Chapter 8: Amplifier Frequency Response

Effect of Coupling Capacitors

Coupling capacitors are in series with the signal and are part of a high-pass filter network. They affect the low-frequency response of the amplifier



Figure 1: Examples of capacitively coupled BJT and FET amplifiers.

For the circuit shown in Figure 1(a), the equivalent circuit for C_1 is a high-pass filter, C_3 and $(R_C + R_L)$ form another high-pass filter.

With FETs, the input coupling capacitor is usually smaller because of the high input resistance. The output capacitor may be smaller or larger depending on the drain and load resistor size. For the circuit shown in Figure 1(b), the equivalent low-pass filter for the input is simply C_1 in series with R_G because the gate input resistance is so high.

Effect of Bypass Capacitors

A bypass capacitor causes reduced gain at low-frequencies and has a high-pass filter response. The resistors "seen" by the bypass capacitor include R_E , $\dot{r_e}$, and the bias resistors. For example, when the frequency is sufficiently high $X_C \cong 0\Omega$ and the voltage gain of the CE amplifier is $A_v = R_c/\dot{r_e}$. At lower frequencies, $X_C \gg 0\Omega$ and the voltage gain $A_v = R_c/(\dot{r_e} + Z_e)$.



Figure 2: Nonzero reactance of the bypass capacitor in parallel with R_E creates an emitter impedance (Z_e), which reduces the voltage gain.

Internal Capacitances

The high-frequency response of an amplifier is determined by internal junction capacitances. These capacitances form low-pass filters with the external resistors. Sometimes a designer will add an external parallel capacitor to deliberately reduce the high frequency response.



Figure 3: Internal transistor capacitances.

Miller's Theorem

Miller's theorem states that, for inverting amplifiers, the capacitance between the input and output is equivalent to separate input and output capacitances to ground.



Figure 4: General case of Miller input and output capacitances, C represents Cbc or Cgd.

 A_v is the absolute value of the gain. For the input capacitance, the gain has a large effect on the equivalent capacitance, which is an important consideration when using inverting amplifiers. Notice that the effect of Miller's theorem is an equivalent capacitance to ground, which shunts high frequencies to ground and reduces the gain as frequency is increased.



Figure 5: Amplifier ac equivalent circuits showing internal and effective Miller capacitances.

Example: What is the input capacitance for a 2N3904 inverting amplifier with a gain of

25? Assume the values of C_{bc} = 4pF and C_{be} = 6pF.

Solution:

$$\begin{split} C_{in} &= C_{bc}(A_v + 1) + C_{be} \\ C_{in} &= 4 \ pF(25 + 1) + 6 \ pF {=} 110 \ pF \end{split}$$

The Decibel

The decibel is a logarithmic ratio of two power levels and is used in electronics work in gain or attenuation measurements. Decibels can be expressed as a voltage ratio when the voltages are measured in the same impedance.

To express power gain in decibels, the formula is

Sometimes, 0 dB is assigned as a convenient reference level for comparison. Then, other power or voltage levels are shown with respect to 0 dB.

Low-Frequency Response

In capacitively coupled amplifiers, the coupling and bypass capacitors affect the low frequency cutoff. These capacitors form a high-pass filter with circuit resistances. A typical BJT amplifier has three high-pass filters. For example, the input coupling capacitor forms a high-pass filter with the input resistance of the amplifier:



Figure 6: A capacitively coupled BJT amplifier.

The input RC circuit for the BJT amplifier in Figure 6 is formed by C_1 and the amplifier's input resistance and is shown in Figure 7. The total input resistance is expressed by the following formula:

$$\mathbf{R}_{in(tot)} = \mathbf{R}_1 \| \mathbf{R}_2 \| \mathbf{R}_{in(base)}$$



Figure 7: Input RC circuit formed by the input coupling capacitor and the amplifier's input resistance.

The output RC circuit is composed of the series combination of the collector and load resistors with the output capacitor. The cutoff frequency due to the output circuit is

$$f_{\rm c} = \frac{1}{2\pi({\rm R}_{\rm C}+R_L){\rm C}_3}$$

Example: For the circuit in the following Figure, calculate the lower critical frequency due to the input RC circuit. Assumed $r'_e = 9.6\Omega$ and $\beta = 200$. Notice that a swamping resistor, R_{E1} , is used.



Solution: The input resistance is

$$R_{in} = R_1 ||R_2|| (\beta(\dot{r}_e + R_{E1})) = 68\Omega ||22\Omega|| (200(9.6\Omega + 33\Omega)) = 5.63k\Omega$$

The lower critical frequency is

$$f_{\rm cl(input)} = \frac{1}{2\pi R_{\rm in}C_1} = \frac{1}{2\pi (5.63 \text{k}\Omega)(0.1 \mu \text{F})} = 282 \text{Hz}$$

The Bode plot

The Bode plot is a plot of decibel voltage gain verses frequency. The frequency axis is logarithmic; the decibel gain is plotted on a linear scale. The -3dB point is the critical frequency.



Figure 8: Bode plot. (Blue is ideal; red is actual.)

The Bypass RC Circuit

The bypass *RC* circuit response can be found by observing the charge/discharge paths. For this circuit, there is one path through R_{E2} . A second path goes through R_{E1} , \dot{r}_e , and the parallel combination of bias and source resistances (source resistance not shown). The lower critical frequency for this equivalent bypass RC circuit is

$$f_{\rm cl(bypass)} = \frac{1}{2\pi (R_{in(emitter)} \parallel R_{E2})C_2}$$

 R_{th} is an equivalent resistance, the resistance in the emitter $R_{in(emitter)}$ bypass circuit is

$$R_{in(emitter)} = \dot{r}_e + R_{E1} + \frac{R_{th}}{\beta_{ac}}$$

Example: For the circuit in the following Figure, calculate the lower critical frequency due to the bypass *RC* circuit. Assume $\dot{r}_e = 9.6\Omega$ and $\beta = 200$.



Solution: The resistance in the emitter bypass circuit is

$$R_{in(emitter)} = \dot{r}_{e} + R_{E1} + \frac{R_{th}}{\beta_{ac}} = 9.6\Omega + 33\Omega + \frac{68k\Omega \parallel 22k\Omega \parallel 600\Omega}{200} = 45.5\Omega$$

The lower critical frequency is

$$f_{\rm cl(bypass)} = \frac{1}{2\pi (R_{\rm in(emitter)} \parallel R_{\rm E2})C_2} = \frac{1}{2\pi (45.5\Omega \parallel 1.5k\Omega)(100\mu F)} = 36 \text{Hz}$$

The Input RC Circuit

The input *RC* circuit for a FET is a basic high-pass filter consisting of the bias resistor (or resistors) and the input coupling capacitor. The FET gate circuit has such high resistance, it can be ignored.



Figure 9: Input RC circuit.

High-Frequency Response

The high frequency response of inverting amplifiers is primarily determined by the transistor's internal capacitance and the Miller effect. The equivalent high-frequency ac circuit is shown for a voltage-divider biased CE amplifier with a fully bypassed emitter resistor.



Figure 10: High-frequency equivalent circuit after applying Miller's theorem.

If there is an unbypassed emitter resistor, such as R_{E1} it is shown in the emitter circuit and acts to increase \dot{r}_e and thus reduce f_c . At high frequencies, the input circuit is as shown in Figure 11(a), where $\beta_{ac}\dot{r}_e$ is the input resistance at the base of the transistor because the bypass capacitor effectively shorts the emitter to ground. By combining C_{be} and $C_{in(Miller)}$ in parallel and repositioning, you get the simplified circuit shown in Figure 11(b). Next, by the venizing the circuit to the left of the capacitor, as indicated, the input RC circuit is reduced to the equivalent form shown in Figure 11(c).

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Figure 11: Development of the equivalent high-frequency input RC circuit.

If there is an unbypassed emitter resistor (R_{E1} in this case), the thevenin resistance is modified to $R_{th} = R_s ||R_1||R_2||\beta_{ac}(\dot{r}_e + R_{E1})$.

The high frequency analysis of FETs is similar to that of BJTs. Like the CE amplifier, the CS amplifier inverts the signal, so the Miller effect must be taken into account. You may see special circuits such as cascode connections in very high frequency applications to minimize the Miller effect. A high frequency ac model of a CS amplifier shown in figure 12.



Figure12; High-frequency equivalent circuit after applying Miller's theorem.

 C_{gs} simply appears as a capacitance to ac ground in parallel with $C_{in(Miller)}$, as shown in Figure 12. Looking in at the drain, C_{gd} effectively appears in the Miller output capacitance from drain to ground in parallel with R_d ,

$$C_{out(Miller)} = C_{gd} \frac{(A_v + 1)}{A_v}$$

The Miller input capacitance is given in as follows: $C_{in(Miller)}=C_{gd}(A_v+1)$

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Example: What is the upper cutoff frequency due to the input circuit? Assume $R_s=600\Omega$, $\dot{r}_e=3.5\Omega$, $\beta=200$, $C_{be}=6$ pF, $C_{bc}=3.5$ pF, and $A_v=9.7$.



Solution:

$$R_{th} = R_s ||R_1||R_2||\beta(\dot{r}_e + R_{E1})$$

= 600\Omega||10k\Omega||4.7\Omega||200(3.5\Omega + 100\Omega)| = 493\Omega
$$C_{in(tot)} = C_{be} + C_{Miller} = C_{be} + C_{bc}(A_{v(mid)} + 1)$$

= 6pF +3.5 pF(9.7 + 1)=43pF
$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi (493\Omega)(43pF)} = 7.4 \text{MHz}$$

Total Amplifier Frequency Response

The overall frequency response is the combination of three lower critical frequencies due to coupling and bypass capacitors and two upper critical frequencies due to internal capacitances. Figure 13 shows a generalized ideal response curve (Bode plot) for the BJT amplifier. The three break points at the lower critical frequencies (f_{cl1} , f_{cl2} , and f_{cl3}) are produced by the three low-frequency *RC* circuits formed by the coupling and bypass capacitors. The break points at the upper critical frequencies, f_{cu1} and f_{cu2} , are produced by the two high-frequency *RC* circuits formed by the transistor's internal capacitances.



Figure 13: A BJT amplifier and its generalized ideal response curve (Bode plot).

For multistage amplifiers, the individual stages have an effect on the overall response. In general, with different cutoff frequencies, the dominant *lower* cutoff frequency is equal to the *highest* f_{cl} ; the dominant *upper* critical frequency is equal to *lowest* f_{cu} .

When the critical frequencies for multistage amplifiers are equal, the lower critical frequency is higher than any one as given by

$$f_{cl}' = \frac{f_{cl}}{\sqrt{2^{1/n} - 1}}$$

and the upper critical frequency is given by

$$f_{cu} = f_{cu} \sqrt{2^{\frac{1}{n}} - 1}$$