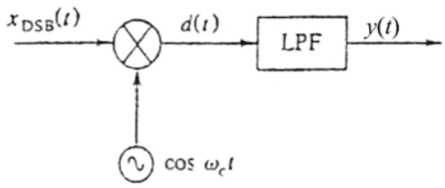


Fig.(2.2)

the output of the multiplier is

$$d(t) = x_{DSB}(t) \cos \omega_c t = [m(t) \cos \omega_c t] \cos \omega_c t$$
$$= m(t) \cos^2 \omega_c t$$
$$= \tfrac{1}{2} m(t) + \tfrac{1}{2} m(t) \cos 2\omega_c t,$$

After low-pass filtering of $d(t)$, we obtain

$$y(t) = \tfrac{1}{2} m(t)$$

Thus by proper amplification (multiplying by 2) we can recover the message, signal $m(t)$. Demodulation of $x_{DSB}(t)$ by the process shown in Fig.(2.2) in frequency domain is illustrated in Fig. (2.3).

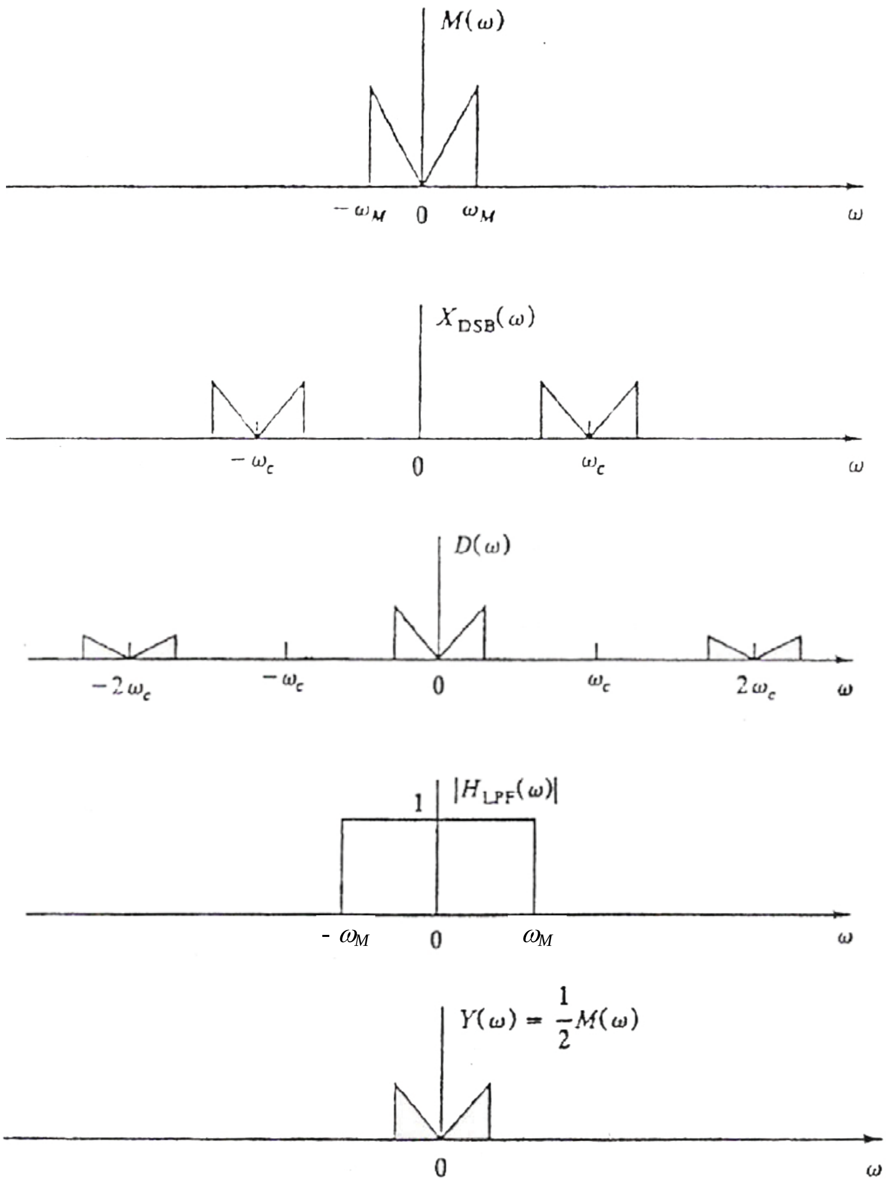


Fig. (2.3)

The basic difficulty associated with the *DSB* modulation is that for demodulation. The receiver must generate a local carrier which is in phase and frequency synchronism with the incoming carrier. This type of demodulation is known as *synchronous demodulation* or *coherent detection*.

Let us evaluate the effect of a phase error in the local oscillator on synchronous *DSB* demodulation shown in Fig. (2.2).

Let the phase error of the local oscillator be ϕ. Then the local carrier is expressed as $cos(\omega_c t + \phi)$. Now

$$x_{DSB}(t) = m(t)\, cos\,\omega_c t$$

and

$$d(t) = [m(t)\, cos\ \omega_c t]\, cos\,(\omega_c t + \phi)$$

$$= \tfrac{1}{2}\, m(t)[cos\ \phi + cos\,(2\omega_c t + \phi)]$$

$$= \tfrac{1}{2}\, m(t)\, cos\ \phi + \tfrac{1}{2}\, m(t)\, cos\,(2\omega_c t + \phi).$$

The second term on the right-hand side is filtered out by the low-pass filter, and we obtain

$$y(t) = \tfrac{1}{2}\, m(t)\, cos\ \phi$$

this output is proportional to $m(t)$ when ϕ is constant. The output is completely lost when $\phi = \pm\ \pi/2$. Thus, the phase error in the local carrier causes attenuation of the output signal without any distortion as long as ϕ is constant and not equal to $\pm\pi/2$. If the phase error ϕ varies randomly with time, then the output also will vary randomly and is undesirable.

To evaluate the effect of a frequency error in the local oscillator on synchronous *DSB* demodulation, let the frequency error of the local oscillator be $\Delta\omega$.The local carrier is then expressed as $cos(\omega_c + \Delta\omega)t$. Then

$$d(t) = m(t)\, cos\ \omega_c t\ cos(\omega_c + \Delta\omega)t$$

$$= \tfrac{1}{2}\, m(t)\, cos(\Delta\omega)t + \tfrac{1}{2}\, m(t)\, cos(2\omega_c + \Delta\omega)t$$

Thus $$y(t) = \tfrac{1}{2}\, m(t)\, cos(\Delta\omega)t$$

The output is the signal $m(t)$ multiplied by a low-frequency sinusoid. This is a “beating” effect and is a very undesirable distortion.

3. ORDINARY AMPLITUDE MODULATION

An ordinary amplitude-modulated signal is generated by adding a large carrier signal to the *DSB* signal. The ordinary *DSB-LC* signal (or simply *AM* signal) has the form

$$x_{AM}(t) = m(t)\cos\omega_c t + A\cos\omega_c t = [A + m(t)]\cos\omega_c t \qquad (3.1)$$

The spectrum of $x_{AM}(t)$ is given by

$$X_{AM}(\omega) = \frac{1}{2}M(\omega - \omega_c) + \frac{1}{2}M(\omega + \omega_c) + \pi A[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] \quad (3.2)$$

An example of an *AM* signal, in both time domain and frequency domain, is shown in Fig. (3.1).

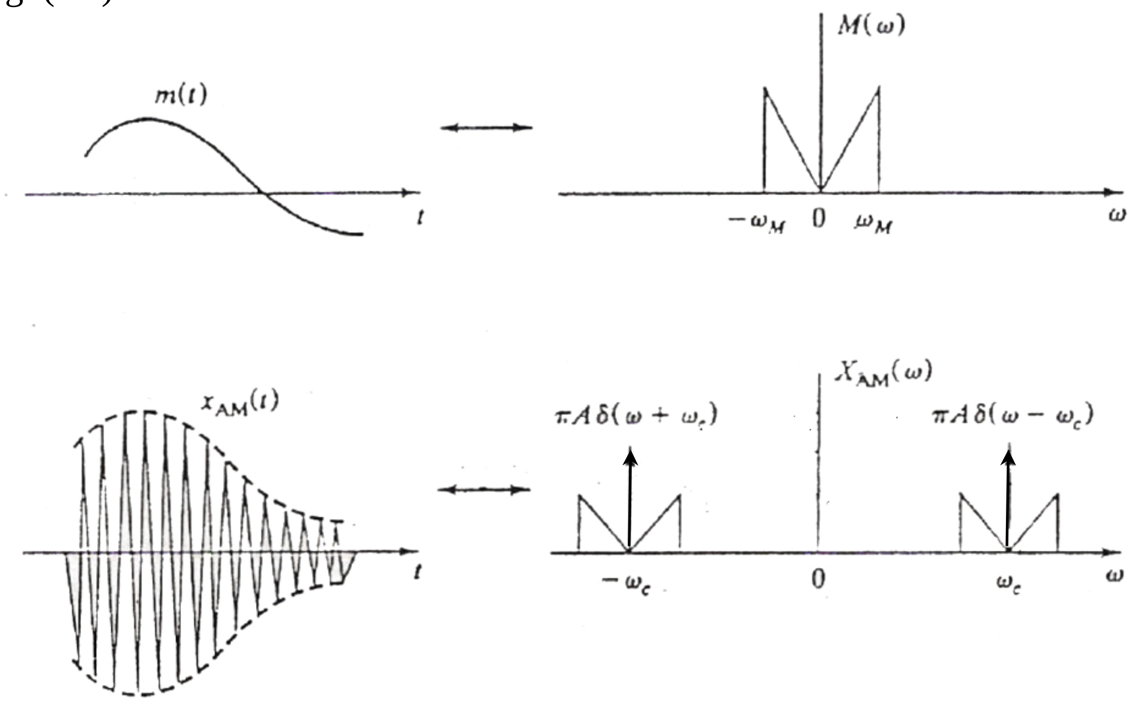


Fig. (3.1)

3.1 Demodulation of AM Signals

The advantage of *AM* over *DSB* modulation is that a very simple scheme, known as *envelope detection*, can be used for demodulation if sufficient carrier power is transmitted. In Eq.(3.1), if A is large enough, the envelope (amplitude) of the modulated waveform given by $A+m(t)$ will be proportional to $m(t)$. Demodulation in this case simply reduces to the detection of the envelope of a modulated carrier with no dependence on the exact phase or frequency of the carrier. If A is not large enough, then the envelope of $x_{AM}(t)$ is not always proportional to $m(t)$, as illustrated in Fig.(3.2). Thus, the condition for demodulation of *AM* by an envelope detector is

$$A + m(t) > 0 \quad for\ all\ t \qquad (3.3)$$

or

$$A \geq |min\,\{m(t)\}| \qquad (3.4)$$

where min $\{m(t)\}$ is the minimum value of $m(t)$.

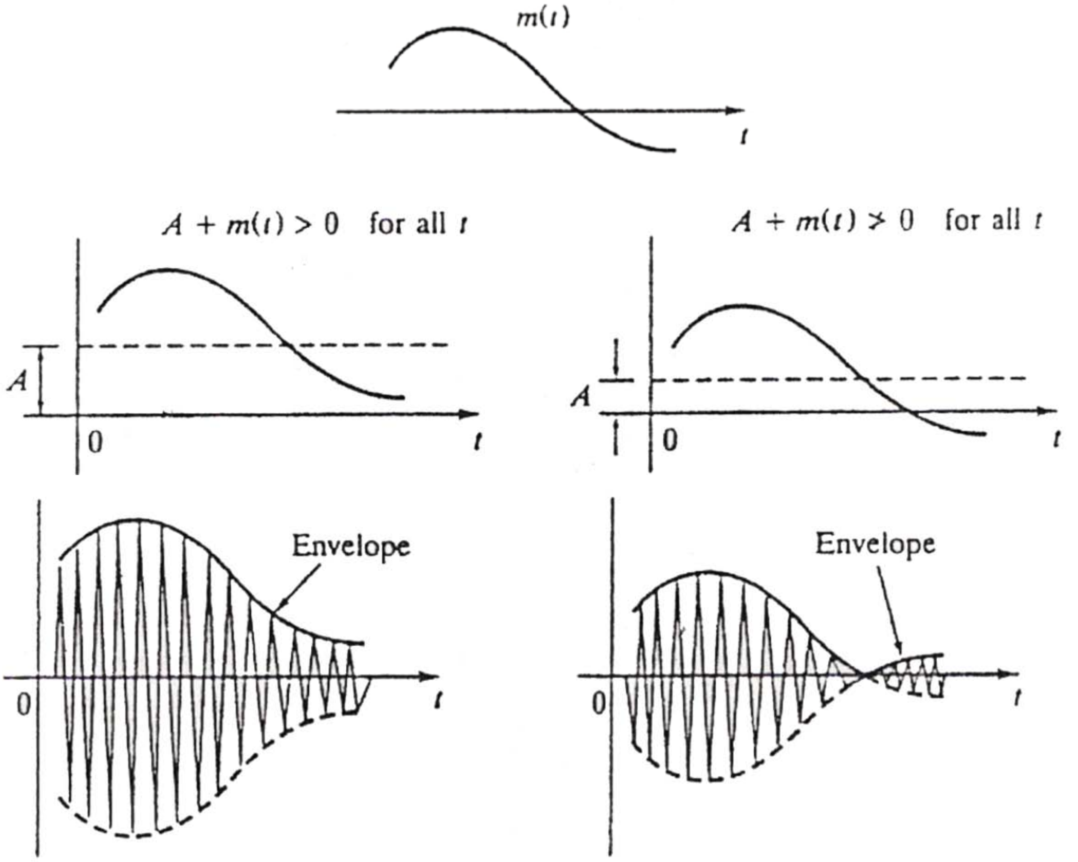


Fig.(3.2)

3.2 Modulation Index

The *modulation index* μ.for *AM* is defined as

$$\mu = \frac{|min\,\{m(t)\}|}{A} \tag{3.5}$$

From Eq.(3.4), the condition for demodulation of *AM* by an envelope detector can be expressed as

$$\mu \leq 1 \tag{3.6}$$

When $\mu > 1$, the carrier is said to be *overmodulated*, resulting in envelope distortion.

3.3 Envelope Detector

Figure (3.3a) shows the simplest form of an envelope detector consisting of a diode and a resistor-capacitor combination. The operation of the envelope detector is as follows. During the positive half-cycle of the input signal, the diode is forward-biased, and the capacitor *C* charges up rapidly to the peak value of the input signal. As the input signal falls below its maximum, the diode turns off. This is followed by a slow discharge of the capacitor through resistor *R* until the next positive half-cycle, when the input signal becomes greater than the capacitor voltage and the diode turns on again. The capacitor charges to the new peak value, and the process is repeated.

For proper operation of the envelope detector, the discharge time constant RC must be chosen properly. In practice, satisfactory operation requires that $1/f_c \ll 1/f_M$ where f_M is the message signal bandwidth

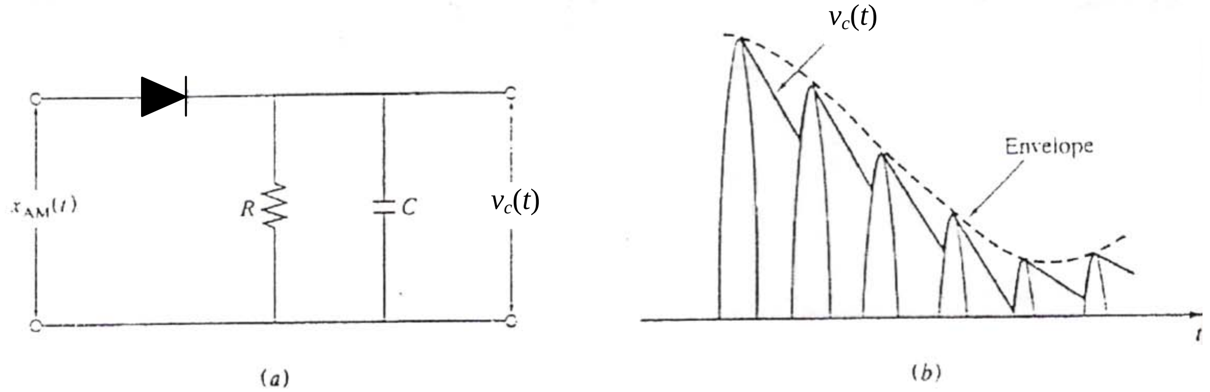


Fig.(3.3)

Example (3.1): Sketch the ordinary *AM* signal for a single-tone modulation with modulation indices of $\mu = 0.5$ and $\mu = 1$.

Sol. For a single-tone modulation

$$m(t) = a_m \cos \omega_m t$$

$$\mu = \frac{|min\,\{m(t)\}|}{A} = \frac{a_m}{A}$$

Hence $\quad m(t) = a_m \cos \omega_m t = \mu A \cos \omega_m t$

and $\quad x_{AM}(t) = [A + m(t)] \cos \omega_c t$

$$= A\,[1 + \mu \cos \omega_m t] \cos \omega_c t$$

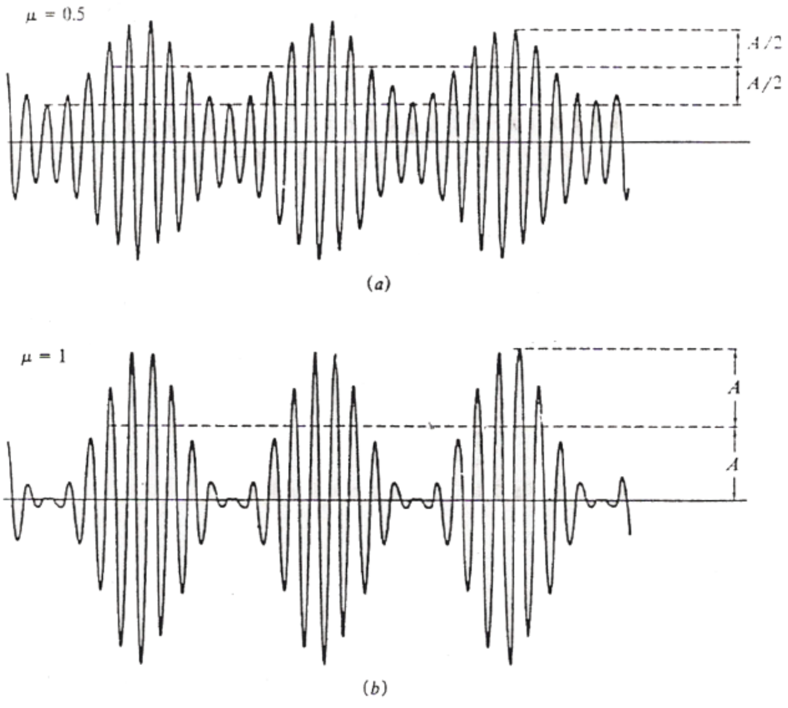


Figure (3.4) shows the envelope and the voltage across the capacitor. The capacitor discharges from the peak value E starting at some arbitrary instant $t = 0$. The voltage v_c across the capacitor is given by

$$v_c = Ee^{-t/RC} \tag{3.7}$$

Because the time constant is much larger than the interval between the two successive cycles of the carrier ($RC \gg 1/\omega_c$), the capacitor voltage v_c discharges exponentially for a short time compared to its time constant. Hence, the exponential can be approximated by a straight line obtained from the first two terms in *Taylor's* series for $Ee^{-t/RC}$

$$v_c \cong E\left(1 - \frac{t}{RC}\right) \tag{3.8}$$

The slope of the discharge is $-E/RC$. In order for the capacitor to follow the envelope $E(t)$, the magnitude of the slope of the RC discharge must be greater than the magnitude of the slope of the envelope $E(t)$. Hence,

$$\left|\frac{dv_c}{dt}\right| = \frac{E}{RC} \geq \left|\frac{dE}{dt}\right| \tag{3.9}$$

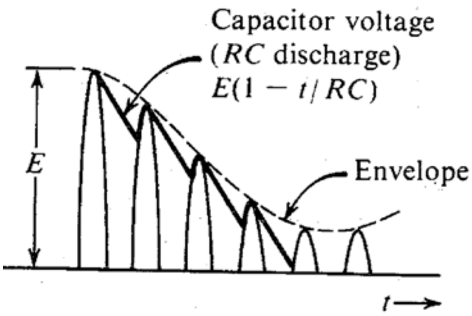


Fig.(3.4)

But the envelope $E(t)$ of a tone-modulated carrier is

$$E(t) = A[1 + \mu\, cos\omega_m t]$$

$$\frac{dE}{dt} = -\mu A\omega_m sin\omega_m t$$

Hence, Eq. (3.9) becomes

$$\frac{A(1 + \mu cos\omega_m t)}{RC} \geq \mu A\omega_m sin\omega_m t \qquad \text{for all } t$$

or

$$RC \leq \frac{1 + \mu cos\omega_m t}{\mu\omega_m sin\omega_m t} \qquad for\ all\ t$$

The worst possible case occurs when the right hand side is the minimum. This is found (as usual. by taking the derivative and setting it to zero) to be when $\cos\omega_m t=-\mu$. For this case, the right-hand side is $\sqrt{(1-\mu^2}/\mu\omega_m$. Hence

$$RC \le \frac{1}{\omega_m}\left(\frac{\sqrt{1-\mu^2}}{\mu}\right)$$

3.4 Efficiency of *AM*

The efficiency η of ordinary *AM* is defined as the percentage of the total power carried by the sidebands that is,

$$\eta = \frac{P_s}{P_t} \times 100\% \tag{3.7}$$

where P_s is the power carried by the sidebands and P_t is the total power of the *AM* signal. For μ=0.5 (50% modulation) single tone *AM* signal, η calculated as follow;

$$x_{AM}(t) = A\cos\omega_c t + \mu A\cos\omega_m t\cos\omega_c t$$

$$= A\cos\omega_c t + \tfrac{1}{2}\mu A\cos(\omega_c-\omega_m)t + \tfrac{1}{2}\mu A\cos(\omega_c+\omega_m)t$$

$$P_c = carrier\ power = \tfrac{1}{2}A^2$$

$$P_S = sideband\ power = \frac{1}{2}\left[\left(\tfrac{1}{2}\mu A\right)^2 + \left(\tfrac{1}{2}\mu A\right)^2\right] = \frac{1}{4}\mu^2A^2$$

The total power P_t

$$P_t = P_c + P_s = \tfrac{1}{2}A^2 + \tfrac{1}{4}\mu^2A^2 = \tfrac{1}{2}\left(1+\tfrac{1}{2}\mu^2\right)A^2$$

Thus

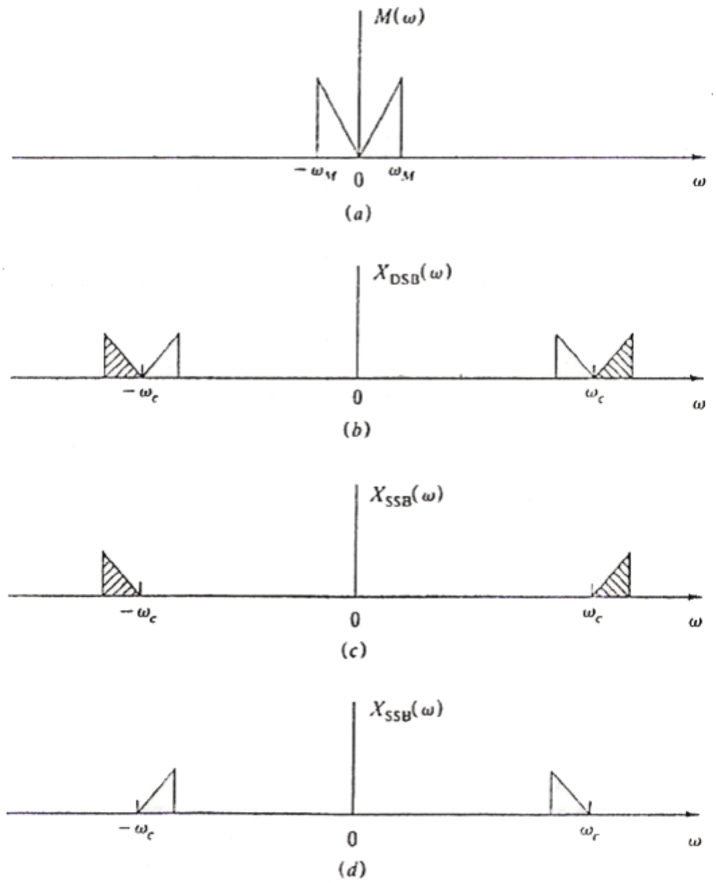
$$\eta = \frac{P_s}{P_t}\times 100\% = \frac{\frac{1}{4}\mu^2A^2}{\left(\frac{1}{2}+\frac{1}{4}\mu^2\right)A^2}\times 100\% = \frac{\mu^2}{(2+\mu^2)}\times 100\%$$

For μ=0.5, $\eta = \frac{(0.5)^2}{2+(0.5)^2} \times 100\% = 11.1\%$

For μ=1, we have maximum value of η (η_{max}), i.e., $\eta_{max} = \frac{1}{3}\times 100\% = 33.3\%$

4. SINGLE-SIDEBAND MODULATION

Ordinary *AM* modulation and *DSB* modulation waste bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth. Since either the upper sideband or the lower sideband contains the complete information of the message signal, only one sideband is necessary for information transmission. When only one sideband is transmitted, the modulation is referred to as *single-sideband* (*SSB*) modulation.

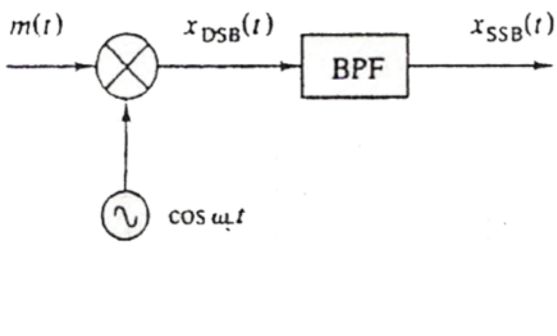


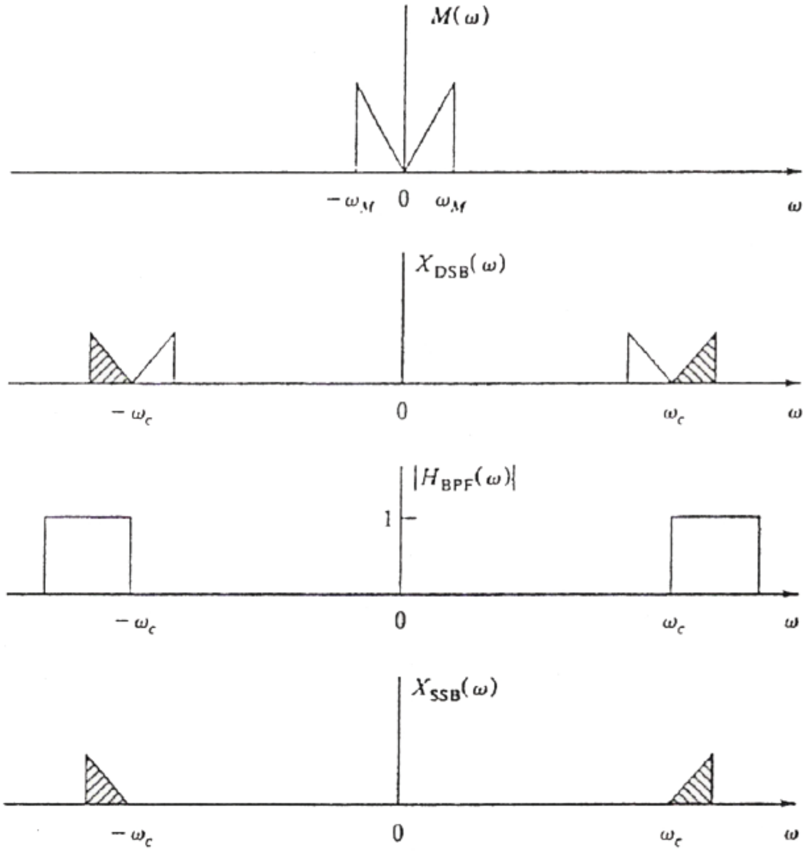
The benefit of *SSB* modulation is the reduced bandwidth requirement, but the principal disadvantages are the cost and complexity of its implementation.

4.1 Generation of *SSB* Signals

4.1.1 *Frequency Discrimination Method*

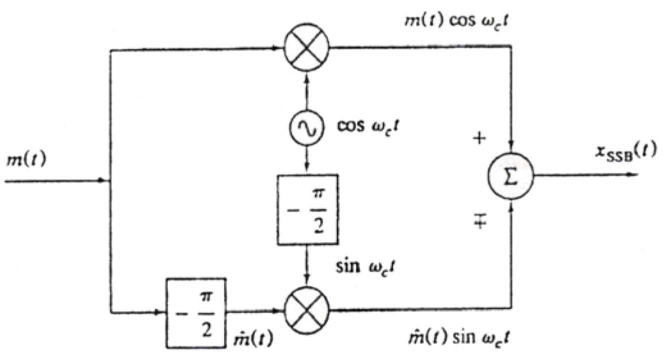
The straightforward way to generate an *SSB* signal is to generate a *DSB* signal first and then suppress one of the sidebands by filtering. This is known as the *frequency discrimination* method. In practice, this method is not easy because the filter must have sharp cutoff characteristics.





4.1.2 Phase-Shift Method

Another method for generating an *SSB* signal, known as the *phase-shift* method, is illustrated below. The box marked –π/2 is a π/2 phase shifter which delays the phase of every frequency component by π/2. An ideal phase shifter is almost impossible to implement exactly. But we can approximate it over a finite frequency band.



If we let $\hat{m}(t)$ be the output of the –π/2 phase shifter due to the input *m*(*t*), then the *SSB* signal x_{SSB}(t) can be represented by

$$x_{SSB}(\mathrm{t}) = m(t)\ cos\ \omega_c t \mp \hat{m}(t)\ sin\ \omega_c t \qquad (4.1)$$

The difference represents the upper-sideband *SSB* signal, and the sum represents the lower-sideband *SSB* signal.

Let $m(t) = \cos\,\omega_m t$

Then $\hat{m}(t) = cos(\omega_m t - \frac{\pi}{2}) = sin\,\omega_m t$

Hence $y(t) = cos\omega_m t\;cos\omega_c t \mp sin\omega_m t\;sin\omega_c t$

$$= cos(\omega_c \pm \omega_m)t$$

Thus with subtraction we have

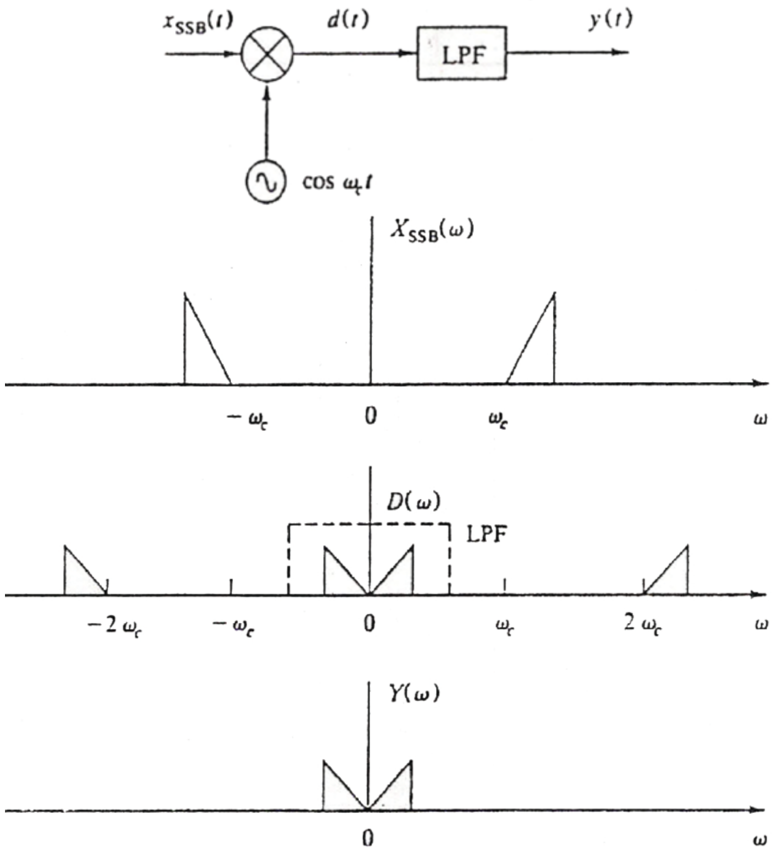
$$y(t) = x_{USB}(t) = cos(\omega_c + \omega_m)t$$

And with addition we have

$$y(t) = x_{LSB}(t) = cos(\omega_c - \omega_m)t$$

4.2 Demodulation of *SSB* Signals

Demodulation of *SSB* signals can be achieved easily by using the coherent detector as used in the *DSB* demodulation, that is, by multiplying $x_{SSB}(t)$ by a local carrier and passing the resulting signal through a low-pass filter.

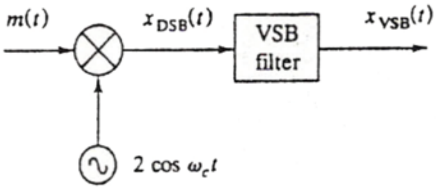


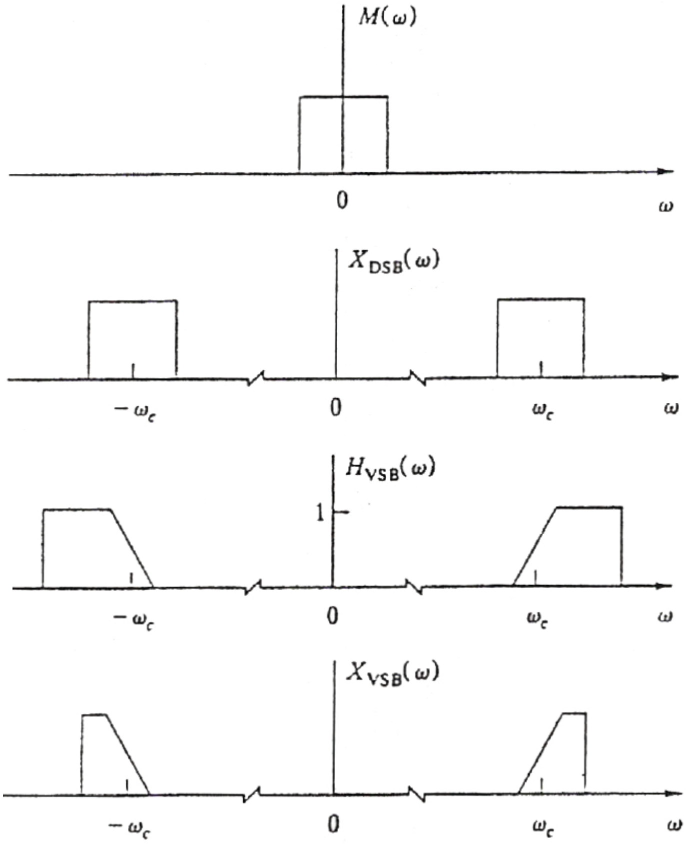
5. VESTIGIAL-SIDEBAND MODULATION

Vestigial-sideband (*VSB*) modulation is a compromise between *SSB* and *DSB* modulations. In this modulation scheme, one sideband is passed almost completely whereas just a trace, or vestige, of the other side band is retained. The typical bandwidth required to transmit a *VSB* signal is about 1.25 that of *SSB*. *VSB* is used for transmission of the video signal in commercial television broadcasting.

5.1 Generation of VSB Signals

A *VSB* signal can be generated by passing a *DSB* signal through a sideband shaping filter (or vestigial filter)

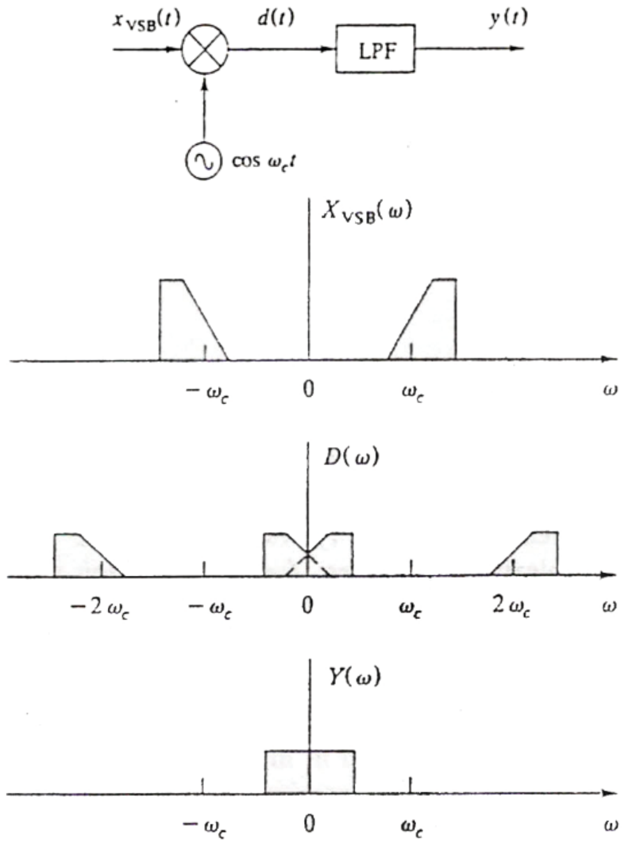




5.2 Demodulation of VSB Signals

For VSB signals, $m(t)$ can be recovered by synchronous or coherent demodulation, this determines the requirements of the frequency response $H(\omega)$. It can be shown that for distortionless recovery of $m(t)$, it is required that

$$H(\omega + \omega_c) + H(\omega - \omega_c) = constant \qquad for\ |\omega| \le \omega_M \qquad (prove)$$



6. FREQUICNCY TRANSLATION AND MIXING

In the processing of signals in communication systems, it is often desirable to translate or shift the modulated signal to a new frequency band. For example, in most commercial *AM* radio receivers, the received radio frequency (*RF*) signal [540 to 1600 kHz] is shifted to the intermediate- frequency (*IF*) (455-kHz) band for processing. The received signal, now translated to a fixed *IF*, can easily be amplified, filtered, and demodulated.

A device that performs the frequency translation of a modulated signal is called a *frequency mixer* (Fig. 6.1). The operation is often called *frequency mixing*, *frequency conversion*, or *heterodyning*.

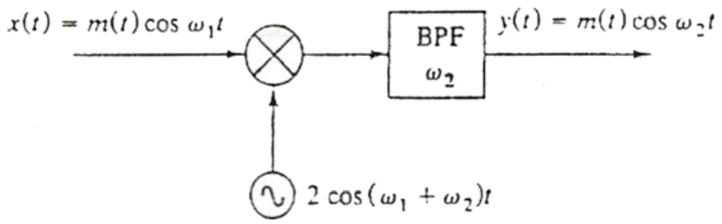
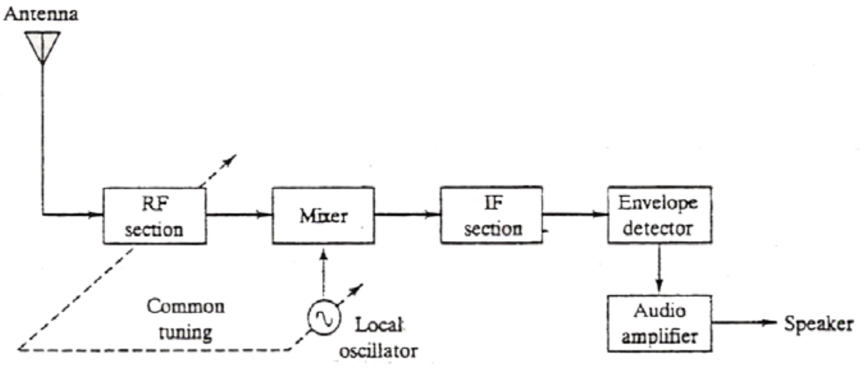


Fig. 6.1



A superheterodyne AM receiver

- When $f_{LO} > f_c$ (*superheterodyne receiver*);

$$540 < f_c < 1600$$
$$f_{LO} - f_c = 455$$

Thus

$$f_{LO} = f_c + 455$$

When f_c = 540 kHz, we get f_{LO} =995 kHz; and when f_c = 1600 kHz, we get f_{LO} = 2055 kHz. Thus the required tuning range of the local oscillator is 995 – 2055 kHz.

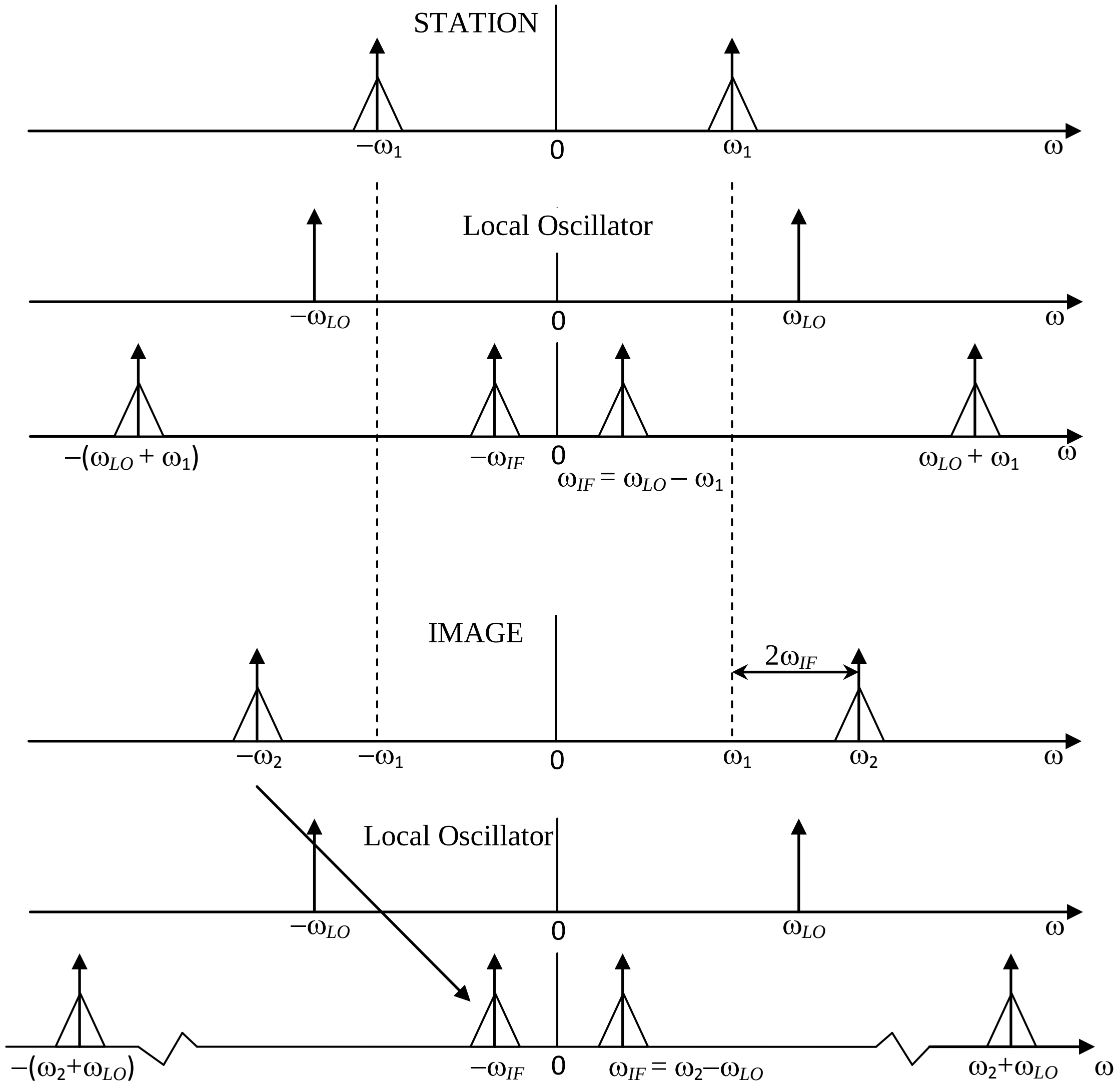
- When $f_{LO} < f_c$

$$f_{LO} = f_c - 455$$

When f_c = 540 kHz, we get f_{LO} = 85 kHz, and when f_c = 1600 kHz, we get f_{LO} = 1145 kHz. Thus the required tuning range of the local oscillator for this case is 85 – 1145 kHz.
The frequency ratio, that is, the ratio of the highest f_{LO} to the lowest f_{LO}, is 2.07 for the case of $f_{LO} > f_c$, and 13.47 for the case of $f_{LO} < f_c$. It is much easier to design an oscillator that is tunable over a smaller frequency ratio; that is the reason why the usual *AM* radio receiver uses the superheterodyne system.

A common problem associated with frequency mixing is the presence of the *image frequency*. For example, in an *AM* superheterodyne receiver (described above), the locally generated frequency is chosen to be 455 kHz higher than the incoming signal. Suppose that the reception of an *AM* station at 600 kHz is desired. Then the locally generated signal is at 1055 kHz. Now if there is another station at 1510 kHz, it also will be received (note that 1510 kHz – 1055 kHz = 455 kHz).

This second frequency, 1510 kHz = 600 kHz + 2(455 kHz), is called the image frequency of the first, and after the heterodyning operation it is impossible to distinguish the two. Note that the image frequency is separated from the desired signal by exactly twice the *IF*. Usually, the image frequency signal is attenuated by a selective *RF* amplifier placed before the mixer.



7. FREQUENCY-DIVISION MULTIPLEXING

Multiplexing is a technique whereby several message signals are combined into a composite signal for transmission over a common channel. To transmit a number of these signals over the same channel, the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiver end.

There are two basic multiplexing techniques: frequency-division multiplexing (*FDM*) and time- division multiplexing (*TDM*). In *FDM* the signals are separated in frequency, whereas in *TDM* the signals are separated in time. The *FDM* scheme is illustrated in Fig.(7.1) with the simultaneous transmission of three message signals. Any type of modulation can be used in *FDM* as long as the carrier spacing is sufficient to avoid spectral overlap.

However, the most widely used method of modulation is *SSB* modulation. At the receiving end of the channel the three modulated signals are separated by bandpass filters (*BPFs*) and then demodulated.

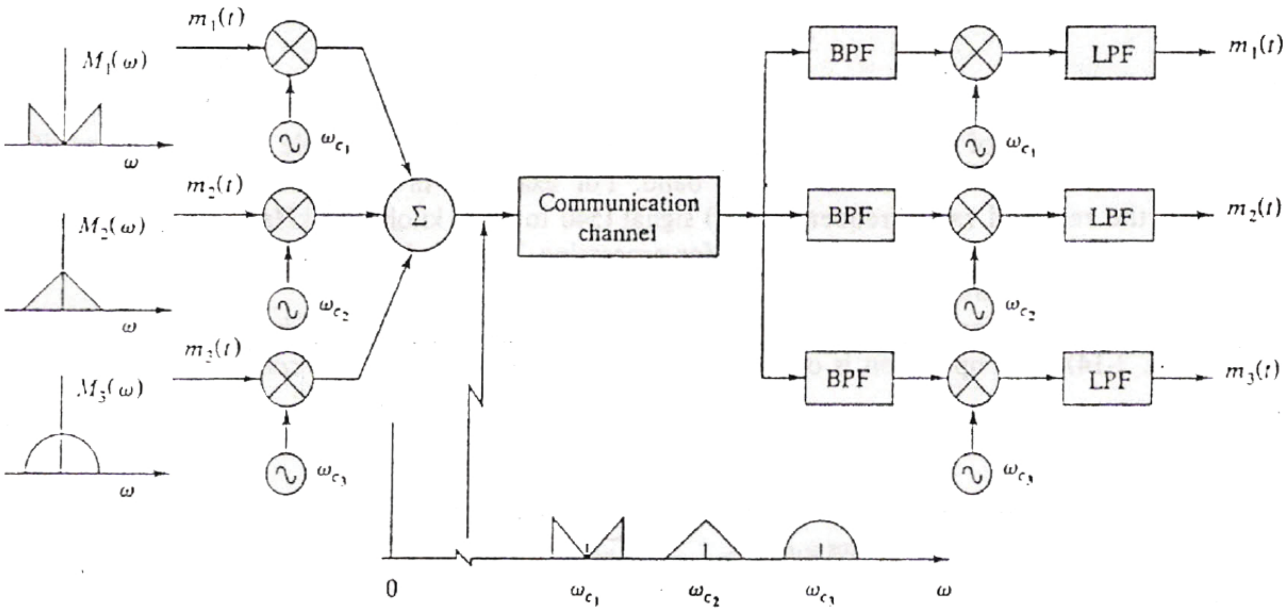


Fig.(7.1)

FDM is used in telephone system, telemetry, commercial broadcast, television, and communication networks. Commercial *AM* broadcast stations use carrier frequency spaced 10 kHz apart in the frequency range from 540 to 1600 kHz. This separation is not sufficient to avoid spectral overlap for *AM* with a reasonably high-fidelity (50 Hz to 15 kHz) audio signal. Therefore, *AM* stations on adjacent carrier frequencies are placed geographically far apart to minimize interference.

8. Performance of Linear Modulation Systems in The Presence of Noise

8.1 Additive Noise and Signal-to-Noise Ratio

For the general communication system shown in Fig.(8.1), if the input is modeled by the random process $X(t)$ and the channel introduces no distortion other than additive noise, the receiver output $Y_o(t)$ can be written as

$$Y_o(t) = X_o(t) + n_o(t) \tag{8.1}$$

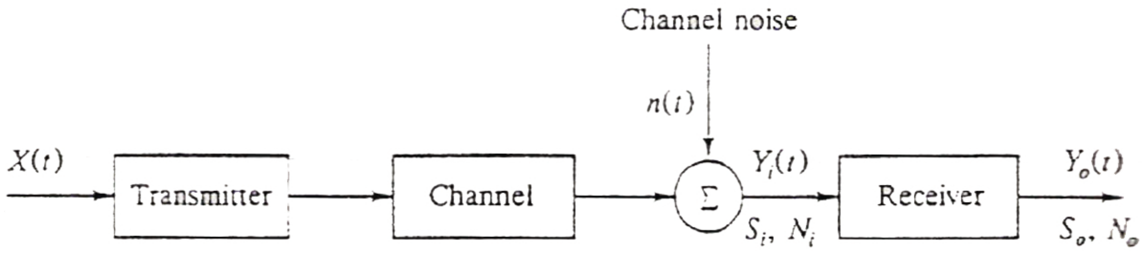


Fig.(8.1)

$X_o(t)$; signal component at receiver output, and
$n_o(t)$; noise component at receiver output
Assume that $n(t)$ is zero mean white Gaussian noise with $S_{nn}(\omega)=\eta/2$
Therefore,

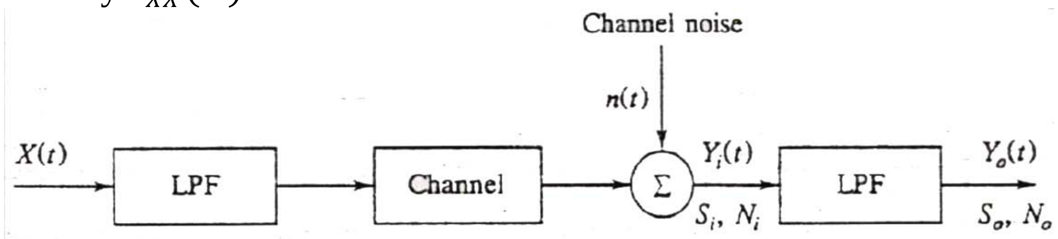
$$E[Y_o^2(t)] = E[X_o^2(t)] + E[n_o^2(t)] = S_o + N_o \tag{8.2}$$

Where S_o and N_o; are the average signal and noise power at the receiver output, respectively.
(Note that the expected value of $X(t)$ is $E[X(t)] = \int_{-\infty}^{\infty} x\, f_X(x;t)\, dx$, and $f_X(x;t)$ is the probability density function of the random process $X(t)$).
The output-signal-to-noise power ratio $(S/N)_o$ is defined as

$$\left(\frac{S}{N}\right)_o = \frac{S_o}{N_o} = \frac{E[X_o^2(t)]}{E[n_o^2(t)]} \tag{8.3}$$

8.2 Noise in Baseband Communication Systems

In baseband communication systems, the signal is transmitted directly without any modulation. Assume that the receiver is ideal LPF with bandwidth $W = 2\pi B$. The message signal $X(t)$ is a zero mean random process band-limited to W with power spectral density $S_{XX}(\omega)$.



The channel is assumed to be distortionless over the message band so that

$$X_o(t) = X(t - t_d) \tag{8.4}$$

Where t_d is the time delay of the system.

$$S_o = E[X_o^2(t)] = E[X^2(t - t_d)]$$
$$= \frac{1}{2\pi}\int_{-W}^{W} S_{XX}(\omega)d\omega = S_X = S_i$$

Where S_X is the average signal power and S_i is the signal power at the input of the receiver. The average output noise power is

$$N_o = E[n_o^2(t)] = \frac{1}{2\pi}\int_{-W}^{W} S_{nn}(\omega)d\omega$$

For the case of additive noise, $S_{nn}(\omega) = \eta/2$, and

$$N_o = \frac{1}{2\pi}\int_{-W}^{W} \frac{\eta}{2} d\omega = \eta \frac{W}{2\pi} = \eta B$$

The output signal-to-noise ratio

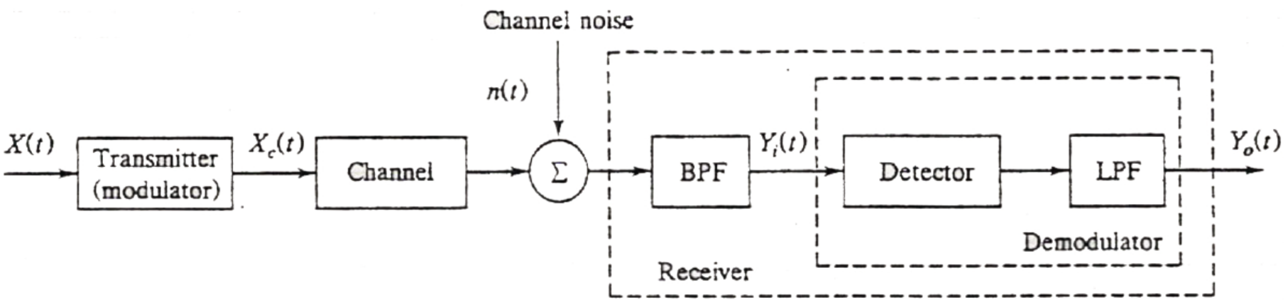
$$\left(\frac{S}{N}\right)_o = \frac{S_o}{N_o} = \frac{S_i}{\eta B} = \gamma \tag{8.5}$$

The parameter γ is directly proportional to S_i. Hence, comparing various systems for the output *SNR* for a given S_i is the same as comparing these systems for the output *SNR* for a given γ

8.3 Noise in Amplitude Modulation Systems

The receiver front end (*RF/IF* stages) is modeled as an ideal bandpass filter with a bandwidth $2W$ centered at ω_c.

$$Y_i(t) = X_c(t) + n_i(t)$$



Where $n_i(t)$ is the narrowband noise, which can be expressed as

$$n_i(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

If the power spectral density of $n(t)$ is $\eta/2$

$$E[n_c^2(t)] = E[n_s^2(t)] = E[n_i^2(t)] = 2\eta B$$

8.3.1 Synchronous Detection

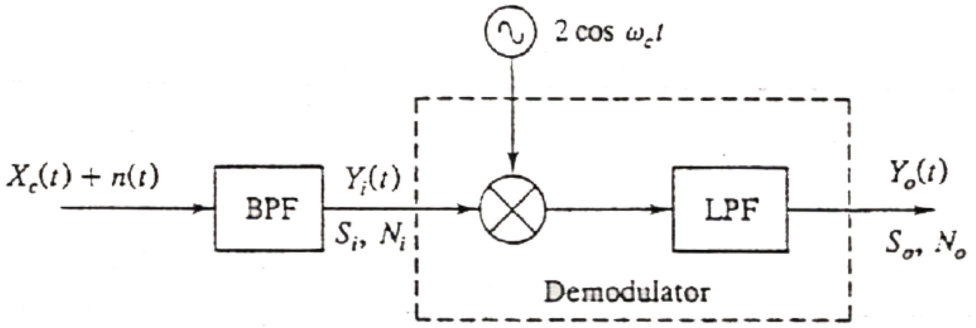
A. DSB Systems

In a *DSB* system, the transmitted signal $X_c(t)$ has the form

$$X_c(t) = A_c X(t) \cos \omega_c t$$

$$Y_i(t) = A_c X(t) \cos \omega_c t + n_i(t)$$

$$= [A_c X(t) + n_c(t)] \cos \omega_c t - n_s(t) \sin \omega_c t$$



Multiplying $Y_i(t)$ by $2\cos \omega_c t$ and using a low-pass filter, we obtain

$$Y_o(t) = A_c X(t) + n_c(t) = X_o(t) + n_o(t)$$

Where $\quad X_o(t) = A_c X(t) \quad$ and $\quad n_o(t) = n_c(t)$

We see that the output signal and noise are additive and the quadrature noise component $n_s(t)$ has been rejected by the demodulator. Now

$$S_o = E[X_o^2(t)] = E[A_c^2 X^2(t)] = A_c^2\, E[X^2(t)] = A_c^2 S_X$$

$$N_o = E[n_o^2(t)] = E[n_c^2(t)] = E[n_i^2(t)] = 2\eta B$$

And the output *SNR* is

$$\left(\frac{S}{N}\right)_o = \frac{S_o}{N_o} = \frac{A_c^2 S_X}{2\eta B} \tag{8.6}$$

The input signal power S_i is given by

$$S_i = E[X_c^2(t)] = \frac{1}{2} A_c^2 S_X$$

Thus, from Eqs. (8.5) and (8.6) we obtain

$$\left(\frac{S}{N}\right)_o = \frac{S_i}{\eta B} = \gamma \tag{8.7}$$

This indicates that *DSB* with ideal synchronous detection has the same performance as baseband system.

The *SNR* at the input of the detector is

$$\left(\frac{S}{N}\right)_i = \frac{S_i}{N_i} = \frac{S_i}{2\eta B}$$

and

$$\frac{(S/N)_o}{(S/N)_i} = \alpha_d = 2$$

The ratio α_d is known as the *detector gain* and is often used as a *figure of merit* for the demodulation.

Similar calculations for an *SSB* system using synchronous detection yield the same noise performance as for a baseband or *DSB* system (prove).

B. AM Systems

In ordinary *AM* systems, *AM* signals can be demodulated by *synchronous detector* or by *envelope detector*. The modulated signal in an *AM* system has the form

$$X_c(t) = A_c[1 + \mu X(t)]cos\omega_c t$$

Assume that $\mu \le 1$ and $|X(t)| \le 1$. The receiver output $Y_o(t)$ is

$$Y_o(t) = A_c\mu X(t) + n_c(t) = X_o(t) + n_o(t) \tag{8.8}$$

Where

$$X_o(t) = A_c\mu X(t) \qquad and \qquad n_o(t) = n_c(t)$$

So

$$\left(\frac{S}{N}\right)_o = \frac{S_o}{N_o} = \frac{A_c^2\mu^2 S_X}{2\eta B}$$

The input signal power S_i is

$$S_i = \frac{1}{2}E[A_c^2[1 + \mu X(t)]^2]$$

Since $X(t)$ is assumed to have a zero mean, (i.e., $E[2\mu X(t)]=0$)

$$S_i = \frac{1}{2}A_c^2(1 + \mu^2 S_X)$$

Thus

$$S_o = A_c^2\mu^2 S_X = \frac{2\mu^2 S_X}{1 + \mu^2 S_X}S_i$$

and

$$\left(\frac{S}{N}\right)_o = \frac{S_o}{N_o} = \frac{\mu^2 S_X}{1 + \mu^2 S_X}\left(\frac{S_i}{\eta B}\right) = \frac{\mu^2 S_X}{1 + \mu^2 S_X}\gamma \tag{8.9}$$

Because $\mu^2 S_X \le 1,$ we have

$$\left(\frac{S}{N}\right)_o \le \frac{\gamma}{2}$$

which indicates that the output *SNR* in *AM* is at least 3 dB worse that in *DSB* systems.

8.3.2 Envelope Detection and Threshold Effect

An ordinary AM signal is usually demodulated by envelope detection. The input to the detector is

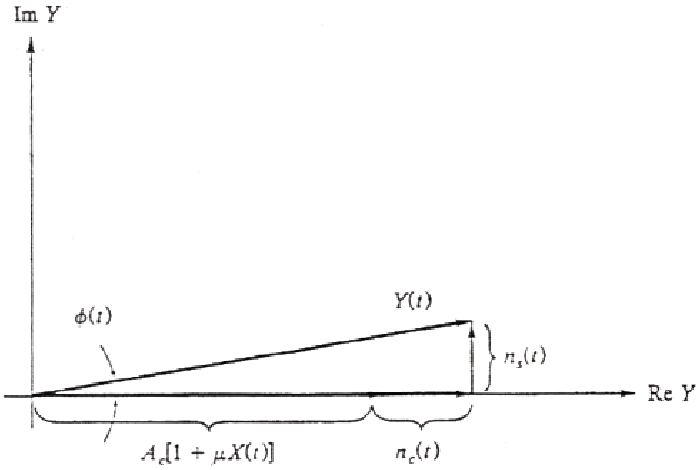
$$\begin{aligned} Y_i(t) &= X_c(t) + n_i(t) \\ &= \{A_c[1 + \mu X(t)] + n_c(t)\}cos\,\omega_c t - n_s(t)sin\,\omega_c \end{aligned}$$

We can analyze the effect of the noise by considering a *phasor representation* of $Y_i(t)$

$$Y_i(t) = Re\left[Y(t)\, e^{j\omega_c t}\right]$$

Where

$$Y(t) = A_c[1 + \mu X(t)] + n_c(t) + jn_s(t)$$



From the phase diagram

$$Y_i(t) = V(t)\, cos[\omega_c t + \phi(t)]$$

$$V(t) = \sqrt{\{A_c[1 + \mu X(t)] + n_c(t)\}^2 + n_s^2(t)}$$

$$\phi(t) = tan^{-1} \frac{n_s(t)}{A_c[1 + \mu X(t)] + n_c(t)}$$

<u>A. Large-SNR (Signal Dominance) case</u>

When $(S/N)_i \gg 1$, $A_c[1 + \mu X(t)] \gg n_i(t)$, and hence $A_c[1 + \mu X(t)] \gg n_c(t)$ and $n_s(t)$ for almost all t. Under this condition, the envelope $V(t)$ can be approximated by

$$A_c[1 + \mu X(t)] + n_c(t)$$

An ideal envelope detector reproduces the envelope $V(t)$ minus its dc component, so

$$Y_o(t) = A_c \mu X(t) + n_c(t)$$

Which is identical to that of a synchronous detector [Eq.(8.8)]. The output *SNR* is then as given in Eq.(8.9).

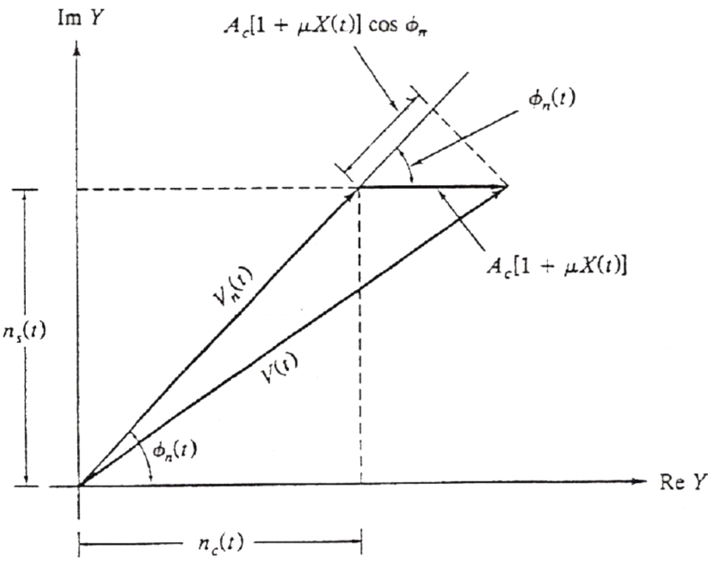
$$\left(\frac{S}{N}\right)_o = \frac{\mu^2 S_X}{1 + \mu^2 S_X}\gamma$$

Therefore, for AM, when $(S/N)_i \gg 1$, the performance of the envelope detector is identical to that of the synchronous detector.

<u>B. Small- SNR (Noise Dominance) Case:</u>

When $(S/N)_i \ll 1$, the envelope of the resultant signal is primarily dominated by the envelope of the noise signal. The envelope of the resultant signal is approximated by

$$V(t) \approx V_n(t) + A_c[1 + \mu X(t)]cos\phi_n(t) \qquad (8.10)$$



Where $V(t)$ and $\phi_n(t)$ are the envelope and the phase of the noise $n_i(t)$. Equation (8.10) indicates that the output contains no term proportional to $X(t)$ and that noise is multiplicative. The signal $X(t)$ is multiplied by noise in the form of $cos\phi_n(t)$ which is random. Thus the information in $X(t)$ has been lost. Under these circumstances, it is meaningless to talk about *SNR*.

Example 8.1

Calculate the transmission bandwidth B_T and the required transmitter power S_T of *DSB*, *SSB*, and *AM* systems for transmitting an audio signal which has a bandwidth of 10 kHz with an output *SNR* of 40 dB. Assume that the channel introduces a 40 dB power loss and channel noise is *AWGN* with *PSD* $(\eta/2)=10^{-9}$ W/Hz. Assume $\mu^2 S_X=0.5$ for *AM*.

Sol.

$$B_T = \begin{cases} 20\ kHz & for\ DSB\ and\ AM \\ 10\ kHz & for\ SSB \end{cases}$$

For *DSB* and *SSB*

$$\left(\frac{S}{N}\right)_O = \frac{S_i}{\eta B} = 10^4 \quad (= 40dB)$$

and $\quad S_i = \eta B(10^4) = 2(10^{-9})(10^4)(10^4) = 0.2\ W$

since the channel power loss is 40 dB, the required transmitted power S_T is

$$S_T = 0.2\ (10^4) = 2000\text{W} = 2\ \text{kW}$$

For an *AM* system with envelop detection

$$\left(\frac{S}{N}\right)_O = \frac{1}{3}\frac{S_i}{\eta B}$$

Thus, the required transmitted power S_T is 3 times that for the *DSB* or *SSB* system, that is, $S_T = 6$ kW