Engineering Mechanics-Statics

References:

1- J. L. Meriam and L. G. Kraige, ‘Engineering Mechanics: Statics (V.1), 7th edition, Wiley 2012.
2- R. C. Hibbeler, Engineering Mechanics: STATICS (SI Edition), Prentice Hall 2004.
3- F. P. Beer and E. R. Johnston, ‘Vector Mechanics for Engineers: Statics (V.1), 3rd SI edition, TMH, 1998

Syllabus:

1- Introduction
2- Force system
3- Two dimensional force systems
4- Rectangular components
5- Moment
6- Couple
7- Resultants
8- Vectors
9- Three dimensional force systems
10-Rectangular components
11-Moment and couple
12-Resultants
13-Modeling of supports and free body diagram
14-Equilibrium
15-Center of mass
16-Centroids of lines, areas, and volumes
17-Centroid of composed figures
18-Truss
19-Frame and machines
20-Friction
21-Flexible belts
22-Moment of inertia (second moment of area)
23-Polar moment of inertia
24-Radius of gyration
25-Transfer of axes(parallel-axis theorems)
26-Moment of inertia of composite areas

Introduction:

Mechanics: is the physical science, which deals with the effects of forces on objects.

Scalar quantities: are those with which only a magnitude is associated. Examples of scalar quantities are time, volume, density, speed, energy,

Vector quantities: on the other hand, possess direction as well as magnitude, and must obey the parallelogram law of addition as de-scribed later in this article. Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum.

Units:

Mass: kilogram, kg -(1Kg=1000g)

Length: meter, m -(1m=1000mm)

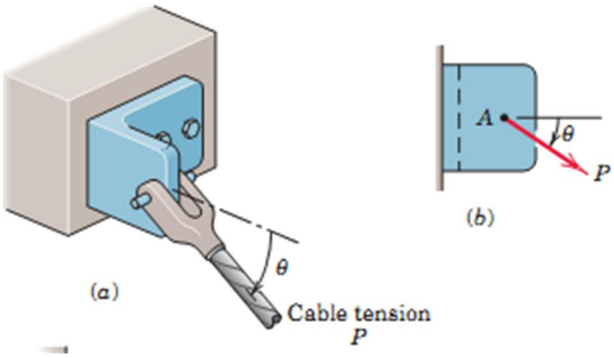
Force: Newton, N-(kN=1000N)

Weight: (N) =m (kg) * g (m/s^2)

Gravitational Acceleration, g=9.81 (m/s^2)

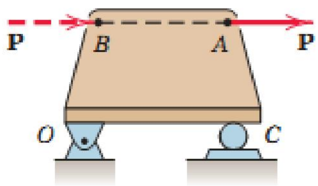
Force:

A force is a vector quantity, because its effect depends on the direction as well as on the magnitude of the action. Thus, forces may be combined according to the parallelogram law of vector addition.



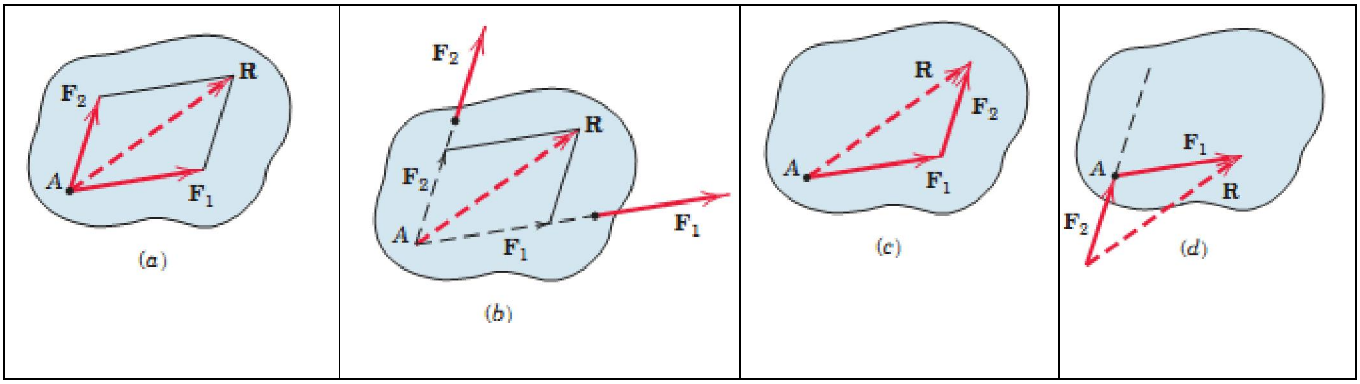
Principle of Transmissibility:

a force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts.



Concurrent Forces:

Two or more forces are said to be concurrent at a point if their lines of action intersect at that point.



Force Components:

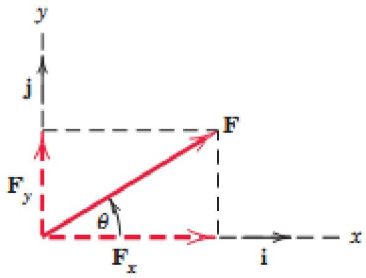
Rectangular Components:

1- Fx, Fy: known

Required: F, resultant force of Fx, Fy

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$



2- F: known

Required: force components Fx, Fy

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

Non-rectangular Components:

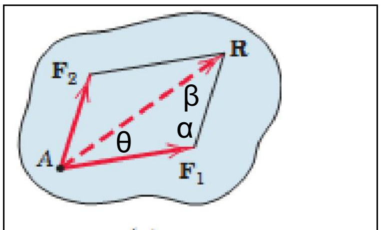
1- F1, F2: known

Required: R, resultant force of F1, F2

Parallelogram low:

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\alpha$$

$$\frac{R}{\sin\alpha} = \frac{F_2}{\sin\theta} \Longrightarrow \theta = \sin^{-1}(\frac{F_2}{R}\sin\alpha)$$



(a)

2- R: known

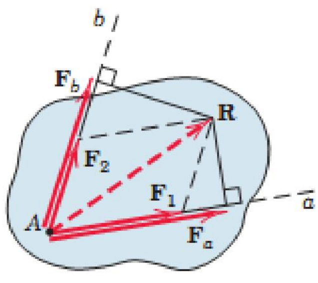
Required: force componentsF1, F2

$$\frac{R}{\sin\alpha} = \frac{F_2}{\sin\theta} \Longrightarrow F_2 = \frac{\sin\theta}{\sin\alpha}R$$

$$\frac{R}{\sin\alpha} = \frac{F_1}{\sin\beta} \Longrightarrow F_1 = \frac{\sin\beta}{\sin\alpha}R$$

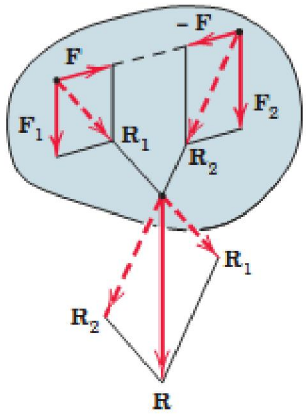
Relationship between force projections and components

The components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore, the vector sum of the projections Fa and Fb is not the vector R, because the parallelogram law of vector addition must be used to form the sum. The components and projections of R are equal only when the axes a and b are perpendicular.



(e)

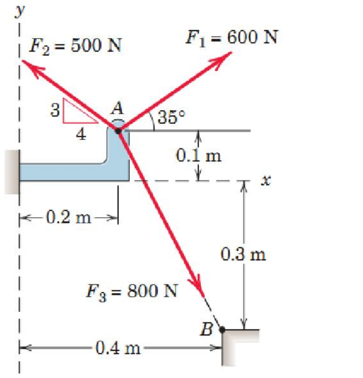
A Special Case of Vector Addition



To obtain the resultant when the two forces F1 and F2 are parallels in Fig. we use a special case of addition. The two vectors are combined by first adding two equal, opposite, and collinear forces F and –F of convenient magnitude, which taken together produce no external effect on the body. Adding F1 and F to produce R1, and combining with the sum R2 of F2 and -F yield the resultant R, which is correct in magnitude, direction, and line of action. This procedure is also useful for graphically combining two forces which have a remote and inconvenient point of concurrency because they are almost parallel.

Example 2-1

The forces F1, F2, and F3, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.



Solution. The scalar components of $\mathbf{F}_1$, from Fig. *a*, are

$$F_{1_x} = 600 \cos 35° = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N}$$

The scalar components of $\mathbf{F}_2$, from Fig. *b*, are

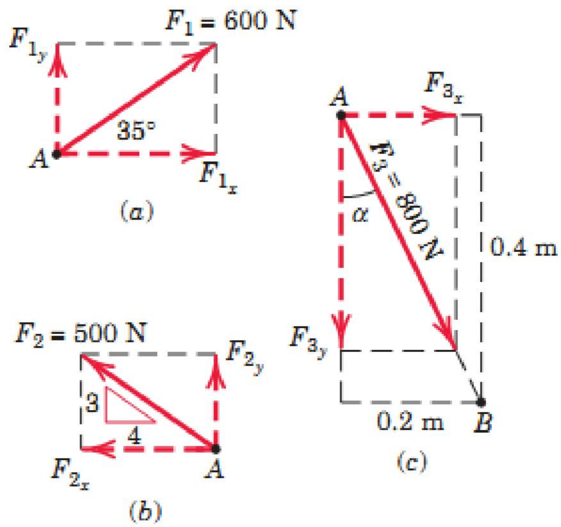
$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

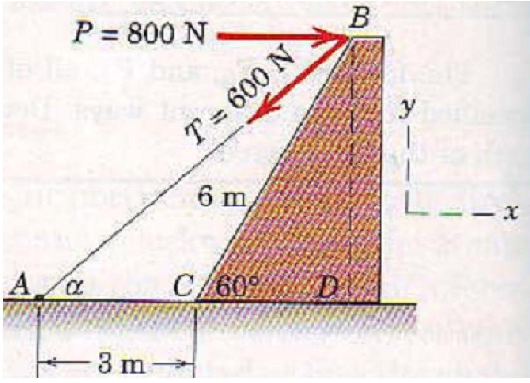
$$F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6° = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6° = -716 \text{ N}$$



Example 2-2

Combine the two forces P and T, which act on the fixed structure at B, into a single equivalent force R.

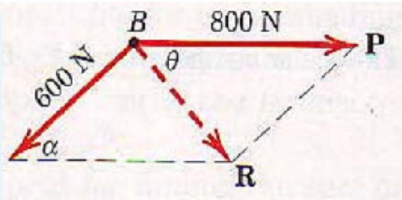


Graphical solution. The parallelogram for the vector addition of forces T and P is constructed as shown in Fig. a.

$$\tan\alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6\sin 60^\circ}{3 + 6\cos 60^\circ} = 0.866 \qquad \alpha = 40.9^\circ$$

Measurement of the length R and direction θ of the resultant force **R** yields the approximate results

$$R = 525\text{ N} \qquad \theta = 49^\circ \qquad \textit{Ans.}$$



(a)

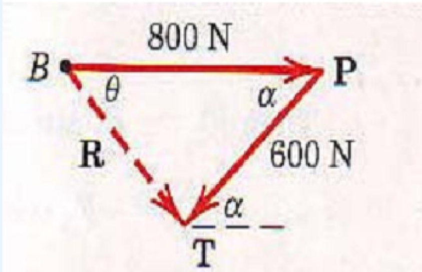
Geometric solution. The triangle for the vector addition of **T** and **P** is shown in Fig. *b*. The angle α is calculated as above. The law of cosines gives

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)\cos 40.9^\circ = 274{,}300$$

$$R = 524\text{ N} \qquad \textit{Ans.}$$

From the law of sines, we may determine the angle θ which orients **R**. Thus,

$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9^\circ} \qquad \sin\theta = 0.750 \qquad \theta = 48.6^\circ \qquad \textit{Ans.}$$

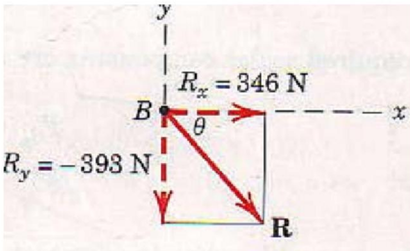


Algebric solution

By using the x-y coordinate system on the given figure, we may write

$$R_x = \Sigma F_x = 800 - 600\cos 40.9^\circ = 346\text{ N}$$

$$R_y = \Sigma F_y = -600\sin 40.9^\circ = -393\text{ N}$$



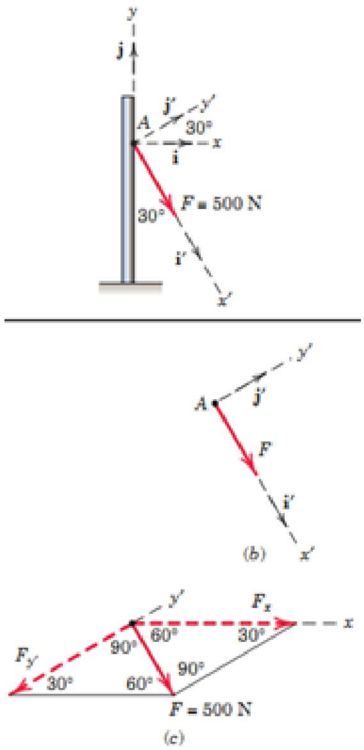
(c)

The magnitude and direction of the resultant force R as shown in Fig, c are then

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{(346)^2 + (-393)^2} = 524\text{ N}$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6^\circ$$

Example 2-3



The 500-N force F is applied to the vertical pole as shown(1) Determine the scalar components of the force vector F along the x'- and y'-axes. (2) Determine the scalar components of F along the x- and y'-axes.

Solution

Part (1).

Fx'= 500 N Fy'= 0

Part (2). The components of F in the x- and y'-directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig. c. The magnitudes of the components may be calculated by the law of sines. Thus,

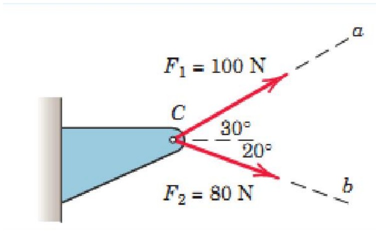
$$\frac{|F_x|}{\sin 90^\circ} = \frac{500}{\sin 30^\circ} \qquad |F_x| = 1000 \text{ N}$$

$$\frac{|F_{y'}|}{\sin 60^\circ} = \frac{500}{\sin 30^\circ} \qquad |F_{y'}| = 866 \text{ N}$$

The required scalar components are then

$$F_x = 1000 \text{ N} \qquad F_{y'} = -866 \text{ N}$$

Example 2-4

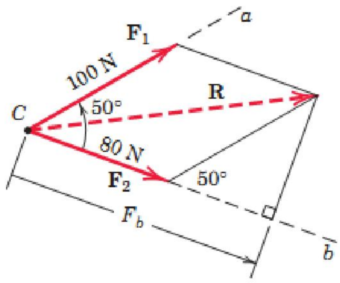


Forces F1 and F2 act on the bracket as shown. Determine the projection F_b of their resultant R onto the b-axis.

Solution:

The parallelogram addition of F1 and F2 is shown in the figure. Using the law of cosines gives us

$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130^\circ \qquad R = 163.4 \text{ N}$$

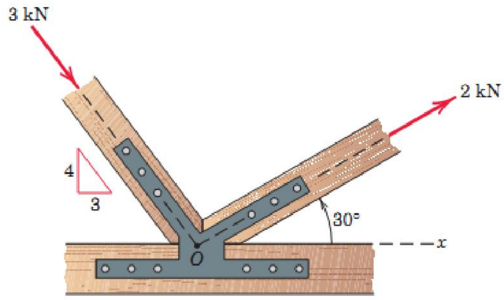


The figure also shows the orthogonal projection F_b of R onto the b-axis. Its length is

$$F_b = 80 + 100\cos 50^\circ = 144.3 \text{ N}$$

Note that the components of a vector are in general not equal to the projections of the vector onto the same axes. If the a-axis had been perpendicular to the b-axis, then the projections and components of R would have been equal.

Problem 2/7: The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant R of the two forces and the angle θ which R makes with the positive x-axis.



Solution:

Procedure 1:

$\beta = \tan^{-1}\frac{4}{3} = 53.13°$

$\alpha = 180 - 53.13 - 30 = 96.87°$

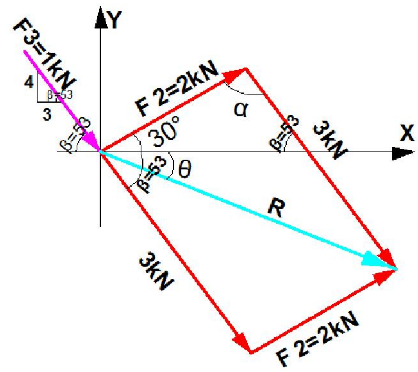
$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\alpha$

$R^2 = 2^2 + 3^3 - 2 * 2 * 3\cos 96.87$

$R = \sqrt{14.35} = 3.8N$

$\frac{R}{\sin\alpha} = \frac{F_1}{\sin(\theta+30)} \Rightarrow \frac{3.8}{\sin 96.87} = \frac{3}{\sin(\theta+30)} \Rightarrow \theta + 30 = \sin^{-1}(0.78)$

$\theta + 30 = 51.6 \Rightarrow \theta = 21.6°$



Procedure 2:

$F_{1x} = F_1\cos 53.13 = 3\cos 53.13 = 1.8N \rightarrow$

$F_{1y} = F_1\sin 53.13 = 3\sin 53.13 = -2.4N \downarrow$

$F_{2x} = F_2\cos 30 = 2\cos 30 = 1.73N \rightarrow$

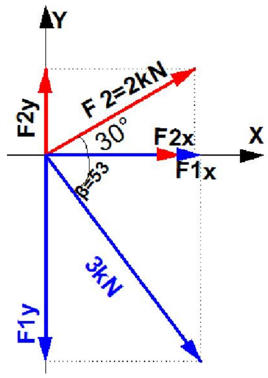
$F_{2y} = F_2\sin 30 = 2\sin 30 = 1N \uparrow$

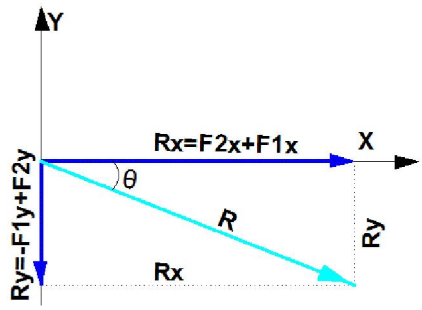
$R_x = F_{1x} + F_{2x} = 1.8 + 1.73 = 3.53N \rightarrow$

$R_y = -F_{1y} + F_{2y} = -2.4 + 1 = -1.4N \downarrow$

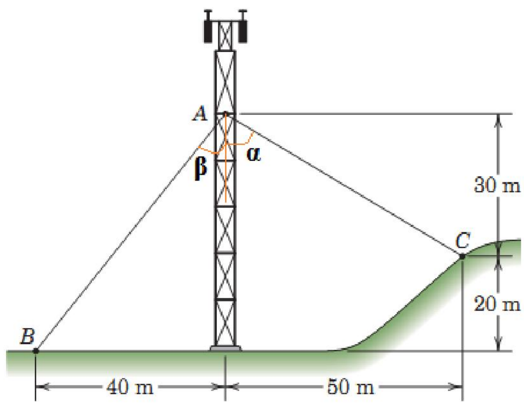
$R = \sqrt{R_1^2 + R_2^2} = \sqrt{3.53^2 + 1.4^2} = 3.8N$

$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{1.4}{3.53} = 21.6°$





Problem 2/13: The guy cables AB and AC are attached to the top of the transmission tower. The tension in cable AB is 8kN. Determine the required tension T in cable AC such that the net effect of the two cable tensions is a *downward* force at point A. Determine the magnitude R of this downward force.



Solution:

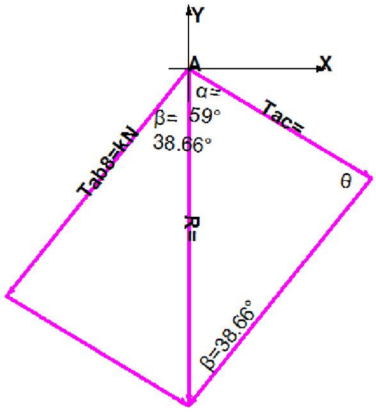
$\beta = \tan^{-1}\frac{40}{20+30} = 38.66°$

$\alpha = \tan^{-1}\frac{50}{30} = 59°$

Procedure 1:

$\theta = 180 - 59 - 38.66 = 82.34°$

$$\frac{T_{ab}}{\sin 59} = \frac{T_{ac}}{\sin 38.66} \Rightarrow T_{ac} = T_{ab}\frac{\sin 38.66}{\sin 59} \Rightarrow T_{ac} = 8 * \frac{\sin 38.66}{\sin 59} = 5.83N \ \ Ans$$



$$R^2 = {T_{ac}}^2 + {T_{ab}}^3 - 2T_{ac}T_{ab}\cos 82.34$$

$$R = \sqrt{85.56} = 9.25N$$

OR

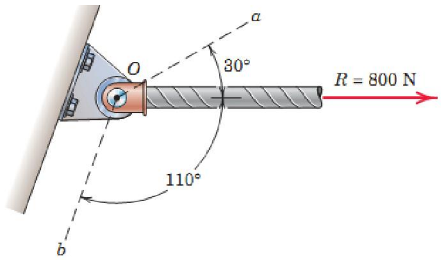
$$\frac{T_{ab}}{\sin 59} = \frac{R}{\sin 82.34} \Rightarrow R = -9.25N \downarrow$$

Procedure 2:

$$R_x = 0 \Rightarrow T_{ac}\sin 59 - T_{ab}\sin 38.66 = 0 \Rightarrow T_{ac}\sin 59 - 8\sin 38.66 = 0 \Rightarrow T_{ac} = 5.83N$$

$$R = R_y = T_{ac}\cos 59 + T_{ab}\cos 38.66 \Rightarrow R = 5.83\cos 59 + 8\cos 38.66 \Rightarrow R = -9.25N \downarrow$$

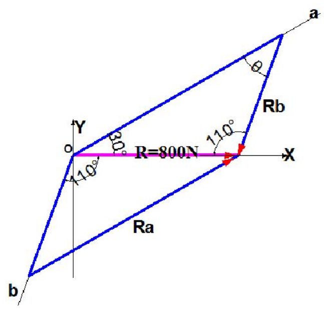
Problem 2/20: Determine the scalar components R_a and R_b of the force R along the *nonrectangular axes* a and b. Also determine the orthogonal projection P_a of R onto axis a.



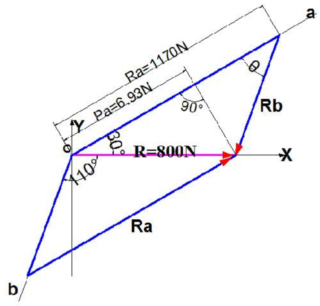
Solution:

$\theta = 180 - 110 - 30 = 40°$

$$\frac{R}{\sin 40} = \frac{R_a}{\sin 110} \Rightarrow \frac{800}{\sin 40} = \frac{R_a}{\sin 110} \Rightarrow R_a = 1170N$$



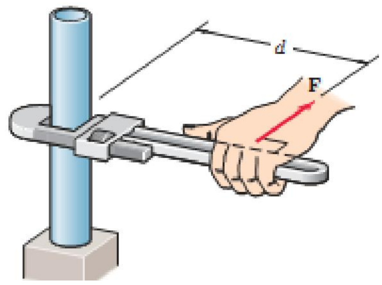
$$\frac{R}{\sin 40} = \frac{R_b}{\sin 30} \Rightarrow \frac{800}{\sin 40} = \frac{R_b}{\sin 30} \Rightarrow R_b = 622N$$



$$P_b = R\cos 30 \Rightarrow P_b = 800\cos 30 \Rightarrow P_b = 693N$$

Moment:

Moment M: is the tendency of a force to rotate a body about an axis in addition to move a body in the direction of its application. The axis may be any line, which neither intersects nor is parallel to the line of action of the force. Moment is also referred to as torque.

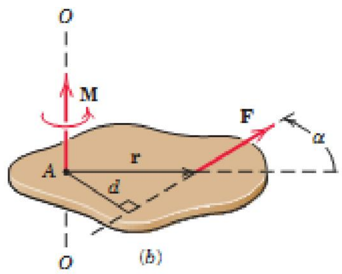


M=F*d

The magnitude of this tendency depends on both the magnitude F of the force and the effective length d of the wrench handle.

Moment about a Point

The magnitude of the moment or tendency of the force to rotate the body about the axis O-O perpendicular to the plane of the body is proportional both to the magnitude of the force F and to the moment arm d, *which is the perpendicular distance from the axis to the line of action of the force*.



M=F*d

Sense of Moment

- minus sign(-) for counterclockwise moments
- a plus sign (+) for clockwise moments

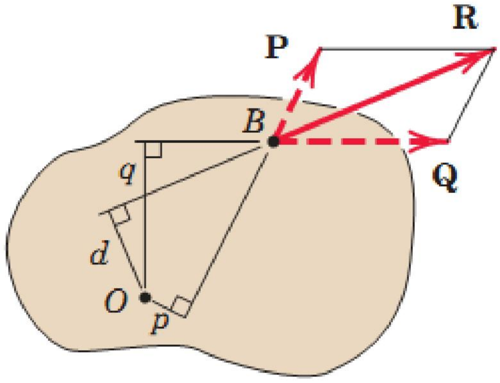
Sign consistency within a given problem is essential.

Basic Units Of moment: N.m

Varignon's Theorem:

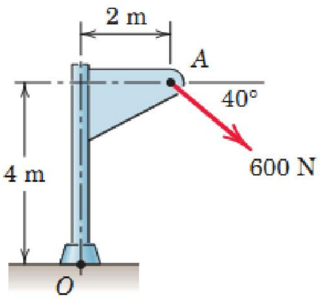
The moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

MO = R*d = Q*q – P*p



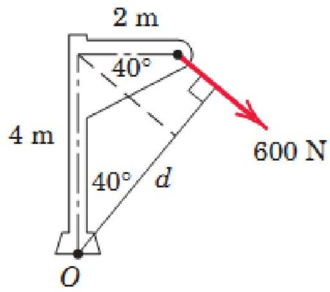
SAMPLE PROBLEM 2/5

Calculate the magnitude of the moment about the base point O of the 600Nforce in five different ways.



Solution:

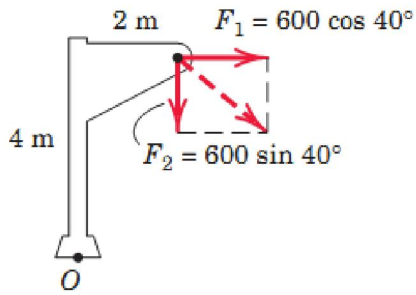
1. By M = F*d the moment is clockwise and has the magnitude



The moment arm to the 600-N force is:

d = 4 cos 40+ 2 sin 40= 4.35 m

Mo = 600(4.35) = 2610 N.m clockwise

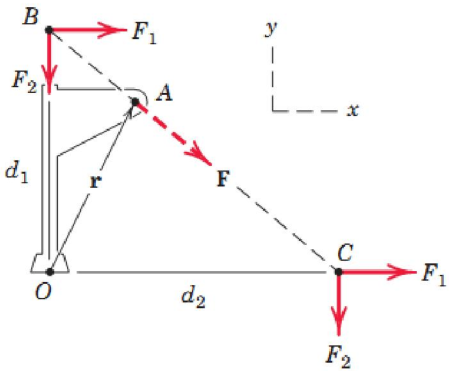


2. By Varignon's theorem,

F1 = 600 cos 40= 460 N,

F2 = 600 sin 40= 386 N

Mo= 460(4) +386(2) = 2610 N.m clockwise



3. By the principle of transmissibility, move the 600N force along its line of action to point B, which eliminates the moment of the component F2.

d1 = 4 + 2 tan 40= 5.68 m

Mo=460(5.68)=2610 N.m clockwise

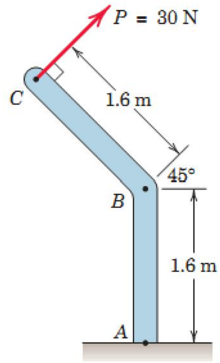
4. Moving the force to point C eliminates the moment of the componentF1. The moment arm of F2 becomes

d2=2+4 cot 40=6.77 m

Mo=386(6.77)=2610 N.m clockwise

5. By the vector expression for a moment

Problem 2.40: The 30-N force P is applied *perpendicular* to the portion BC of the bent bar. Determine the moment of P about point B and about point A.



Solution:

Moment about point B

$\circlearrowright M_B = P.d = 30 * 1.6 = 48\ N.m\ \circlearrowright OR\ c.w$

OR

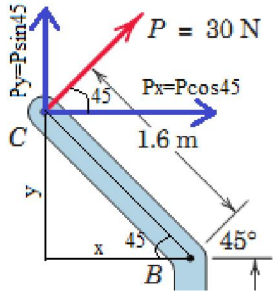
$P_x = P\cos 45 = 30\cos 45 = 21.2N$

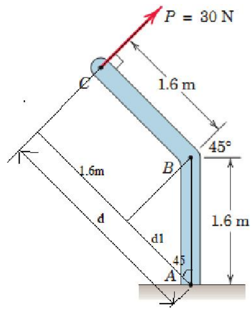
$P_y = P\sin 45 = 30\sin 45 = 21.2N$

$y = 1.6\sin 45 = 1.13m$

$x = 1.6\cos 45 = 1.13m$

$\circlearrowright M_B = P_x.y + P_y.x = 21.2 * 1.13 + 21.2 * 1.13 = 48\ N.m\ \circlearrowright OR\ c.w$

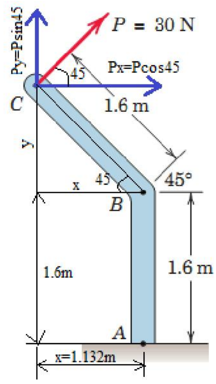




Moment about point A

$d = d_1 + 1.6 = 1.6\cos 45 + 1.6 = 2.73m$

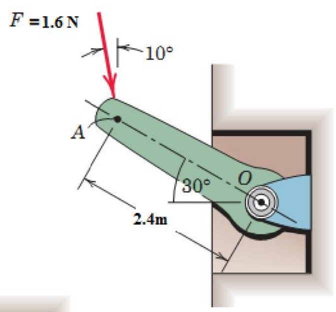
$\circlearrowright M_A = P.d = 30 * 2.73 = 81.9\ N.m\ \circlearrowright$



OR

$\circlearrowright M_A = P_x(y + 1.6) + P_y.x = 21.2(1.13 + 1.6) + 21.2 * 1.13$
$= 81.9\ N.m\ \circlearrowright$

Problem 2/41: Compute the moment of the 1.6 N force about the pivot O of the wall-switch toggle.



Solution:

Procedure1:

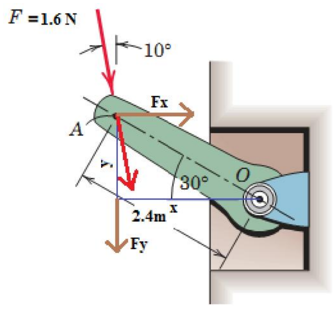
$F_x = F\sin 10 = 1.6\sin 10 = 0.28N$

$F_y = F\cos 10 = 1.6\cos 10 = 1.57N$

$y = 2.4\sin 30 = 1.2m$

$x = 2.4\cos 30 = 2.08m$

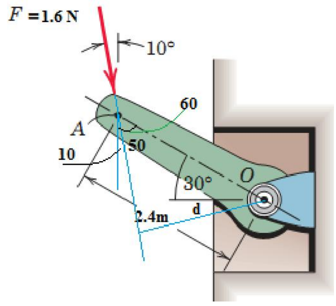
$\circlearrowright M_O = F_x.y - F_y.x = 0.28 * 1.2 - 1.57 * 2.08$
$= -2.93N.m\ \circlearrowleft$



Procedure2:

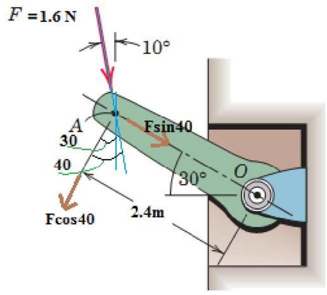
$d = 2.4 \sin 50 = 1.834m$

$\circlearrowleft M_O = F.d = 1.6 * 1.834 = 2.94 N.m \circlearrowleft$

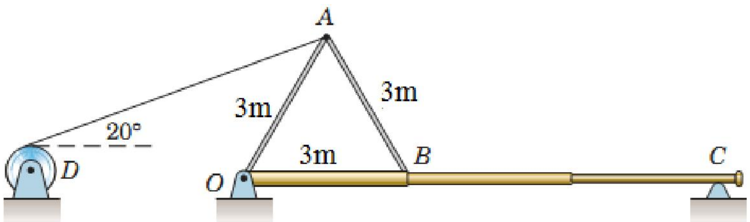


Procedure3:

$\circlearrowleft M_O = (F \cos 40) * 2.4 + (F \sin 40) * 0$
$= (1.6 \cos 40) * 2.4 + 0 = 2.94 N.m \circlearrowleft$



Problem 2/51: In order to raise the flagpole OC, a light frame OAB is attached to the pole and a tension of 1.2 kN is developed in the hoisting cable by the power winch D. Calculate the moment M_o of this tension about the hinge point O.



Solution:

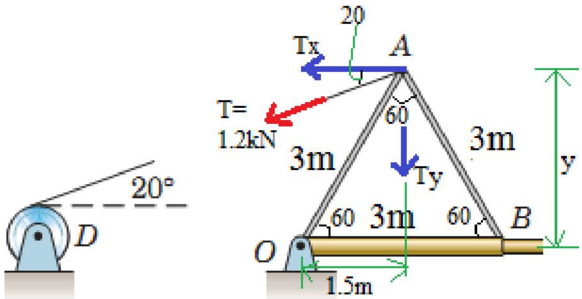
Procedure 1:

$3^2 = 1.5^2 + y^2 \Longrightarrow y = 2.6m \; Or \; y = 3 \sin 60$

$T_x = T \cos 20 = 1.2 * \cos 20 = 1.13 kN$

$T_y = T \sin 20 = 1.2 * \sin 20 = 0.41 kN$

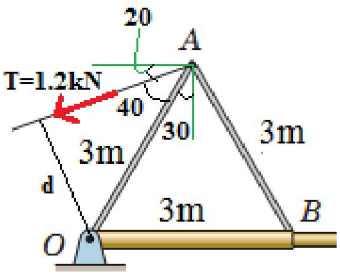
$\circlearrowright M_O = -T_x . y + T_y * 1.5$
$= -1.13 * 2.6 + 0.41 * 1.5$
$= -2.32 kN.m \circlearrowleft$



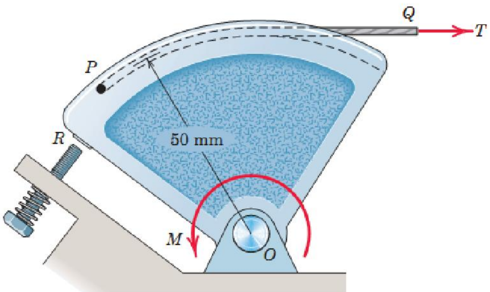
Procedure 2:

$d = 3 \sin 40 = 1.93m$

$\circlearrowright M_O = -T.d = -1.2 * 1.93 = -2.32 kN.m \circlearrowleft$



Problem 2/33: The throttle-control sector pivots freely at O. If an internal torsional spring exerts a return moment M=1.8 N.m on the sector when in the position shown, for design purposes determine the necessary throttle-cable tension T so that the net moment about O is zero. Note that when T is zero, the sector rests against the idle-control adjustment screw at R.



Solution:

$M = T.r$

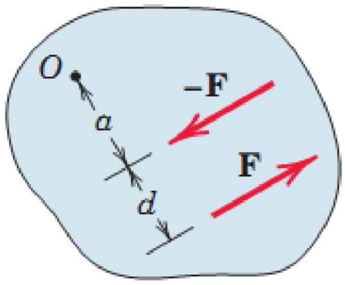
$1.8 = T * 0.05 \Longrightarrow T = 36 \; N$

Couple:

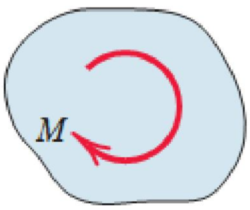
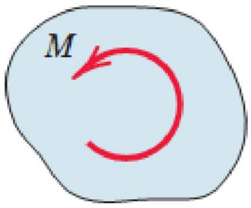
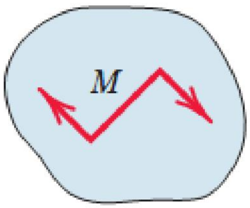
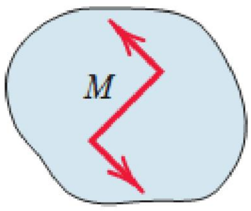
The moment produced by two equal, opposite, and non collinear forces

Mo=F(a+d)-Fa=F*a+F*d-F*a=F*d

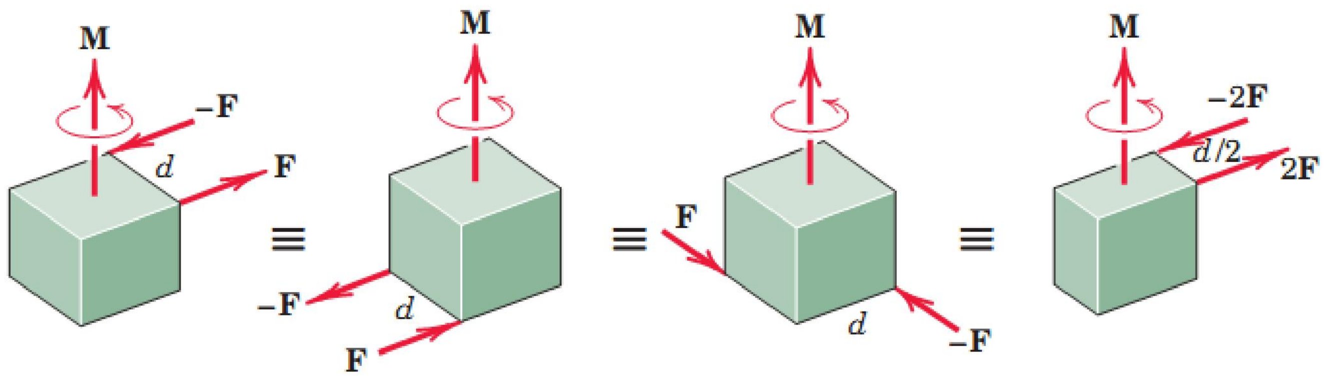
The magnitude of the couple is independent of the distance (a) which locates the forces with respect to the moment center O. It follows that the moment of a couple has the *same value* for all moment centers.



Sense of Couple:

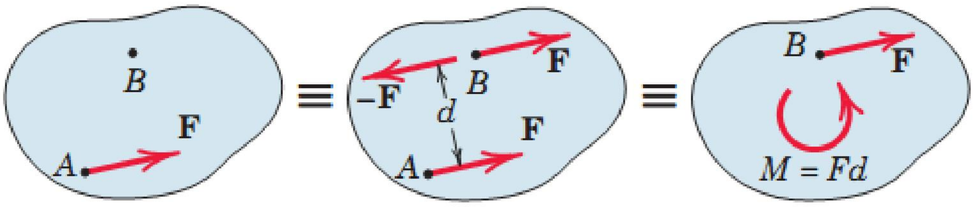
(+) Clockwise	(-) Counterclockwise
	
	

Equivalent Couples



Force–Couple Systems

The force F acting at point A is replaced by an equal force F at some point B and the counterclockwise couple M= Fd.



By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force.

SAMPLE PROBLEM 2/7

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces P and -P, each of which has a magnitude of 400 N. Determine the proper angle θ.

Solution.

The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

M= F*d=100(0.1)=10 N.m counterclockwise

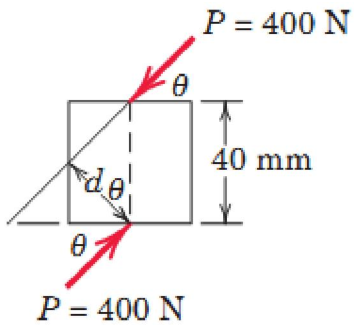
The forces P and -P produce a counterclockwise couple

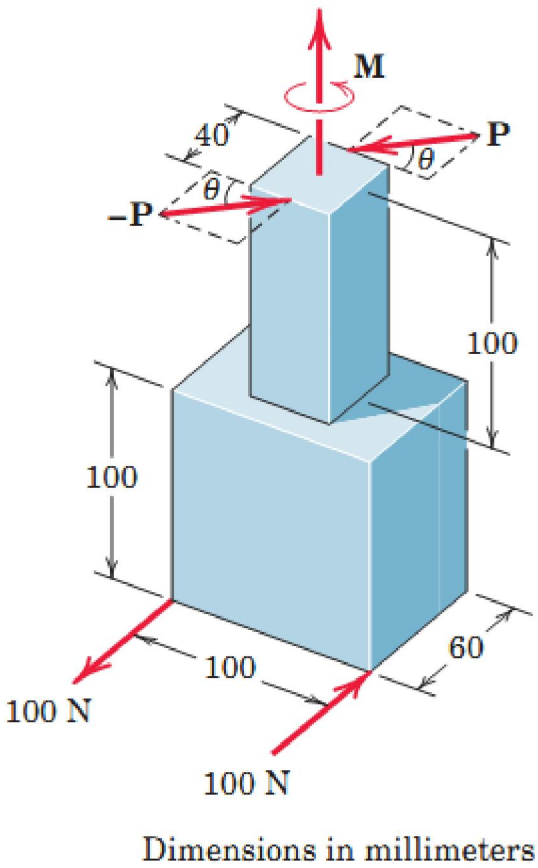
M=400(0.040) cosθ

Equating the two expressions gives

10=(400)(0.040) cosθ

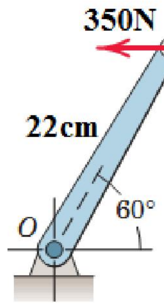
$\theta=\cos^{-1} 10/16=51.3°$





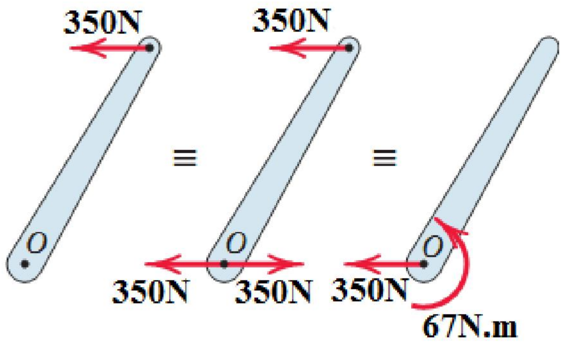
SAMPLE PROBLEM 2/8

Replace the horizontal 350 N force acting on the lever by an equivalent system consisting of a force at O and a couple.

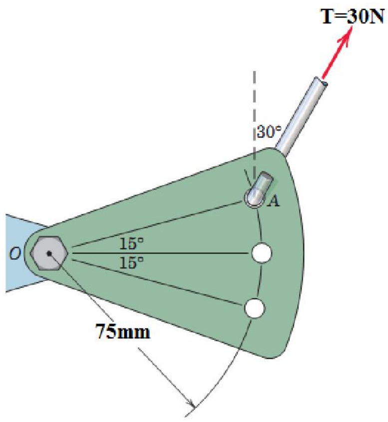


Solution.

We apply two equal and opposite 350N forces at O and identify the counterclockwise couple M=F*d=350(0.22 sin 60°)=67N.m. counterclockwise



Problem 2/65: The 30N force is applied by the control rod on the sector as shown. Determine the equivalent force–couple system at O.

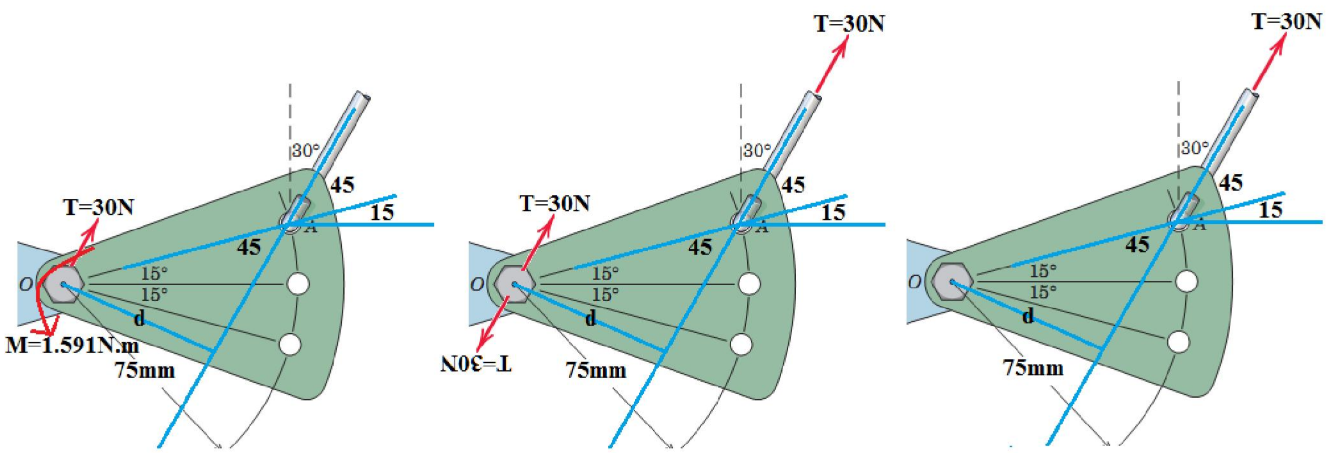


Solution:

Procedure1:

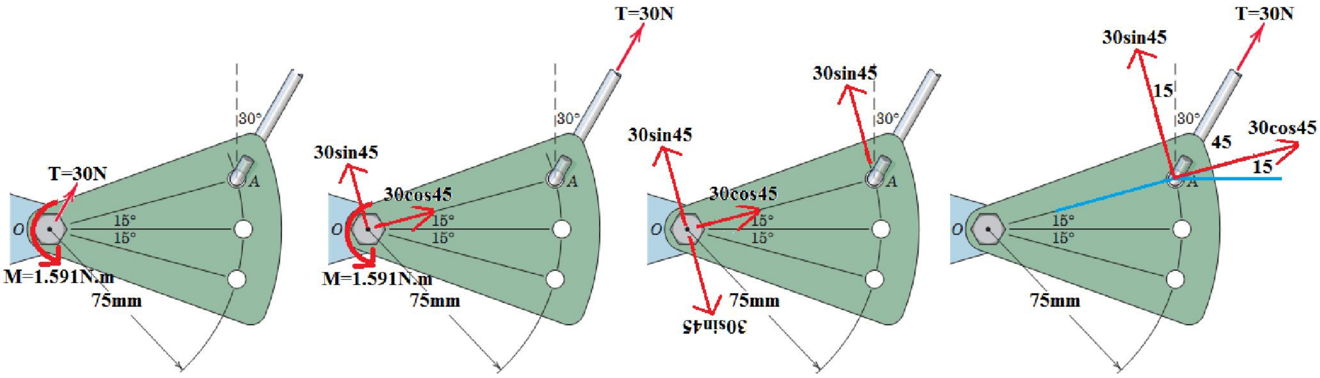
$d = 75 \sin 45 = 53mm$

$M = T.d = 30 * 0.53 = 1.591N.m$

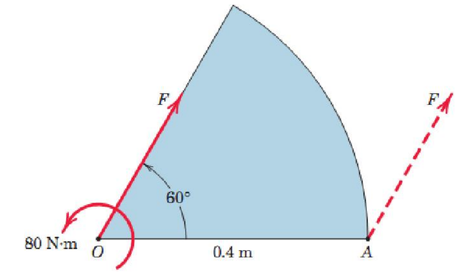


Procedure2:

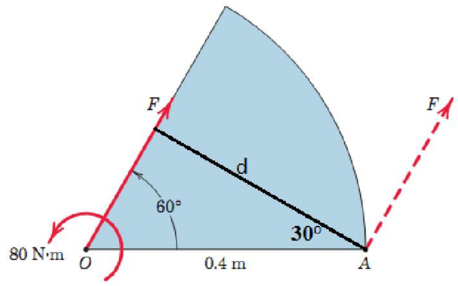
$M = (30 \sin 45) * 0.075 = 1.591 N.m$



Problem 2/70: A force–couple system acts at O on the 60° circular sector. Determine the magnitude of the force F if the given system can be replaced by a stand-alone force at corner A of the sector.



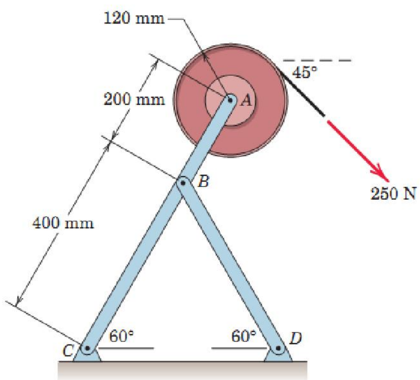
Solution:



$d = 0.4 \sin 60 = 0.346 m$

$M_o = F.d \Longrightarrow 80 = F * 0.346 \Longrightarrow F = 231 N$

Problem 2/74: The 250-N tension is applied to a cord which is securely wrapped around the periphery of the disk. Determine the equivalent force–couple system at point C. Begin by finding the equivalent force–couple system at A.



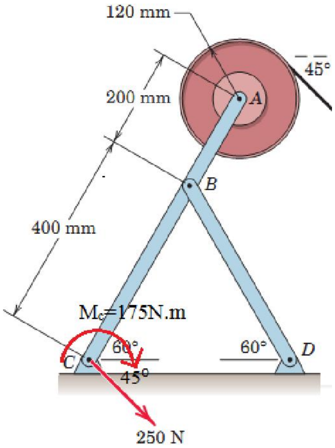
Solution:

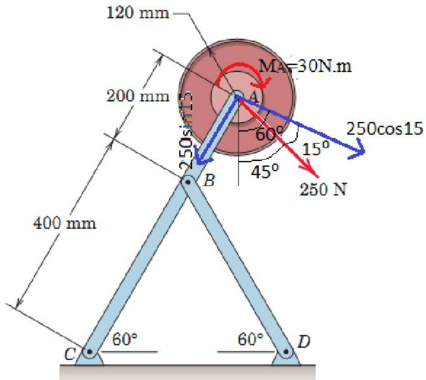
$M_A = F.r = 250 * 0.12 = 30 N.m \circlearrowright$

Procedure1:

$M_C = (250 \cos 15) * (0.4 + .02) + (250 \sin 15) * 0 + 30$

$= 175 N.m \circlearrowright$

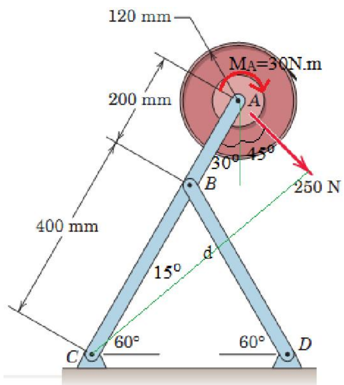




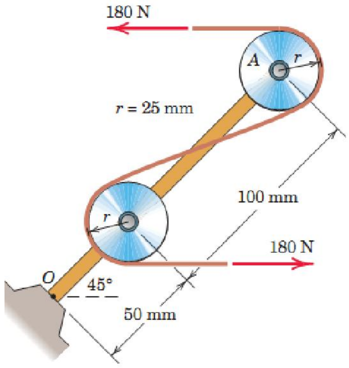
Procedure2:

$d = 0.6\cos 15 = 0.58m$

$M_C = 250 * 0.58 + 30 = 175N.m \circlearrowright$



Problem 2/75: The system consisting of the bar OA, two identical pulleys, and a section of thin tape is subjected to the two 180-N tensile forces shown in the figure. Determine the equivalent force–couple system at point O.



Solution:

Procedure 1:

$M_o = 180(r + 0.1\sin 45 + r)$

$M_o = 180(0.025 + 0.1\sin 45 + 0.025) = 21.74N.m \circlearrowleft$

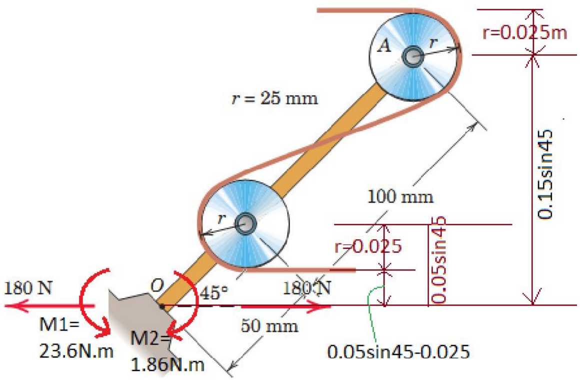
Procedure 2:

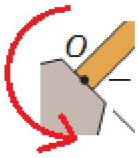
$\circlearrowright M_1 = -180(0.15\sin 45 + 0.025)$

$= -23.6N.m \circlearrowleft$

$\circlearrowright M_2 = 180(0.05\sin 45 - 0.025) = 1.86N.m \circlearrowleft$

$\circlearrowright M_o = 1.86 - 23.6) = -21.74N.m \circlearrowleft$





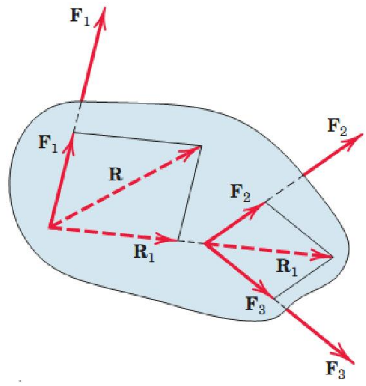
Mo=21.7N.m

Resultants:

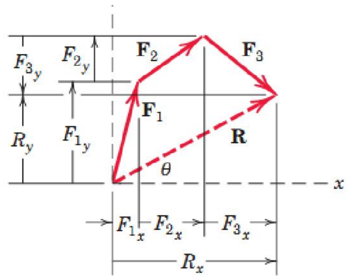
The resultant of a system of forces is the simplest force combination, which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Resultant force can be determinate by using one of the following:

1. Using parallelogram law.



2. Forming the force polygon



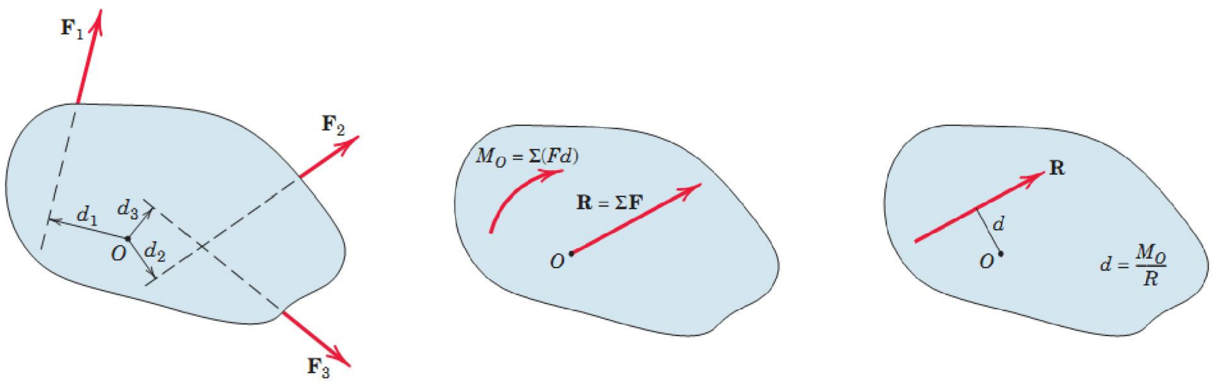
3. Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

- Resolve each force into horizontal component (Fx) and vertical component (Fy).
- Calculate $Rx = F1x + F2x + \cdots = \sum Fx$
- Calculate $Ry = F1y + F2y + \cdots = \sum Fy$
- Calculate $R = \sqrt{Rx^2 + Ry^2}$
- Calculate $\theta = \tan^{-1}(\frac{Rx}{Ry})$
- Find the line of action of R

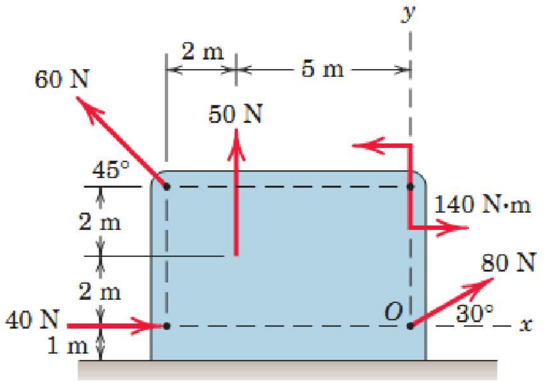
Mo=ΣM=F1*d1+F2*d2+F3*d3+…..=Σ(Fd)=R*d

d=Mo/R



SAMPLE PROBLEM 2/9

Determine the resultant of the four forces and one couple, which act on the plate shown.



Solution.

Rx=ΣFx=40+80 cos 30-60 cos 45=66.9 N⟶

Ry=ΣFy=50+80 sin 30+60 cos 45=132.4 N↑

$$R = \sqrt{Rx^2 + Ry^2} == \sqrt{66.9^2 + 132.4^2} = 148.3\ N$$

$$\theta = \tan^{-1}\left(\frac{Rx}{Ry}\right) = \tan^{-1}\left(\frac{66.9}{132.4}\right) = 63.2^{\circ}$$

Point O is selected as a convenient reference point for the force–couple system which is to represent the given system.

↺Mo=Σ(Fd)= 140-50(5)+60 cos 45*(4) -60 sin 45*(7)= -237 N.m↻

Mo=R*d⟹d=Mo/R=237/148.2=1.6m

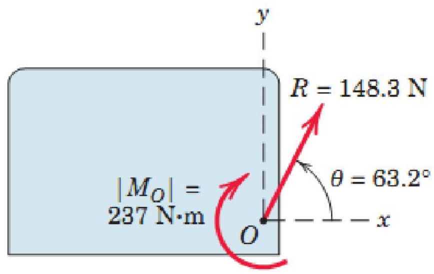
Intercept distance (b) to point C on the x-axis

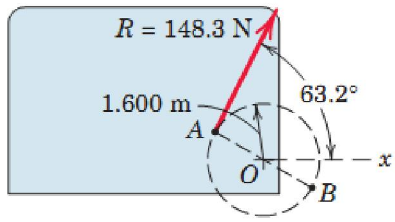
Mo=Ry*b⟹b=237/132.4=1.79m

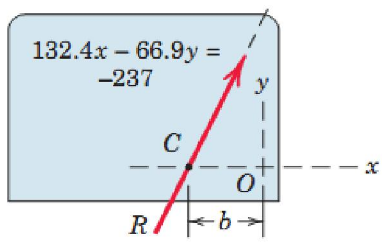
OR

Intercept distance (y) to point withe y-axis

Mo=Rx*y⟹y=237/66.9=3.55m







Problem 2/84: Determine and locate the resultant R of the two forces and one couple acting on the I-beam.

Solution:

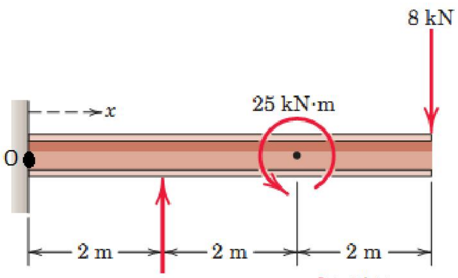
$$R_y = \uparrow^{+} \sum F_y = 5 - 8 = -3kN \downarrow$$

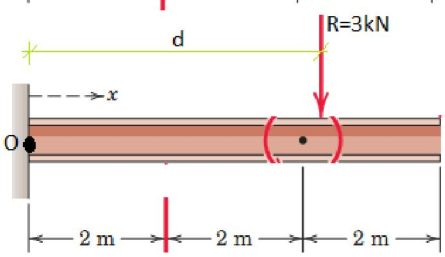
$$R_x = \rightarrow^{+} \sum F_x = 0$$

$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{0 + 3^2} = 3kN \downarrow$$

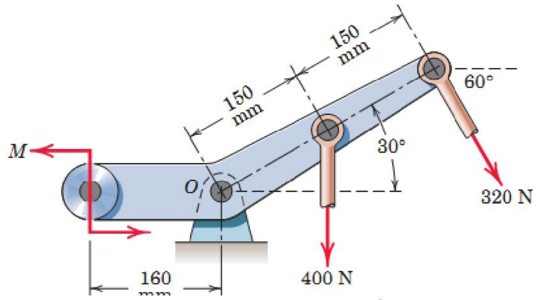
$$\circlearrowright^{+} \sum M_o = 8 * 6 - 5 * 2 - 25 = 13kN.m \circlearrowright$$

$$M_o = R.d \Longrightarrow d = \frac{13}{3} = 4.33m$$





Problem 2/88: If the resultant of the two forces and couple M passes through point O, determine M.

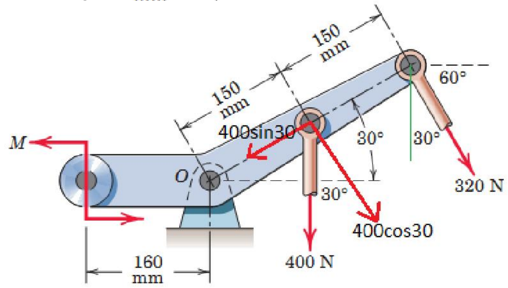


Solution:

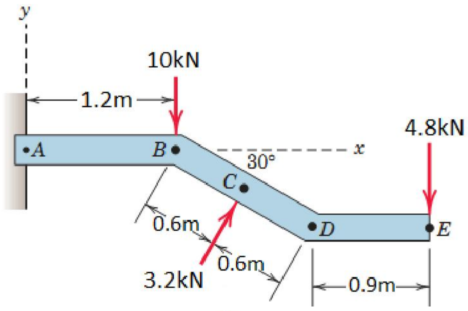
Since the resultant force pass through point O, hence, the summation of the moment about O equal to zero.

$$\circlearrowleft^{+} \sum M_o = (400\cos 30) * 0.15 + 320 * 0.3 - \mathrm{M} = 0$$

$$\mathrm{M} = 148\mathrm{N.m} \circlearrowleft$$



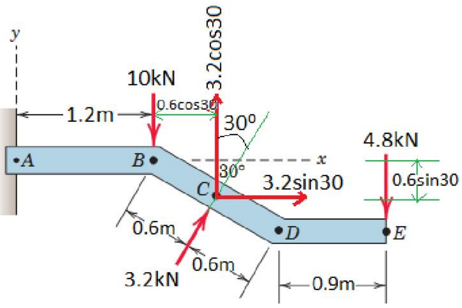
Problem 2/89: Replace the three forces which act on the bent bar by a force–couple system at the support point A. Then determine the x-intercept of the line of action of the stand-alone resultant force R.



Solution:

$$R_y = \uparrow^{+} \sum F_y = -10 + 3.2\cos 30 - 4.8 = -12.03kN \downarrow$$

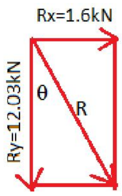
$$R_x = \rightarrow^{+} \sum F_x = 3.2\sin 30 = 1.6kN \longrightarrow$$



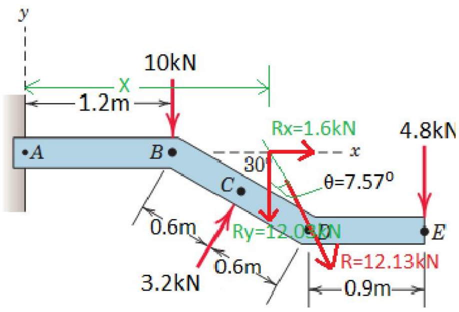
$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{1.6^2 + 12.03^2}$$

$$= 12.13kN$$

$$\theta = \tan^{-1}\frac{R_x}{R_y} = \tan^{-1}\frac{1.6}{12.03} = 7.576°$$

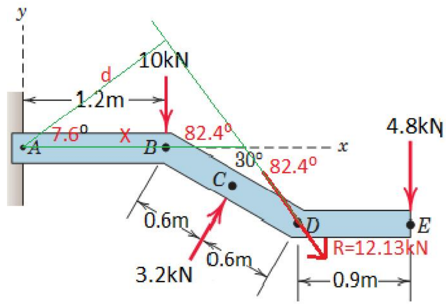


$$\circlearrowleft^{+} \sum M_A = 10 * 1.2 - (3.2\cos 30)(1.2 + 0.6\cos 30) - (3.2\sin 30)(0.6\sin 30) + 4.8(1.2 + 1.2\cos 30 + 0.9) = 21.82\mathrm{kN.m} \circlearrowleft$$



$$M_A = R_y * x \Longrightarrow x = \frac{21.82}{12.03} = 1.814m$$

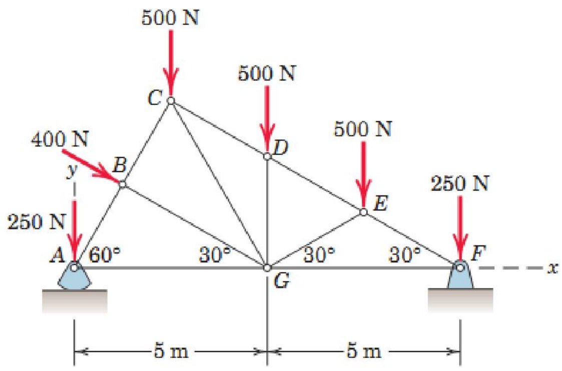
OR



$$M_A = R * d \Longrightarrow d = \frac{21.82}{12.13} = 1.799m$$

$$x = \frac{d}{\cos 7.6} = \frac{1.799}{\cos 7.6} = 1.814m$$

Problem 2/94: The asymmetric roof truss is of the type used when a near normal angle of incidence of sunlight onto the south-facing surface ABC is desirable for solar energy purposes. The five vertical loads represent the effect of the weights of the truss and supported roofing materials. The 400-N load represents the effect of wind pressure. Determine the equivalent force–couple system at A. Also, compute the x-intercept of the line of action of the system resultant treated as a single force R.

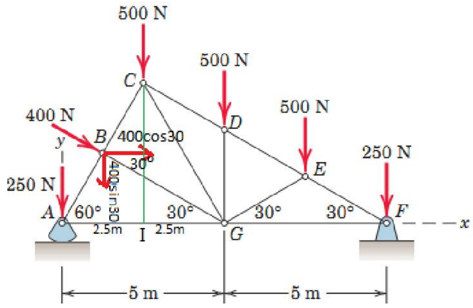


Solution:

$$AC = 10\cos 60 = 5m$$

$$CG^2 = AC^2 + AG^2 - 2AG * AC * \cos 60 \Longrightarrow CG = 5m$$

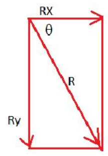
$$\therefore AI = 2.5m$$

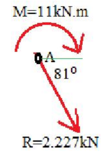


$$R_y = \downarrow^+ \sum F_y = 250 * 2 + 500 * 3 + 400\sin 30 = 2200N \downarrow$$

$$R_x = \rightarrow^+ \sum F_x = 400\cos 30 = 346N \longrightarrow$$

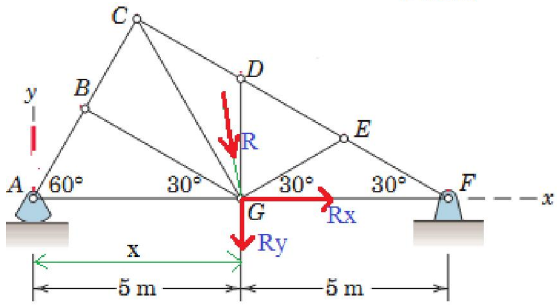
$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{346^2 + 2200^2} = 2227N$$





$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{2200}{346} = 81°$$

$$\circlearrowright^+ \sum M_A = 400 * 2.5 + 500(2.5 + 5 + 7.5) + 250 * 10 = 11000N.m$$



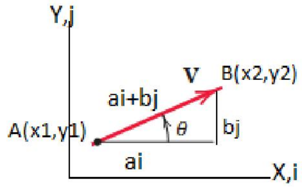
$$M_A = R_y * x \Longrightarrow x = \frac{11000}{2200} = 5m$$

Vectors:

A vector is represented by a line segment, having the direction of the vector and having an arrowhead to indicate thesense.

$\overrightarrow{AB} = ai + bj$

Length, $|\overrightarrow{AB}| = \sqrt{a^2 + b^2}$

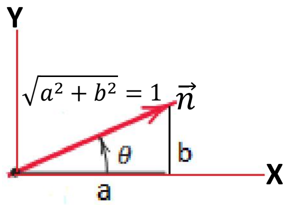


Unite Vector, $\vec{n}$

$a = \vec{n}_x = 1\cos\theta = \cos\theta$

$b = \vec{n}_y = 1\sin\theta = \sin\theta$

$\vec{n} = ai + bj = \cos\theta\ i + \underbrace{\sin\theta}_{\cos(90-\theta)}\ j$



To determine unite vector for $\overrightarrow{AB}$

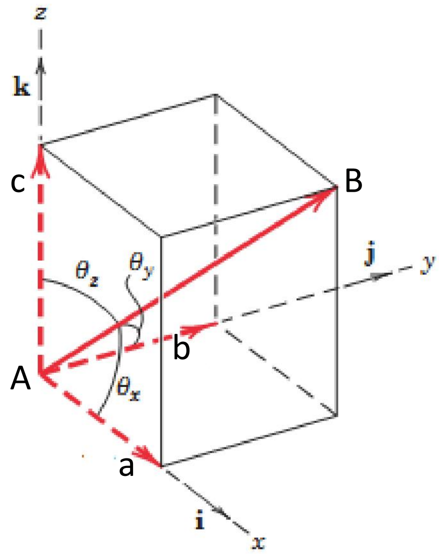
$$\vec{n}_{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\overbrace{a}^{x2-x1}\ i + \overbrace{b}^{y2-y1}\ j}{\sqrt{a^2+b^2}}$$

$$= \underbrace{\boxed{\frac{a}{\sqrt{a^2+b^2}}}}_{\cos\theta} i + \underbrace{\boxed{\frac{b}{\sqrt{a^2+b^2}}}}_{\sin\theta} j = \cos\theta\ i + \sin\theta\ j$$

Three Dimensional Vector,

$\overrightarrow{AB} = ai + bj + ck$

Length, $|\overrightarrow{AB}| = \sqrt{a^2 + b^2 + c^2}$



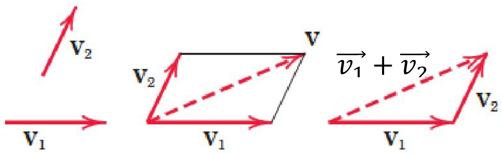
$$\vec{n}_{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\overbrace{a}^{x2-x1}\ i + \overbrace{b}^{y2-y1}\ j + \overbrace{c}^{z2-z1}\ k}{\sqrt{a^2+b^2+c^2}}$$

$$= \underbrace{\boxed{\frac{a}{\sqrt{a^2+b^2+c^2}}}}_{\cos\theta_x} i + \underbrace{\boxed{\frac{b}{\sqrt{a^2+b^2+c^2}}}}_{\cos\theta_y} j + \underbrace{\boxed{\frac{c}{\sqrt{a^2+b^2+c^2}}}}_{\cos\theta_z} k$$

$= \cos\theta_x\ i + \cos\theta_y\ j + \cos\theta_z\ k$

Note: $(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1$

Vector addition:

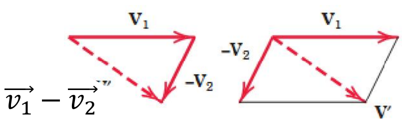


$\vec{v_1} = a_1 i + b_1 j + c_1 k$

$\vec{v_2} = a_2 i + b_2 j + c_2 k$

$\vec{v_1} + \vec{v_2} = (a_1 + a_2)i + (b_1 + b_2)j + (c_1 + c_2)k$

Vector subtraction:

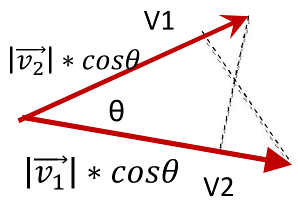


$\vec{v_1} - \vec{v_2} = (a_1 - a_2)i + (b_1 - b_2)j + (c_1 - c_2)k$

Dot or scalar product

$\overrightarrow{v_1}.\overrightarrow{v_2} = (a_1 i + b_1 j + c_1 k).(a_2 i + b_2 j + c_2 k) = a_1 a_2 + b_1 b_2 + c_1 c_2$

Also $\overrightarrow{v_1}.\overrightarrow{v_2} = |\overrightarrow{v_1}| * \underbrace{|\overrightarrow{v_2}| * cos\theta}_{proj\,\overrightarrow{v_2}\,on\,\overrightarrow{v_1}} = |\overrightarrow{v_2}| * \underbrace{|\overrightarrow{v_1}| * cos\theta}_{proj\,\overrightarrow{v_1}\,on\,\overrightarrow{v_2}}$



$$\cos\theta = \frac{\overrightarrow{v_1}.\overrightarrow{v_2}}{|\overrightarrow{v_1}| * |\overrightarrow{v_2}|} = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2} * \sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$$

$$= \cos\theta_{x1}\cos\theta_{x2} + \cos\theta_{y1}\cos\theta_{y2} + \cos\theta_{z1}\cos\theta_{z2}$$

Cross or vector product

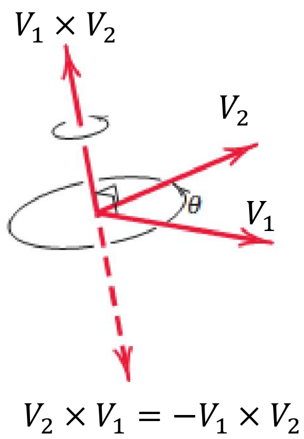
$v_1 \times v_2 = (a_1 i + b_1 j + c_1 k) \times (a_2 i + b_2 j + c_2 k)$

$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$= +(b_1 c_2 - b_2 c_1)i - (a_1 c_2 - a_2 c_1)j + (a_1 b_2 - a_2 b_1)k$

Also $\overrightarrow{v_1} \times \overrightarrow{v_2} = |\overrightarrow{v_1}| * |\overrightarrow{v_2}| * sin\theta$

Note, $\overrightarrow{v_2} \times \overrightarrow{v_1} = -\overrightarrow{v_1} \times \overrightarrow{v_2}$



THREE-DIMENSIONAL FORCE SYSTEMS

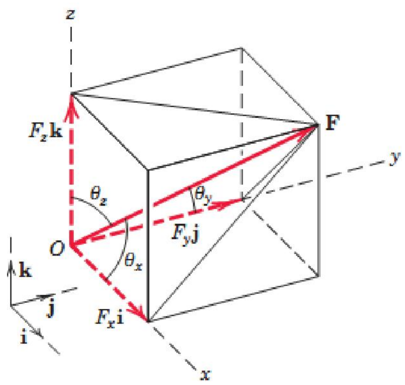
Rectangular Components

$F_x = F\cos\theta_x$

$F_y = F\cos\theta_y$

$F_z = F\cos\theta_z$

$F = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$



Or in vector notations:

$\vec{F} = F\,\vec{n}_F = F\,(\mathrm{i}\cos\theta_x + \,j\cos\theta_y + \mathrm{k}\cos\theta_z)$

i, j, and k : the unit vectors in the x-, y-, and z-directions, respectively.

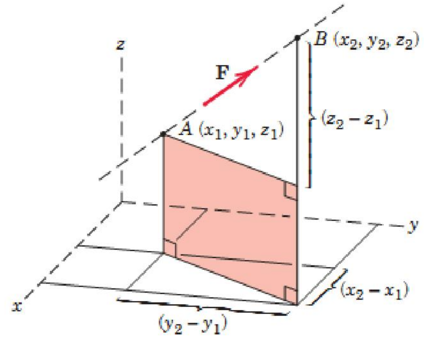
The choice of orientation of the coordinate system, use aright-handed set of axes in three-dimensional work to be consistent with the right-hand-rule definition of the cross product. When we rotate from the x- to the y-axis through the 90° angle, the positive direction for z-axis in a right-handed system is that of the advancement of aright-handed screw rotated in the same sense. This is equivalent to the right-hand rule.

The direction of a force is described:

(a) Specification by two points on the line of action of the force.

$$\vec{F} = F\,\vec{n}_F = F\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = F\frac{(x2-x1)i+(y2-y1)j+(z2-z1)k}{\sqrt{(x2-x1)^2+(y2-y1)^2+(z2-z1)^2}}$$

Thus the x, y, and z scalar components of F are the scalar coefficients of the unit vectors i, j, and k, respectively.



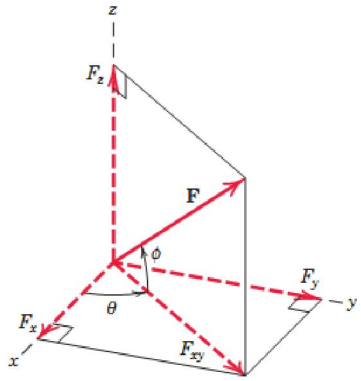
(b) Specification by two angles which orient the line of action of the force.

F_z= F sin Ø

F_{xy}= F cosØ

F_x=F_{xy}cosθ=F cosØ cosθ

F_y=F_{xy}sinθ= F cosØ sinθ



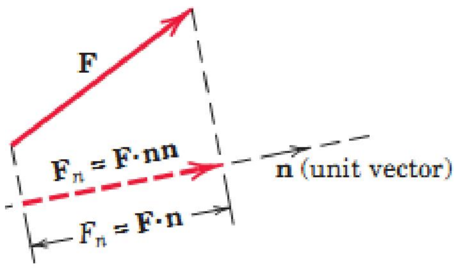
The projection of vector force along oblique line: (benefit of dot product)

If$\vec{n}$ is a unit vector in a specified direction,the projection of $\vec{F}$ in the n-direction, has the magnitude

$F_n = \vec{F}.\vec{n}$ Scalar quantity

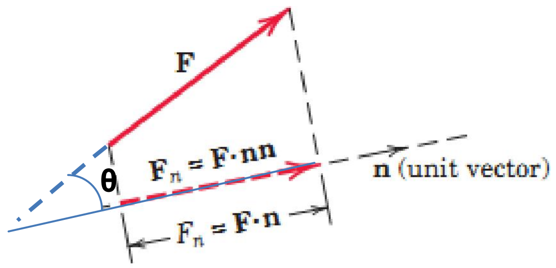
The projection in the n-direction as a vector quantity,

$\vec{F}_n = (\vec{F}.\vec{n})\vec{n}$ Vector quantity



Angle between Two Vectors

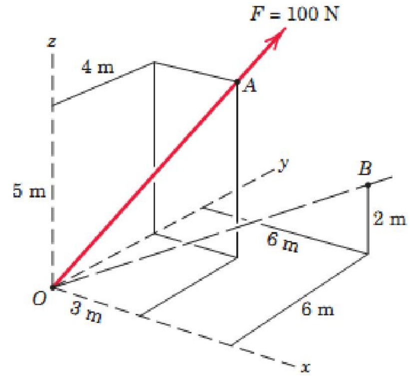
$$\theta = \cos^{-1}\left(\frac{\vec{F}.\vec{n}}{|\vec{F}|}\right) = \cos^{-1}\left(\frac{\vec{F}.\vec{n}}{\sqrt{a^2+b^2+c^2}}\right) = \cos^{-1}\left(\frac{\vec{F}.\vec{n}}{F}\right)$$



SAMPLE PROBLEM 2/10

A force F with a magnitude of 100 N is applied at the origin O of the axes x-y-z as shown. The line of action of F passes through a point A whose coordinates are 3 m, 4 m, and 5 m.

Determine (a) the x, y, and z scalar components of **F**, (b) the projection F_{xy} of **F** on the x-y plane, and (c) the projection FOB of **F** along the line OB.



Solution.

Part (a). We begin by writing the force vector F as its magnitude F times a unit vector $\vec{n}_{OA}$.

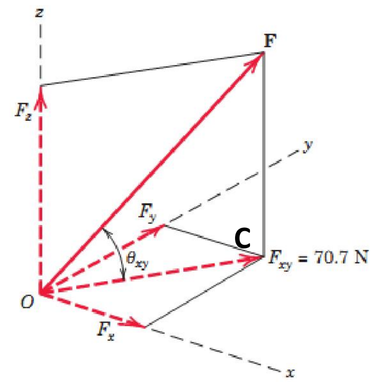
$$\vec{n}_{OA} = \frac{\overrightarrow{OA}}{|\overrightarrow{OA}|} = \frac{(x2-x1)i+(y2-y1)j+(z2-z1)k}{\sqrt{(x2-x1)^2+(y2-y1)^2+(z2-z1)^2}}$$

$$= \frac{(3-0)i+(4-0)j+(5-0)k}{\sqrt{(3-0)^2+(4-0)^2+(5-0)^2}} = 0.424i + 0.565j + 0.707k$$

$$\vec{F} = F\,\vec{n}_{OA} = 100\left(\overbrace{0.424}^{\cos\theta_x}i + \overbrace{0.565}^{\cos\theta_y}j + \overbrace{0.707}^{\cos\theta_z}k\right)$$

$$= 42.4i + 56.5j + 70.7k$$

Fx=42.4 N, Fy=56.6 N, Fz=70.7 N



Part (b).

$$\vec{n}_{OC} = \frac{\overrightarrow{OC}}{|\overrightarrow{OC}|} = \frac{(3-0)i+(4-0)j+(0-0)k}{\sqrt{(3-0)^2+(4-0)^2+(0-0)^2}} = 0.6i + 0.8j$$

Projection of $\vec{F}$ on vector OC (i.e on X-Y plane):

$$F_{OC} = \vec{F}.\vec{n}_{OC} = (42.4i + 56.5j + 70.7k).(0.6i + 0.8j)$$

$$= (42.4 * 0.6) + (56.5 * 0.8) + 0 = 70.72\ N$$

Or ,the tan of the angle θ_{xy} between $\vec{F}$ and the x-y plane is

$$\theta_{xy} = \tan^{-1}\frac{5}{\sqrt{(3)^2+(4)^2}} = 45^\circ$$

$$F_{xy} = F\cos\theta_{xy} = 100\ cos45 = 70.7\ N$$

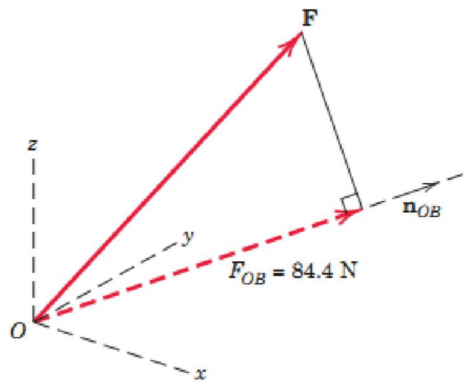
Part (c):

$$\vec{n}_{OB} = \frac{\overrightarrow{OB}}{|\overrightarrow{OB}|} = \frac{(6-0)i+(6-0)j+(2-0)k}{\sqrt{(6-0)^2+(6-0)^2+(2-0)^2}} = 0.688i + 0.688j + 0.229k$$

$$F_{OB} = \vec{F}_{OA}.\vec{n}_{OB} = (42.4i + 56.5j + 70.7k).(0.688i + 0.688j + 0.229k)$$

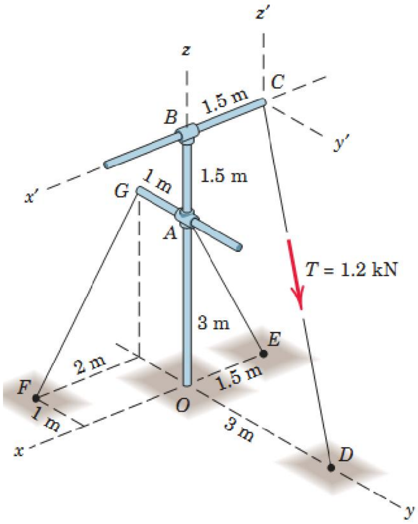
$$= (42.4 * 0.688) + (56.5 * 0.688) + (70.07 * 0.229) = 84.3\ N$$

$$\vec{F}_{OB} = F_{OB} * \vec{n}_{OB} = 84.3 * (0.688i + 0.688j + 0.229k) = 58i + 58j + 19.3k$$



Prob. 2/107

The rigid pole and cross-arm assembly is supported by the three cables shown. A turnbuckle at D is tightened until it induces a tension T in CD of 1.2 kN. Express T as a vector. Does it make any difference in the result which coordinate system is used?



Prob. 2/108

Use the result cited for Prob. 2/107 and determine the magnitude T_{GF} of the projection of T onto line GF.

Solution:

1.

C(-1.5,0,4.5), D(0,3,0)

$$\vec{n}_{cd} = \frac{\overrightarrow{cd}}{|\overrightarrow{cd}|} = \frac{(0+1.5)i + (3-0)j + (0-4.5)k}{\sqrt{(1.5)^2 + (3)^2 + (4.5)^2}}$$

$$= 0.267i + 0.534j - 0.962k$$

$$\vec{T}_{cd} = T\,\vec{n}_{cd} = 1.2(0.267i + 0.534j - 0.962k) = 0.32i + 0.64j - 0.962k$$

2.

G(0,-1,3), F(2,-1,0)

$$\vec{n}_{GF} = \frac{\overrightarrow{GF}}{|\overrightarrow{GF}|} = \frac{(2-0)i + (-1+1)j + (0-3)k}{\sqrt{(2)^2 + (0)^2 + (3)^2}} = 0.555i + 0j - 0.832k$$

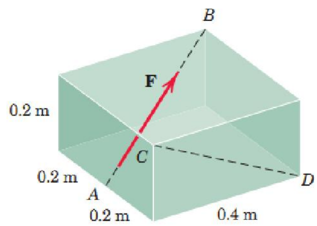
$$T_{GF} = \vec{T}_{cd}.\vec{n}_{GF} = (0.32i + 0.64j - 0.962k).(\,0.555i + 0j - 0.832k)$$

$$= (0.32 * 0.555) + (0) + (-0.962 * -0.832) = 0.978\ kN$$

$$\vec{T}_{GF} = T_{GF} * \vec{n}_{GF} = 0.978 * (0.555i - 0.832k) = 0.542i - 0.813k$$

Prob. 2/110

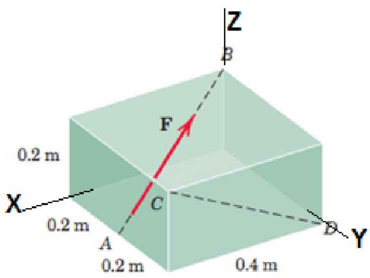
The force F has a magnitude of 2 kN and is directed from A to B. Calculate the projection of F onto line CD and determine the angle between F and CD.



Solution:

A(0.4,0.2,0), B(0,0,0.2)

$$\vec{n}_{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{(0-0.4)i+(0-0.2)j+(0.2-0)k}{\sqrt{(0.4)^2+(0.2)^2+(0.2)^2}} = -0.816i - 0.408j + 0.408k$$



$$\vec{F}_{AB} = F\,\vec{n}_{AB} = 2(-0.816i - 0.408j + 0.408k)$$

$$= -1.633i - 0.816j + 0.816k$$

C(0.4,0.4,0.2), D(0,0.4,0)

$$\vec{n}_{CD} = \frac{\overrightarrow{CD}}{|\overrightarrow{CD}|} = \frac{(0-0.4)i + (0.4-0.4)j + (0-0.2)k}{\sqrt{(0.4)^2 + (0)^2 + (0.2)^2}}$$

$$= -0.894i - 0.447k$$

$$F_{CD} = \vec{F}_{AB}.\vec{n}_{CD} = (-1.633i - 0.816j + 0.816k).(\,-0.894i - 0.447k)$$

$$= (-1.633 * -0.894) + (0) + (0.816 * -0.447) = 1.095\ N$$

$$\vec{F}_{CD} = F_{CD} * \vec{n}_{CD} = 1.095 * (-0.894i - 0.447k) = -0.979i - 0.489k$$

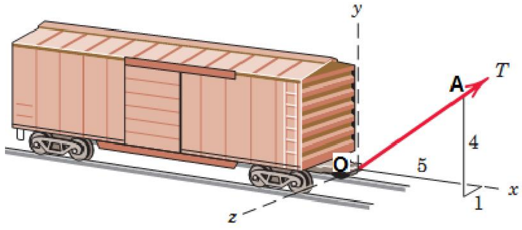
$$\vec{F}_{AB}.\vec{n}_{CD} = |\vec{F}_{AB}| * |\vec{n}_{CD}| * cos\theta \rightarrow$$

$$cos\theta = \frac{\vec{F}_{AB}.\vec{n}_{CD}}{|\vec{F}_{AB}| * |\vec{n}_{CD}|} = \frac{\vec{F}_{AB}.\vec{n}_{CD}}{\underbrace{|\vec{F}_{AB}|}_{\sqrt{1.633^2+0.816^2+0.816^2}=2} * \underbrace{|\vec{n}_{CD}|}_{=1}} = \frac{1.095}{2*1} = 0.547 \rightarrow$$

$$\theta = \cos^{-1} 0.547 = 56.8°$$

Prob. 2/115

An overhead crane is used to reposition the boxcar within a railroad car-repair shop. If the boxcar begins to move along the rails when the x-component of the cable tension reaches 3 kN, calculate the necessary tension T in the cable. Determine the angle θ_{xy} between the cable and the vertical x-y plane.



Solution:

O(0,0,0), A(5,4,1)

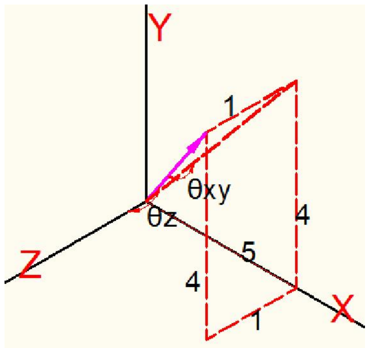
$$\vec{n}_T = \frac{\overrightarrow{OA}}{|\overrightarrow{OA}|} = \frac{5i + 4j + 1k}{\sqrt{5^2 + 4^2 + 1^2}} = 0.771i + 0.671j + 0.154k$$

$$\vec{T} = T\,\vec{n}_T = T(0.771i + 0.671j + 0.154k)$$

$$T_x = 0.771T = 3 \rightarrow T = \frac{3}{0.771} = 3.89\ kN$$

$$\cos\theta_z = 0.154 \rightarrow \theta_z = \cos^{-1} 0.154 = 81.14°$$

$$\theta_{xy} = 90 - \theta_z = 90 - 81.14 = 8.86$$

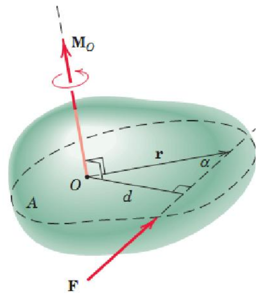


Moment and Couple (3D):

In two-dimensional analyses it is often convenient to determine a moment magnitude by scalar multiplication using the moment-arm rule. In three dimensions, however, the determination of the perpendicular distance between a point or line and the line of action of the force can be a tedious computation. A vector approach with cross-product multiplication then becomes advantageous.

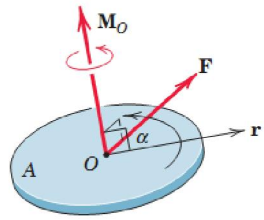
Moments in Three Dimensions

Consider a force F with a given line of action acting on a body, and any point O not on this line. Point O and the line of F establish a plane A. The moment Mo of F about an axis through O normal to the plane has the magnitude Mo=Fd, where d is the perpendicular distance from O to the line of F. This moment is also referred to as the moment of F about the point O.



The vector Mo is normal to the plane and is directed along the axis through O. We can describe both the magnitude and the direction of Mo by *the vector cross-product* relation. The vector **r** *runs from O to any point on the line of action of F*. the cross product of rand F is written **r×F** and has the magnitude (r sin α)F, which is the same as F*d, the magnitude of Mo.

The correct direction and sense of the moment are established by the right-hand rule. Thus, with r and F treated as free vectors emanating from O. The thumb points in the direction of Mo if the fingers of the right hand curl in the direction of rotation from r to F through the angle α. Therefore, we may write the moment of F about the axis through O as:



$$Mo = \vec{r} \times \vec{F}$$

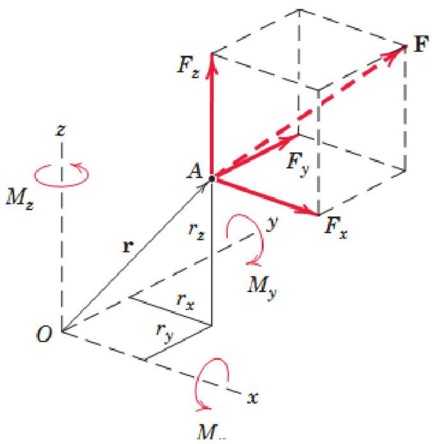
Note:

$$\vec{r} \times \vec{F} = Mo \quad \text{while} \quad \vec{F} \times \vec{r} = -Mo$$

Evaluating the Cross Product

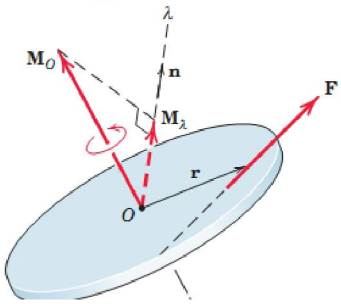
$$Mo = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= +\left(\underbrace{r_yF_z - r_zF_y}_{M_x}\right)i - \left(\underbrace{r_xF_z - r_zF_x}_{M_y}\right)j + \left(\underbrace{r_xF_y - r_yF_x}_{M_z}\right)k$$



Moment about an Arbitrary Axis

To expression for the moment $\overrightarrow{M_\lambda}$of $\vec{F}$ about any axis λ through O, as shown in Fig. If $\vec{n}_\lambda$ is a unit vector in the λ-direction, then we can use the dot-product expression for the component of a vector to obtain $\overrightarrow{M_o}.\vec{n}_\lambda$ (the component of $\overrightarrow{M_o}$ in the direction of λ). This scalar is the magnitude of the moment$M_\lambda$of $\vec{F}$ about λ



$$M_\lambda = \underbrace{\vec{r} \times \vec{F}}_{Mo}.\vec{n}_\lambda \qquad \text{Scalar quantity}$$

To obtain the vector expression for the moment $\overrightarrow{M_\lambda}$ of $\vec{F}$ about λ, multiply the magnitude by the directional unit vector $\vec{n}_\lambda$ to obtain

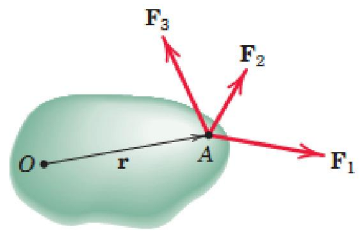
$$\overrightarrow{M_\lambda} = (\underbrace{\vec{r} \times \vec{F}}_{Mo} . \vec{n}_\lambda)\vec{n}_\lambda$$ Vector quantity

Varignon's Theorem in Three Dimensions

The sum of the moments of concurrent forces F1, F2, F3,...about O of these forces is

$$\vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \vec{r} \times \vec{F}_3 + \cdots = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots\right) = \vec{r} \times \underbrace{\sum \vec{F}}_{=Resltant}$$

$$\therefore \vec{M}_o = \sum(\vec{r} \times \vec{F}) = \vec{r} \times \vec{R}$$

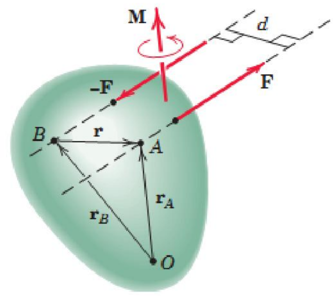


Couples in Three Dimensions

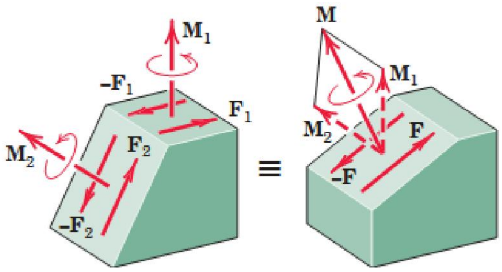
The two equal and opposite forces $\vec{F}$ and -$\vec{F}$ acting on a body. The vector $\vec{r}$ runs from any point Bon the line of action of -$\vec{F}$ to any point A on the line of action of $\vec{F}$.Points A and B are located by position vectors $\vec{r}_A$ and $\vec{r}_B$ from any pointO. The combined moment of the two forces about O is

$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right) = (\underbrace{\vec{r}_A - \vec{r}_B}_{=\vec{r}\text{ vector subtraction}}) \times \vec{F}$$

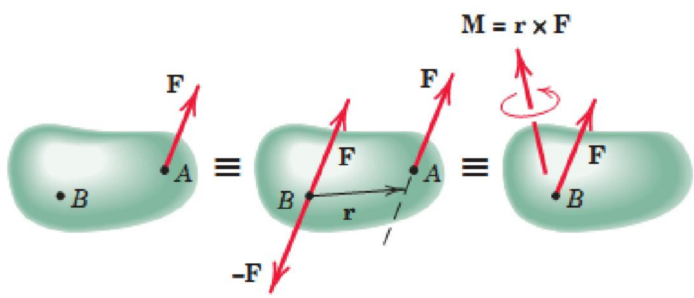
$$\therefore \vec{M} = \vec{r} \times \vec{F}$$



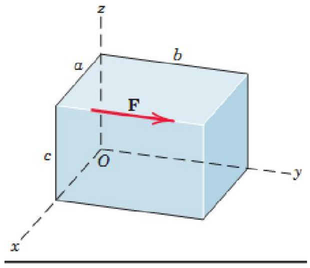
- Thus, the moment of a couple is the same about all points.
- Couples may be added



- Replace a force by its equivalent force–couple system.



SAMPLE PROBLEM 2/11

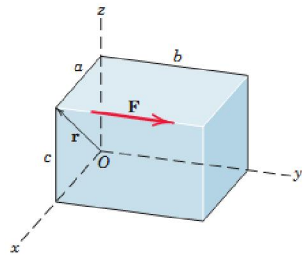


Determine the moment of force $\vec{F}$ about point O (a) by inspection and (b) bythe formal cross-product definition$\vec{M}_O = \vec{r} \times \vec{F}$.

Solution:

(a)

$$\vec{M}_O = \underbrace{-}_{\text{right-hand rule}} cFi + aFk = F(-ci + ak)$$



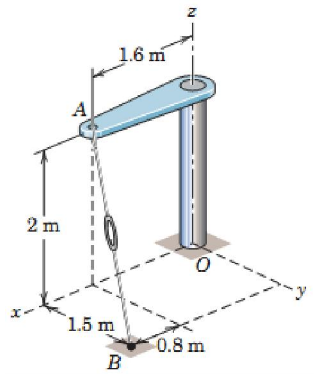
(b) $\vec{M}_O = \vec{r} \times \vec{F} = (ai + ck) \times (Fj) = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

$$\vec{M}_0 = \begin{vmatrix} i & j & k \\ a & 0 & c \\ 0 & F & 0 \end{vmatrix}$$

$$= +(0*0 - c*F)i - (a*0 - c*0)j + (a*F - 0*0)k = -cFi - 0j + aFk = F(-ci + ak)$$

SAMPLE PROBLEM 2/12

The turnbuckle is tightened until the tension in cable AB is 2.4 kN. Determine the moment about point O of the cable force acting on point A and the magnitude of this moment.



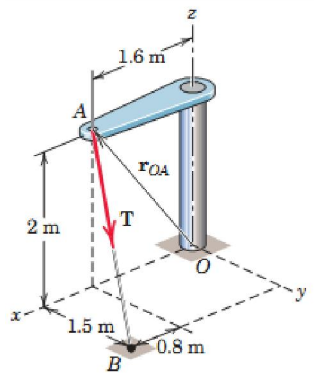
Solution: We begin by writing the described force as a vector.

$$\vec{T}_{AB} = T\,\vec{n}_{AB} = T\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = T\frac{(x2 - x1)i + (y2 - y1)j + (z2 - z1)k}{\sqrt{(x2 - x1)^2 + (y2 - y1)^2 + (z2 - z1)^2}}$$

$$= 2.4\left[\frac{(2.4 - 1.6)i + (1.5 - 0)j + (0 - 2)k}{\sqrt{0.8^2 + 1.5^2 + 2^2}}\right] = 0.732i + 1.37j - 1.824k$$

$$\vec{r}_{OA} = (1.6 - 0)i + 0j + (2 - 0)k = 1.6i + 2k$$

$$\vec{M}_O = \vec{r}_{OA} \times \vec{T}_{AB} = (1.6i + 2k) \times (0.732i + 1.37j - 1.824k)$$



$$= \begin{vmatrix} i & j & k \\ 1.6 & 0 & 2 \\ 0.732 & 1.37 & -1.824 \end{vmatrix}$$

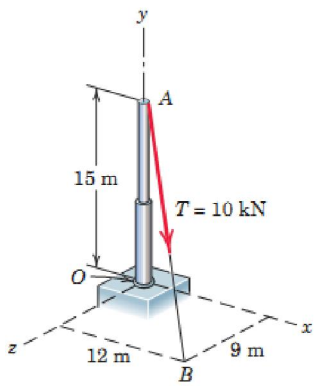
$$= (0*-1.824 - 2*1.37)i - (1.6*-1.824 - 2*0.732)j + (1.6*1.37 - 0*0.732)k = -2.74i + 4.38j + 2.19k$$

$$M_O = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{2.74^2 + 4.38^2 + 2.19^2} = 5.61\ kN.m$$

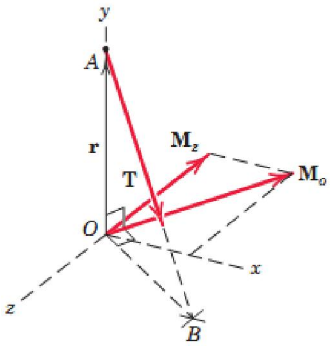
OR, use force components.

SAMPLE PROBLEM 2/13

A tension T of magnitude 10 kN is applied to the cable attached to the top A of the rigid mast and secured to the ground at B. Determine the moment Mz of **T** about the z-axis passing through the base O.



Solution (a): use vector approach.



$$\vec{T}_{AB} = T\,\vec{n}_{AB} = T\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = 10\left[\frac{(12)i + (0-15)j + (9-0)k}{\sqrt{12^2+15^2+9^2}}\right]$$
$$= 5.65i - 7.07j + 4.24k$$

$$\vec{M}_O = \vec{r}_{OA} \times \vec{T}_{AB} = (15j) \times (5.65i - 7.07j + 4.24k)$$

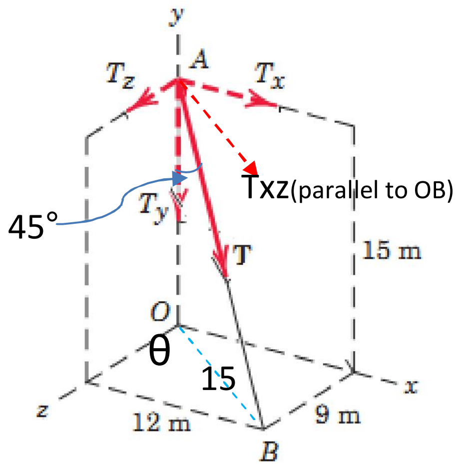
$$= \begin{vmatrix} i & j & k \\ 0 & 15 & 0 \\ 5.65 & -7.07 & 4.24 \end{vmatrix}$$

$$= +(15*4.24 - 0*-7.07)i - (0*4.24 - 0*5.65)j + (0*-7.07 - 15*5.65)k$$

$$= \underbrace{63.6}_{M_x}\,i - \underbrace{84.75}_{M_z}\,k$$

$$M_z = -84.75\ kN.m$$

Solution (b): use force components



Ty=-T*cos45=-10*cos45=-7.07 kN

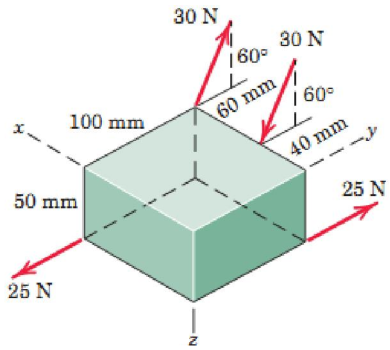
Txz=T*sin45=10*sin45=7.07 kN

Tx=Txz*sinθ=7.07*(12/15)=5.65 kN

Tz=Txz*cosθ=7.07*(9/15)=4.242 kN

$$M_z = Tx * 15 = 5.65 * 15 = -84.75\ kN.m$$

SAMPLE PROBLEM 2/14



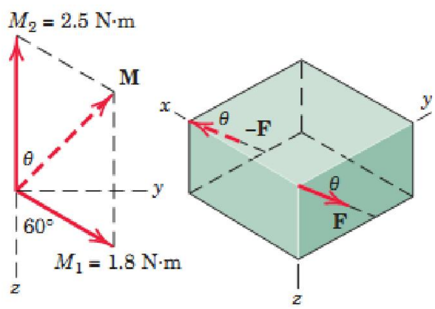
Determine the magnitude and direction of the couple **M** which will replace the two given couples and still produce the same external effect on the block. Specify the two forces F and -F, applied in the two faces of the block parallel to the y-z plane, which may replace the four given forces. The 30-N forces act parallel to the y-z plane.

Solution.

The couple due to the 30-N forces has the magnitude M1= 30(0.06)=1.80 N.m. The direction of M1 is normal to the plane defined by the two forces, and the sense, shown in the figure, is established by the right-hand convention. The couple due to the 25-N forces has the magnitude M2= 25(0.10)= 2.50 N.m with the direction and sense shown in the same figure.

M1= 30(0.06)=1.80 N.m

M2= 25(0.10)= 2.50 N.m



The two couple vectors combine to give the components

My= 1.80 sin 60= 1.559 N.m

Mz=-2.50+ 1.80 cos 60= -1.600 N.m

$$M = \sqrt{1.559^2 + (1.6)^2} = 2.23\ N.m$$

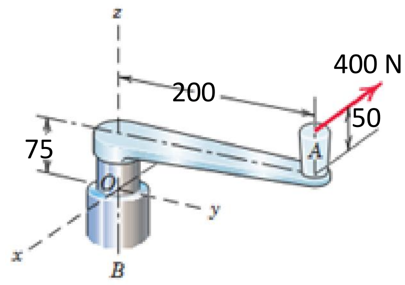
$$\theta = \tan^{-1}\left(\frac{My}{Mz}\right) = \tan^{-1}\left(\frac{1.559}{1.6}\right) = \tan^{-1}(0.974) = 44.3°$$

The forces F and -F lie in a plane normal to the couple M, and their moment arm as seen from the right-hand figure is 100 mm. Thus, each force has the magnitude

M = Fd→F=(2.23/0.1)=22.3 N

SAMPLE PROBLEM 2/15

A force of 400 N is applied at A to the handle of the control lever which is attached to the fixed shaft OB. In determining the effect of the force on the shaft at a cross section such as that at O, we may replace the force by an equivalent force at O and a couple. Describe this couple as a vector M.



Solution:

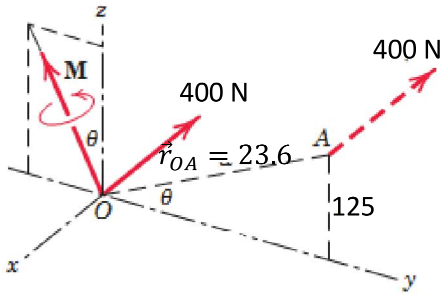
$\vec{r}_{OA} = 0i + 0.2j + 0.125k$

$\vec{F}$= -400i

$\vec{M}_O = \vec{r}_{OA} \times \vec{F} \quad = (0.2j + 0.125k) \times (-400i)$

$$= \begin{vmatrix} i & j & k \\ 0 & 0.2 & 0.125 \\ -400 & 0 & 0 \end{vmatrix}$$

$= +(0.2 * 0 - 0.125 * 0)i - (0 * 0 - 0.125 * -400)j + (0 * 0 - 0.2 * -400)k = -50j + 80k \ \ N.m$



$\vec{M}_O = \sqrt{50^2 + (80)^2} = 94.34 \ N.m$

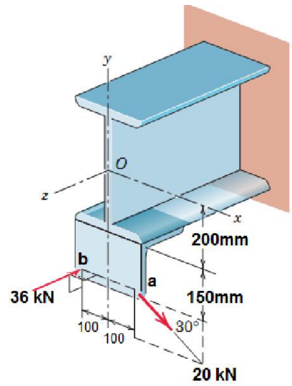
$\theta = \tan^{-1}\left(\frac{My}{Mz}\right) = \tan^{-1}\left(\frac{50}{80}\right) = 32°$

OR

$\theta = \tan^{-1}\left(\frac{125}{200}\right) = 32°$

Prob. 2/125

A right-angle bracket is welded to the flange of the I-beam to support the 36 kN force, applied parallel to the axis of the beam, and the 20 kN force, applied in the end plane of the beam. In analyzing the capacity of the beam to withstand the applied loads in the design stage, it is convenient to replace the forces by an equivalent force at O and a corresponding couple M. Determine the x-, y-, and z-components of M.



Solution

$\vec{F}_{36} = -36k$

$\vec{r}_{Ob} = (-0.1 - 0)i + (-0.35 - 0)j + 0k = -0.1i - 0.35j$

$\vec{M}_{O36} = \vec{r}_{Ob} \times \vec{F}_{36} = (-0.1i - 0.35j) \times (-36k)$

$$= \begin{vmatrix} i & j & k \\ -0.1 & -0.35 & 0 \\ 0 & 0 & -36 \end{vmatrix}$$

$= +(-0.35 * -36 - 0 * 0)i - (-0.1 * -36 - 0 * 0)j + (-0.1 * 0 + 0.35 * 0))k = 12.6i - 3.6j \ \ N.m$

$\vec{F}_{20} = 20[i\left(\cos \overbrace{-30}^{or\ 360-30}\right) + j\left(\cos \overbrace{-120}^{or\ 270-30}\right) + k\cos 90\Big] = 17.33i - 10j$

$\vec{r}_{Oa} = (0.1 - 0)i + (-0.35 - 0)j + 0k = 0.1i - 0.35j$

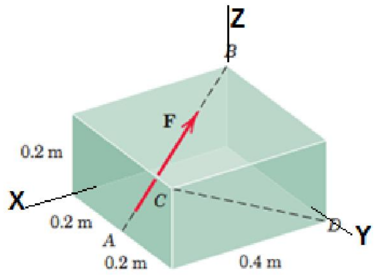
$\vec{M}_{O20} = \vec{r}_{Oa} \times \vec{F}_{20} = (0.1i - 0.35j) \times (17.33i - 10j)$

$$= \begin{vmatrix} i & j & k \\ 0.1 & -0.35 & 0 \\ 17.33 & -10 & 0 \end{vmatrix}$$

$= +(-0.35 * 0 - 0 * -10)i - (0.1 * 0 - 0 * 17.33)j + (0.1 * -10 + 0.35 * 17.33)k = 5.055k \ \ N.m$

$\sum \vec{M}_O = \vec{M}_{O36} + \vec{M}_{O20} = (12.6i - 3.6j) + (5.055k) = 12.6i - 3.6j + 5.055k$

Prob. 2/139



If the magnitude of the moment of F about line CD is 50 N.m, determine the magnitude of F.

Solution

$$\vec{F}=F\,\vec{n}_{AB}=F\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}=F\left[\frac{(0-0.4)i+(0-0.2)j+(0.2-0)k}{\sqrt{0.4^2+0.2^2+0.2^2}}\right]$$

$$=F(-0.816i-0.408j+0.408k)$$

$$\vec{r}_{CA}=0i+(0.2-0.4)j+(0-0.2)k=-0.2j-0.2k$$

$$\vec{M}_C=\vec{r}_{CA}\times\vec{F}=(-0.2j-0.2k)\times F(-0.816i-0.408j+0.408k)$$

$$=\begin{vmatrix} i & j & k\\ 0 & -0.2 & -0.2\\ -0.816 & -0.408 & 0.408\end{vmatrix}_F$$

$$=+(-0.2*0.408+0.2*-0.408)i-(0*0.408+0.2*-0.816)j$$
$$+(0*-0.408+0.2*-0.816)k=F(-0.1632i+0.1632j-0.1632k)\ N.m$$

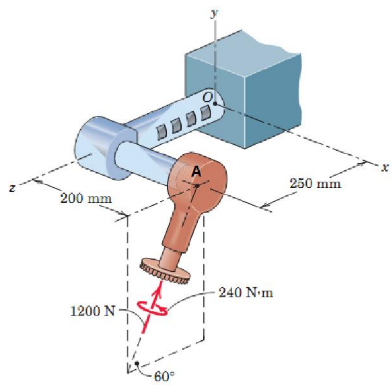
$$\vec{n}_{CD}=\frac{(0-0.4)i+0j+(0-0.2)k}{\sqrt{0.4^2+0+0.2^2}}=-0.894i-0.447k$$

$$M_{CD}=\vec{M}_c.\vec{n}_{CD}=F(-0.1632i+0.1632j-0.1632k).(-0.894i-0.447k)$$
$$=F[(0.1632*-0.894)+(0.1632*2)+(-0.1632*-0.447)]=0.2188F$$

$50=0.2188F\rightarrow F=228\,N$ Ans.

$$\vec{M}_{CD}=M_c*\vec{n}_{CD}=50(-0.894i-0.447k)$$

Prob. 2/144



The special-purpose milling cutter is subjected to the force of 1200 N and a couple of 240 N.m as shown. Determine the moment of this system about point O.

Solution:

$$\vec{n}_F=i\cos 90+j\cos(270+60)+k\cos(180+60)$$
$$=0i+0.866j-0.5k$$

$$\vec{F}=F\,\vec{n}_F=1200(0.866j-0.5k)=1039j-600k\ \ N$$

$$\vec{r}_{OA}=(0.2-0)i+0j+(0.25-0)k=0.2i+0.25k$$

$$\vec{M}_O=\vec{r}_{OA}\times\vec{F}=(0.2i+0.25k)\times(1039j-600k)$$

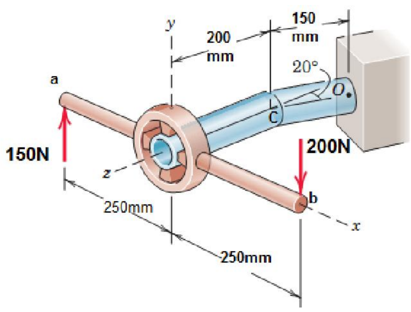
$$=\begin{vmatrix} i & j & k\\ 0.2 & 0 & 0.25\\ 0 & 1039 & -600\end{vmatrix}$$

$$=(0*-600-0.25*1039)i-(0.2*-600-0.25*0)j$$
$$+(0.2*1039-0*0)k=-260i+120j+208k\ \ N.m$$

$$\vec{M}_{24\,0}=\begin{cases}\vec{M}_z=-240\cos 60=-120\,k\\ \vec{M}_y=240\sin 60=208j\end{cases}\ N.m$$

$$\sum\vec{M}_O=-260\mathrm{i}+(120+208)\mathrm{j}+(208-120)\mathrm{k}=-260\mathrm{i}+328\mathrm{j}+88\mathrm{k}\ \ \mathrm{N.m}$$

Prob. 2/148



The threading die is screwed onto the end of the fixed pipe, which is bent through an angle of 20º. Replace the two forces by an equivalent force at O and a couple M. Find M and calculate the magnitude of **M'** the moment which tends to screw the pipe into the fixed block about its angled axis through O.

Solution:

$$\vec{F}_{15\,0}=150j$$

$\vec{r}_{Oa} = (-0.25 - 0.15\sin 20)i + 0j + [0 + (0.2 + 0.15\cos 20)]k = -0.301i + 0.341k$

$\vec{M}_{O15\ 0} = \vec{r}_{Oa} \times \vec{F}_{150} = (-0.301i + 0.341j) \times (150j)$

$$= \begin{vmatrix} i & j & k \\ -0.301 & 0 & 0.341 \\ 0 & 150 & 0 \end{vmatrix}$$

$$= (0 * 0 - 0.341 * 150)i - (-0.301 * 0 - 0.341 * 0)j + (-0.301 * 150 - 0 * 0)\mathrm{k} = -51.15\mathrm{i} - 45.15\mathrm{k}$$

$\vec{F}_{200} = -200j$

$\vec{r}_{Ob} = (0.25 - 0.15\sin 20)i + 0j + [0 + (0.2 + 0.15\cos 20)]k = 0.198i + 0.341k$

$\vec{M}_{O200} = \vec{r}_{Ob} \times \vec{F}_{200} = (0.198i + 0.341k) \times (-200j)$

$$= \begin{vmatrix} i & j & k \\ 0.198 & 0 & 0.341 \\ 0 & -200 & 0 \end{vmatrix}$$

$$= (0 * 0 - 0.341 * -200)i - (0.198 * 0 - 0.341 * 0)j + (0.198 * 200 - 0 * 0)\mathrm{k} = 68.2\mathrm{i} - 39.6\mathrm{k}$$

$\sum \vec{M}_O = \vec{M}_{O15\ 0} + \vec{M}_{O200} = (-51.15\mathrm{i} - 45.15\mathrm{k}) + (68.2\mathrm{i} - 39.6\mathrm{k}) = 17.05\mathrm{i} - 84.75\mathrm{k}\ \ \mathrm{N.m}$

$F = \uparrow \sum Fy = 150 - 200 = -50\ N \downarrow$

$$\left.\begin{array}{r} \vec{M}_O = 17.05\mathrm{i} - 84.75\mathrm{k}\ \ \mathrm{N.m} \\ F = 50\ N \downarrow \end{array}\right\} Ans.$$

$$\vec{n}_{oc} = \frac{(0 - 0.15\sin 20)i + 0j + [-0.2 + (0.2 + 0.15\cos 20)]k}{\sqrt{0.051^2 + 0.141^2}} = -0.342i + 0.94k$$

$\grave{M} = \vec{M}_o . \vec{n}_{oc} = (17.05\mathrm{i} - 84.75\mathrm{k}).(-0.342i + 0.94k)$

$$= [17.05 * (-0.342)] + (-84.75 * 0.94) = -85.5\ N.m$$

OR

$\grave{M} = 17.05\cos 70 + 84.75\cos 20 = 85.5\ N.m$

Resultants 3D:

The resultant of a system of forces is the simplest force combination, which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

In the two-dimensional force system the magnitude and direction of the resultant force found by a vector summation of forces, and the location of the line of action of the resultant force by applying the principle of moments. These same principles can be extended to three dimensions.

- a force could be moved to a parallel position by adding a corresponding couple (force-couple system).

for the system of *concurrent* forces F1, F2, F3 . . . acting on a rigid body in Fig. may move each of them in turn to the arbitrary point O, a couple for each force transferred is introduced. These couples are:

$\vec{M}_1 = \vec{r}_1 \times \vec{F}_1, \quad \vec{M}_2 = \vec{r}_2 \times \vec{F}_2, \quad \vec{M}_3 = \vec{r}_3 \times \vec{F}_3,$

$\vec{r}$ is a vector from O to any point on the line of action of the force(F1,F2,F3)

The concurrent forces and the couples may be added to produce a resultant force **R**, and a resultant couple M (vector addition).

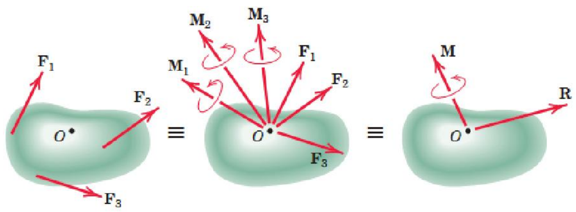
$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F}$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \cdots = \sum \vec{M}$$

The couple vectors are shown through point O, but because they are free vectors, they may be represented in any parallel positions. The magnitudes of the resultants and their components are

$$\vec{R}_x = \sum \vec{F}_x, \quad \vec{R}_y = \sum \vec{F}_y, \quad \vec{R}_z = \sum \vec{F}_z$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$



$$\vec{M}_x = \sum(\vec{r} \times \vec{F})_x, \quad \vec{M}_y = \sum(\vec{r} \times \vec{F})_y, \quad \vec{M}_z = \sum(\vec{r} \times \vec{F})_z$$

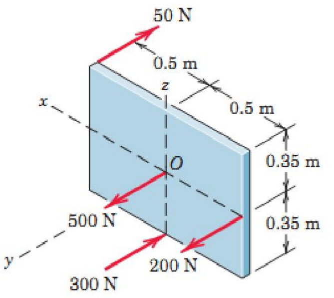
$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

The resultants for several special force systems

Concurrent Forces. When forces are concurrent at a point, only the resultant $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F}$ needs to be used because there are no moments about the point of concurrency.

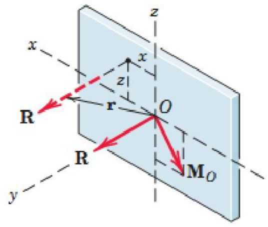
Coplanar Forces explained in resultant of 2D force system.

Parallel Forces. For a system of parallel forces not all in the same plane, the magnitude of the parallel resultant force R is simply the magnitude of the algebraic sum of the given forces.



$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F}$$

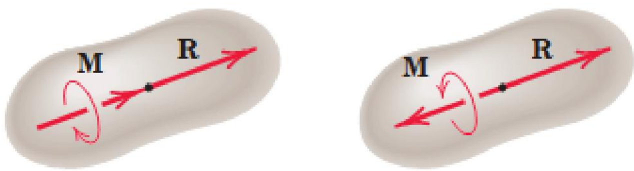
The position of its line of action is obtained from the principle of moments about O.



$$\vec{M}_O = \sum (\vec{r} \times \vec{F})_O = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \cdots$$

$$\vec{M}_O = \vec{r} \times \vec{R} \Rightarrow \vec{r} = \frac{\vec{M}_O}{\vec{R}}$$

Wrench Resultant. When the resultant couple vector M is parallel to the resultant force R, as shown in Fig., the resultant is called a *wrench*. By definition, a wrench is positive if the couple and force vectors point in the same direction and negative if they point in opposite directions.

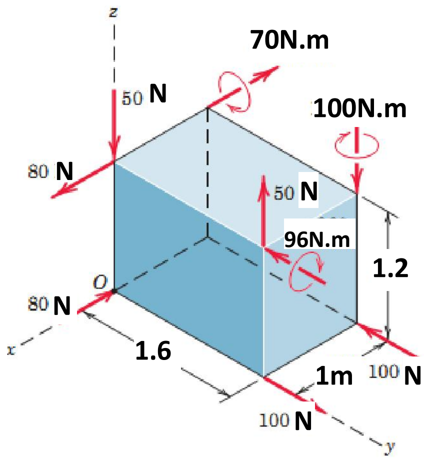


SAMPLE PROBLEM 2/16

Determine the resultant of the force and couple system which acts on the rectangular solid.

Solution:

Choose point O as a convenient reference point for the initial step of reducing the given forces to a force–couple system. The resultant force is



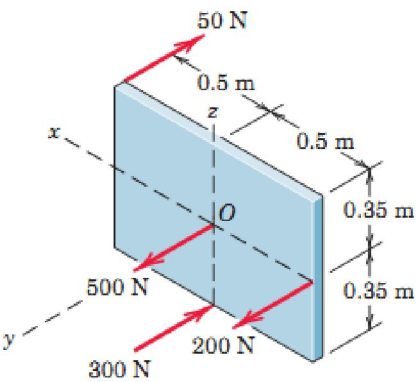
$$\vec{R} = \sum \vec{F} = (80 - 80)i + (100 - 100)j + (50 - 50)k = 0$$

Mo=(50*1.6-70)i+(80*1.2-96)j+(100*1-100)k=10i N.m

Hence, the resultant consists of a couple, which of course may be applied at any point on the body or the body extended.

SAMPLE PROBLEM 2/17

Determine the resultant of the system of parallel forces which act on the plate. Solve with a vector approach.



Solution:

Transfer of all forces to point O results in the force–couple system

$$\vec{R} = \sum \vec{F} = (200 + 500 - 50 - 300)\mathrm{j} = 350\mathrm{j} \ \ \mathrm{N}$$

$$\vec{M}_O = \sum \vec{M}_O$$

$$= (50 * 0.35 - 300 * 0.35)i + (0)j + (-50 * 0.5 - 200 * 0.5)k = -87.5i - 125k \quad N.m$$

The placement of R so that it alone represents the above force–couple system is determined by the principle of moments in vector form

$$\vec{M}_O = \vec{r} \times \vec{R} = (xi + yj + zk) \times (350j) = \begin{vmatrix} i & j & k \\ x & y & z \\ 0 & 350 & 0 \end{vmatrix}$$

$$= (y * 0 - z * 350)i - (x * 0 - z * 0)j + (x * 350 - y * 0)k = -350zi + 350xk \quad N.m$$

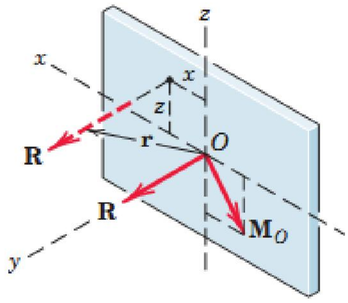
$$-350zi + 350xk = -87.5i - 125k$$

From the one vector equation we may obtain the two scalar equations

-350z=-87.5 ⇒z=0.25 m

350x=-125 ⇒x=-0.357 m

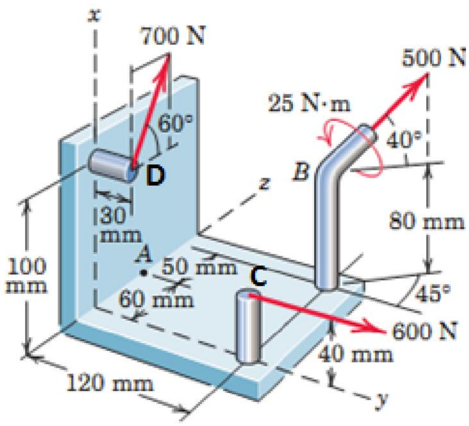
y =any value according to principle of transmissibility



SAMPLE PROBLEM 2/18

Replace the two forces and the negative wrench by a single force **R** applied at A and the corresponding couple **M**.

Solution:



The resultant force has the components

$$\vec{R}_x = \sum \vec{F}_x = 700 \sin 60 + 500 \sin 40 = 928\ N$$

$$\vec{R}_y = \sum \vec{F}_y = 600 + 500 \cos 40 \cos 45 = 871\ N$$

$$\vec{R}_z = \sum \vec{F}_z = 700 \cos 60 + 500 \cos 40 \sin 45 = 621\ N$$

$$R = \sqrt{928^2 + 871^2 + 621^2} = 1416\ N$$

<u>The couple to be added as a result of moving the 500-N force is</u>

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F}_{500} =$$

$$\vec{M}_{500} = (0.08 - 0)i + (0.12 - 0)j + (0.11 - 0.06)k \times 500(i\sin 40 + j \cos 40 \cos 45 + k \cos 40 \sin 45)$$

$$= \begin{vmatrix} i & j & k \\ 0.08 & 0.12 & 0.05 \\ \sin 40 & \cos 40 \cos 45 & \cos 40 \sin 45 \end{vmatrix}_{*500}$$

$$= 500(0.12 * \cos 40 \sin 45 - 0.05 * \cos 40 \cos 45)i - 500(0.08 * \cos 40 \sin 45 - 0.05 \sin 40)j + 500(0.08 * \cos 40 \cos 45 - 0.012 \sin 40)k$$

$\vec{M}_{500} = 18.96i - 5.59j - 16.9k \;\; N.m$

<u>The moment of the 600-N force about A is</u>

- written by inspection of its x- and z-components

$\vec{M}_{6\ 00} = 600 * 0.06i + 0j + 600 * 0.04k = 36i + 24k \;\; N.m$

- Or use vector approach

$\vec{M}_{6\ 00} = \vec{r}_{AC} \times \vec{F}_{6\ 00} = (0.04i + 0.12j - 0.06k) \times 600(0i + 1j + 0k)$

$= \begin{vmatrix} i & j & k \\ 0.04 & 0.12 & -0.06 \\ 0 & 600 & 0 \end{vmatrix}_{*6\ 00}$

$= (0.12 * 0 + 0.06 * 600)i - (0.04 * 0 + 0.06 * 0)j + (0.04 * 600 - 0.12 * 0)k$

$= 36i + 24k \;\; N.m$

<u>The moment of the 700-N force about A is</u>

- written by inspection of its x- and z-components

$\vec{M}_{7\ 00} = (700 \cos 60 * 0.03)i + (-700 \cos 60 * 0.1 - 700 \sin 60 * 0.06)j$
$+ (-700 \sin 60 * 0.03)k = 10.5i - 71.4j - 18.19k \;\; N.m$

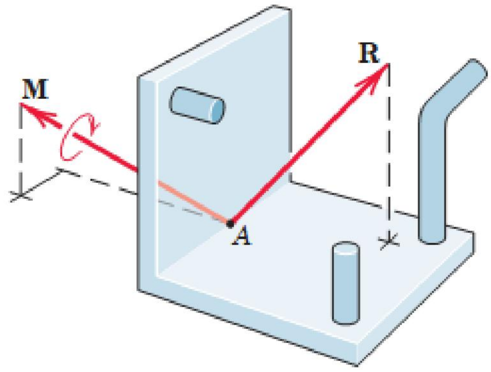
- Or use vector approach

$\vec{r}_{AD} = (0.1 - 0)i + (0.03 - 0)j + (0 - 0.06)k = 0.1i + 0.03j - 0.06k$

$\vec{M}_{7\ 00} = \vec{r}_{AD} \times \vec{F}_{7\ 00} = (0.1i + 0.03j - 0.06k) \times 700(\sin 60\, i + 0j + \cos 60\, k)$

$= \begin{vmatrix} i & j & k \\ 0.1 & 0.03 & -0.06 \\ \sin 60 & 0 & \cos 60 \end{vmatrix}_{*7\ 00} = 10.5i - 71.4j - 18.19k \;\; N.m$

<u>The couple of the given wrench</u>



$\vec{M}' = 25(-\sin 40\, i - \cos 40 \cos 45\, j$
$- \cos 40 \sin 45\, k)$

$= -16.07i - 13.54j - 13.54k \;\; N.m$

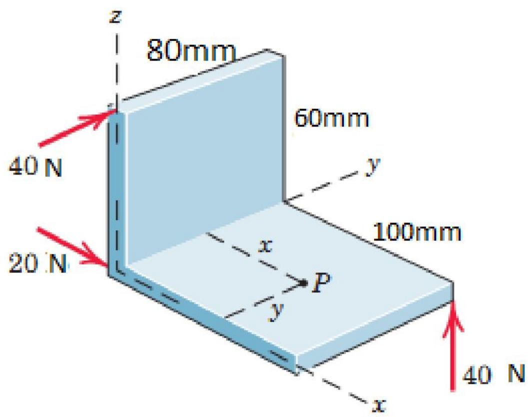
$\vec{M}_A = \vec{M}_{500} + \vec{M}_{6\ 00} + \vec{M}_{7\ 00} + \vec{M}'$
$= 49.4i - 90.5j - 24.6k \;\; N.m$

$M_A = \sqrt{49.4^2 + 90.5^2 + 24.6^2} = 106 \; N.m$

SAMPLE PROBLEM 2/19

Determine the wrench resultant of the three forces acting on the bracket.

Calculate the coordinates of the point P in the x-y plane through which the resultant force of the wrench acts. Also find the magnitude of the couple M of the wrench.



Solution:

The direction cosines of the couple M of the wrench must be the same as those of the resultant force R, assuming that the wrench is positive. The resultant force is

$\vec{R} = \sum \vec{F}_x\, i + \sum \vec{F}_y\, j + \sum \vec{F}_z\, k = 20i + 40j + 40k$

$R = \sqrt{20^2 + 40^2 + 40^2} = 60 \; N$

The direction cosines for $\vec{R}$ are:

$\cos\theta_x = \frac{R_x}{R} = \frac{20}{60} = \frac{1}{3}, \quad \cos\theta_y = \frac{R_y}{R} = \frac{40}{60} = \frac{2}{3}, \quad \cos\theta_z = \frac{R_z}{R} = \frac{40}{60} = \frac{2}{3}$

The moments of the given forces about point P through which R passes (point P).

$\vec{M}_{R_x} = 20y\,k \quad N.mm$

$\vec{M}_{R_y} = -40 * 60i - 40x\,k \quad N.mm$

$\vec{M}_{R_z} = 40 * (80 - y)i - 40(100 - x)j \quad N.mm$

$\vec{M} \;= \sum \vec{M} \;= [-40 * 60 + 40(80 - y)]i + [-40(100 - x)]j + [20y - 40x]k$

$= (800 - 40y)i + (-400 + 40x)j + (20y - 40x)k$

The direction cosines for $\vec{M}$ are:

$\cos\theta_x = \frac{8\ 00-4\ 0y}{M}, \; \cos\theta_y = \frac{-4\ 00+4\ 0x}{M}, \; \cos\theta_z = \frac{20y-4\ 0x}{M}$

Direction cosines for $\vec{R}$= direction cosines for $\vec{M}$

$\frac{1}{3} = \frac{8\ 00-4\ 0y}{M}$……1

$\frac{2}{3} = \frac{-4\ 00+4\ 0x}{M}$……2

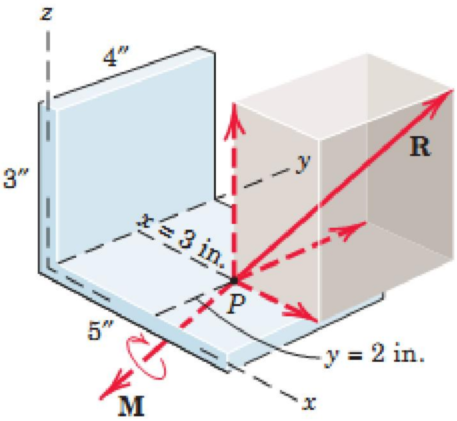
$\frac{2}{3} = \frac{20y-4\ 0x}{M}$…….3

Solution of the three equations gives

M=-2400 N.mm

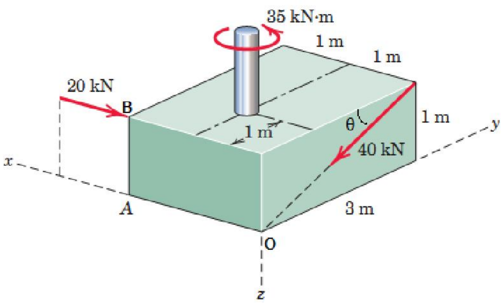
x=60 mm

y=40 mm



Prob. 2/158

Replace the two forces and single couple by an equivalent force–couple system at point A.



Solution:

$\theta = \tan^{-1}\frac{1}{3} = 18.435°$

$\vec{R} = \sum \vec{F_x}\, i + \sum \vec{F_y}\, j + \sum \vec{F_z}\, k$

$= -20i - 40\cos 18.435\, j + 40\sin 18.435\, k$

$= -20i - 38j + 12.6k\ kN \;\; Ans.$

$R = \sqrt{20^2 + 38^2 + 12.6^2} = 44.76\ kN$

$\vec{M}_{A_x} = (-40\cos 18.435) * 1 + (40\sin 18.435) * 3 = 0$

$\vec{M}_{A_y} = 20 * 1 + (40\sin 18.4) * 2 = 45.25\ kN.m$

$\vec{M}_{A_z} = (40\cos 18.4) * 2 - 35 = 40.91\ kN.m$

$\vec{M}_A = 0i + 45.25j + 40.91k \;\; kN.m$ Ans.

OR use vector approach:

$\vec{r}_{AO} = -2i$

$\vec{F}_{4\ 0} = 40\cos(180 - 18.435)\, j + 40\cos(90 - 18.435)\, k = -38j + 12.6k$

$\vec{M}_{A4\ 0} = \vec{r}_{AO} \times \vec{F}_{4\ 0} = (-2i) \times (-38j + 12.6k)$

$= \begin{vmatrix} i & j & k \\ -2 & 0 & 0 \\ 0 & -38 & 12.6 \end{vmatrix}$

$= (0 * 12.6 - 0 * -38)i - (-2 * 12.6 - 0 * 0)j + (-2 * -38 - 0 * 0)k$

$= 25.2j + 76k \;\; kN.m$

$\vec{r}_{AB} = -1k$

$\vec{F}_{20} = -20i$

$\vec{M}_{A20} = \vec{r}_{AB} \times \vec{F}_{20} = (-1k) \times (-20i)$

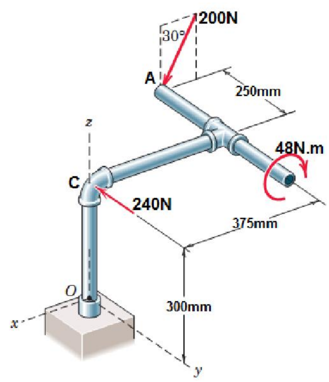
$= \begin{vmatrix} i & j & k \\ 0 & 0 & -1 \\ -20 & 0 & 0 \end{vmatrix} = 0i - (0-20)j + 0k = 20j \ kN.m$

$\vec{M}_A = \sum \vec{M}_A = 25.2j + 76k + 20j - 35k = 45j + 41k \ kN.m \ \ Ans.$

$\vec{R} = \sum \vec{F} = \vec{F}_{4\ 0} + \vec{F}_{20} = -38j + 12.6k - 20i = -20i - 38j + 12.6k$

Prob. 2/162

Replace the two forces and one couple acting on the rigid pipe frame by their equivalent resultant force R acting at point O and a couple M_o.



Solution:

$\vec{R} = \sum \vec{F}_x \, i + \sum \vec{F}_y \, j + \sum \vec{F}_z \, k$

$= 200\cos 30\, i - 240j + 200\cos 120\, k$

$= 173.2i - 240j - 100k \ N \ \ Ans$

$\vec{M}_{O_x} = 240 * 03 + 200\sin 30 * 0.25 = 97 \ N.m$

$\vec{M}_{O_y} = 200\cos 30 * 0.3 - 200\sin 30 * 0.375 - 48 = -33.54 \ N.m$

$\vec{M}_{O_z} = 200\cos 30 * 0.25 = 43.3 \ N.m$

$\vec{M}_O = \sum \vec{M}_O = 97i - 33.5j + 43.3k \ N.m \ \ Ans.$

OR use vector approach:

$\vec{r}_{OA} = -0.375i - 0.25j + 0.3k$

$\vec{F}_{200} = 200\cos 30\, i + 200\sin 120 \ k = 173.2i - 100k$

$\vec{M}_{O200} = \vec{r}_{OA} \times \vec{F}_{200} = (-0.375i - 0.25j + 0.3k) \times (173.2i - 100k)$

$= \begin{vmatrix} i & j & k \\ -0.375 & -0.25 & 0.3 \\ 173.2 & 0 & -100 \end{vmatrix}$

$= (-0.25 * -100 - 0.3 * 0)i - (0.375 * -100 - 0.3 * 173.2)j$
$+ (-0.375 * 0 + 0.25 * 173.2)k = 25i + 14.46j + 43.3k \ \ kN.m$

$\vec{r}_{OC} = 0.3k$

$\vec{F}_{24\ 0} = -240j$

$\vec{M}_{O200} = \vec{r}_{OC} \times \vec{F}_{24\ 0} = (0.3k) \times (-240j)$

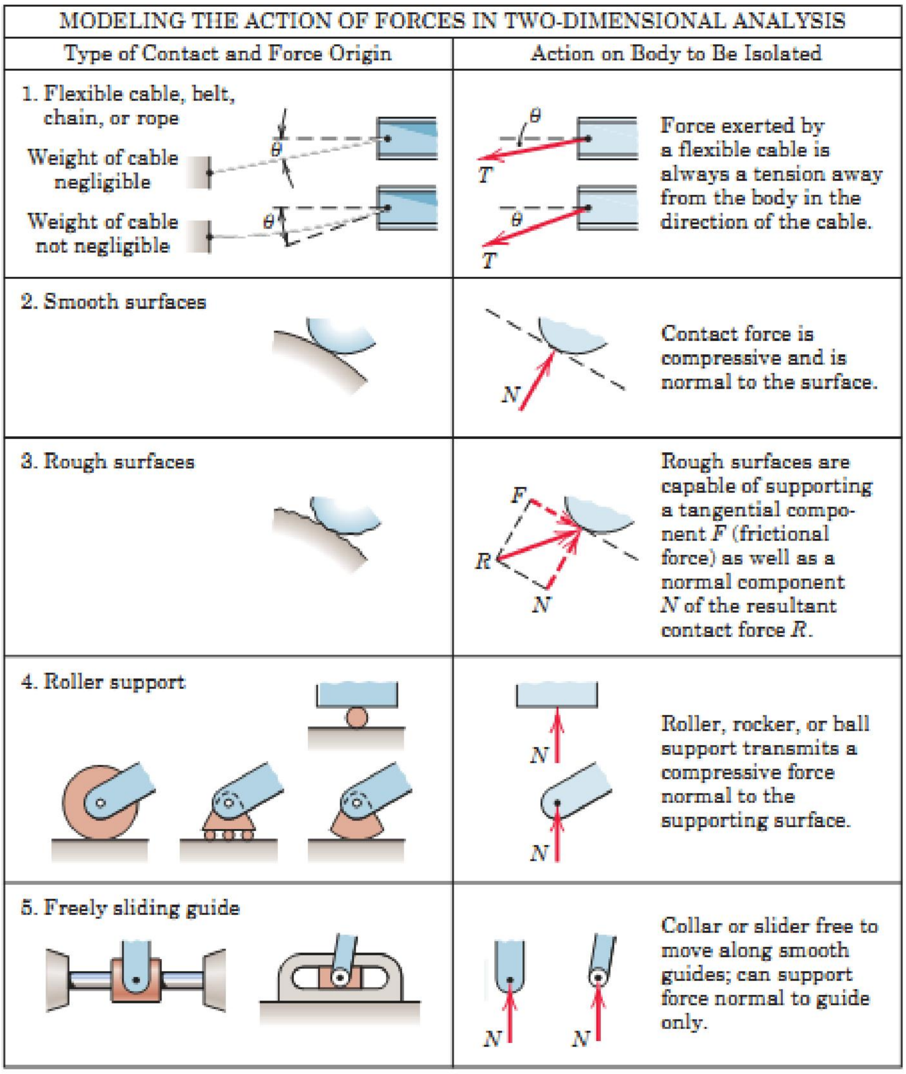
$= \begin{vmatrix} i & j & k \\ 0 & 0 & 0.3 \\ 0 & -240 & 0 \end{vmatrix} = 72i \ \ kN.m$

$\vec{M}_O = \sum \vec{M}_O = 25i + 14.46j + 43.3k + 72i - 48j = 97i - 33.5j + 43.3k \ N.m \ \ Ans.$

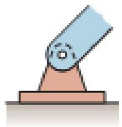
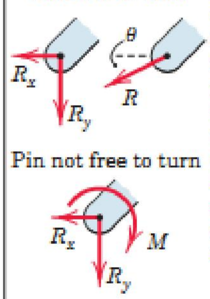
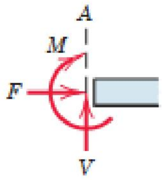
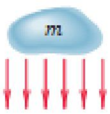
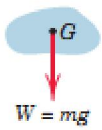
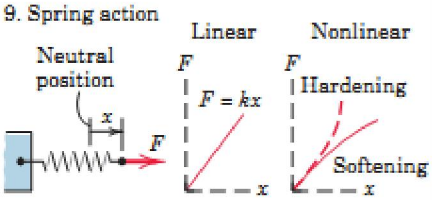
$\vec{R} = \sum \vec{F} = \vec{F}_{200} + \vec{F}_{24\ 0} = 173.2i - 100k - 240j = 173.2i - 240j - 100k \ N \ \ Ans.$

Modeling the Action of Forces

Figure below shows the common types of force application on mechanical systems for analysis in two dimensions. Each example shows the force exerted on the body to be isolated, by the body to be removed. Newton's third law, which notes the existence of an equal and opposite reaction to every action, must be carefully observed. The force exerted on the body in question by a contacting or supporting member is always in the sense to oppose the movement of the isolated body which would occur if the contacting or supporting body were removed.



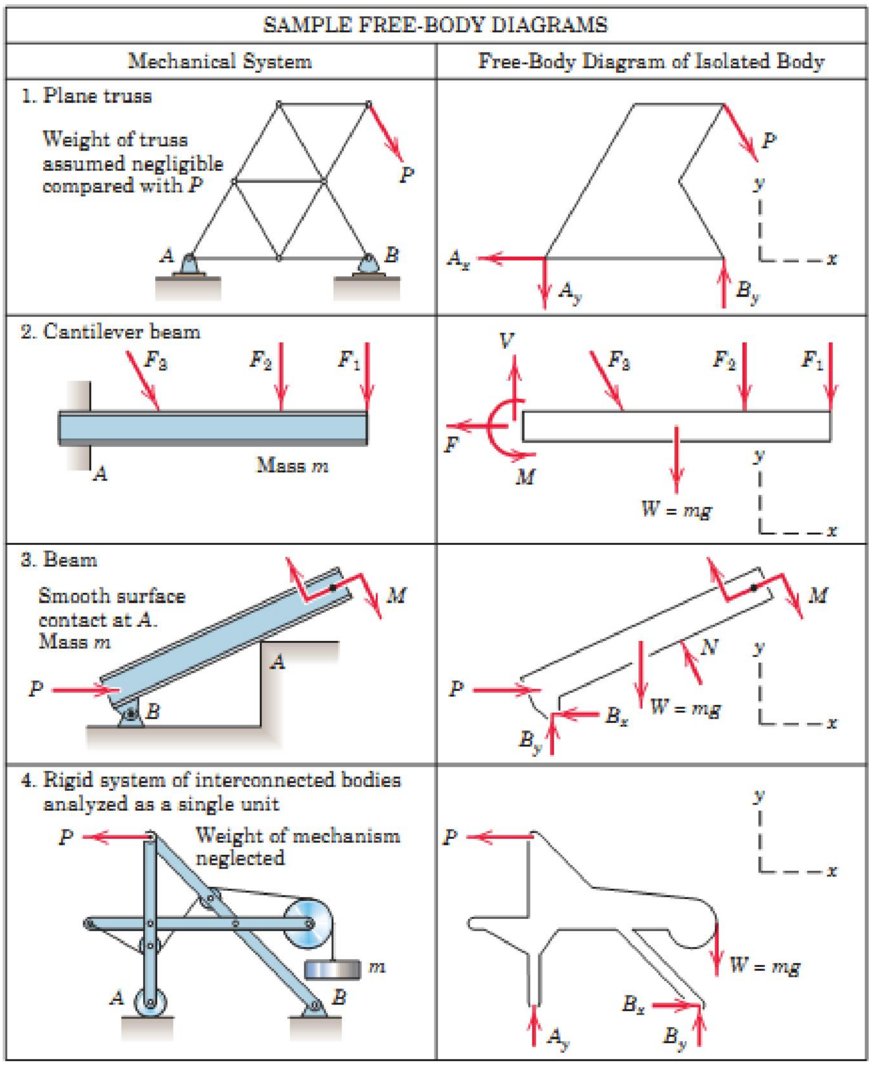
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)

Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection 	Pin free to turn Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Free-Body Diagram

A body or combination of connected bodies as a single body *isolated* from all *surrounding* bodies.

Examples of Free-Body Diagrams



Equilibrium

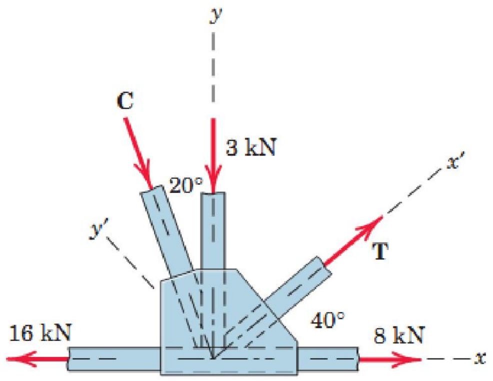
The condition in which the resultant of *all* forces and moments acting on a body is *zero*. Stated in another way, a body is in equilibrium if all forces and moments applied to it are in *balance*. These requirements are contained in the vector equations of equilibrium, which in two dimensions may be written in scalar form as

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_O = 0$$

The third equation represents the zero sum of the moments of all forces about any point O on or off the body.

SAMPLE PROBLEM 3/1

Determine the magnitudes of the forces C and T, which, along with the other three forces shown, act on the bridge-truss joint.



Solution:

The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which are in equilibrium.

$$[\Sigma F_x = 0] \qquad 8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$$

$$0.766T + 0.342C = 8 \qquad (a)$$

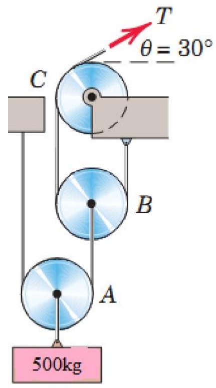
$$[\Sigma F_y = 0] \qquad T\sin 40^\circ - C\cos 20^\circ - 3 = 0$$

$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

SAMPLE PROBLEM 3/2

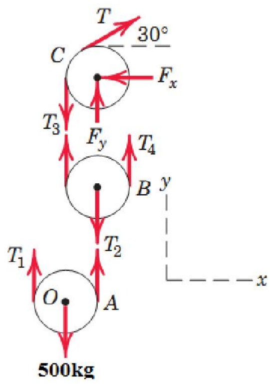


Calculate the tension T in the cable which supports the 500kg load with the pulley arrangement shown. Each pulley is free to rotate about its bearing (i.e. frictionless), and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.

Solution:

$W = mg = 500 * 9.81 = 4900N = 4.9kN$

The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley A, which includes the only known force. With the unspecified pulley radius designated by r, the equilibrium of moments about its center O and the equilibrium of forces in the vertical direction require



$$\circlearrowright \sum M_O = 0 \Rightarrow T_1 r - T_2 r = 0 \Rightarrow T_1 = T_2$$

$$+\uparrow \sum F_y = 0 \Rightarrow T_1 + T_2 - 4.9 = 0 \Rightarrow 2T_1 = 4.9 \Rightarrow T_1 = T_2 = 2.45kN$$

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$T_3 = T_4 = T_2/2 = 2.45/2 = 1.225kN$

For pulley C the angle θ=30° in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

$T = T_3 = 1.225kN$

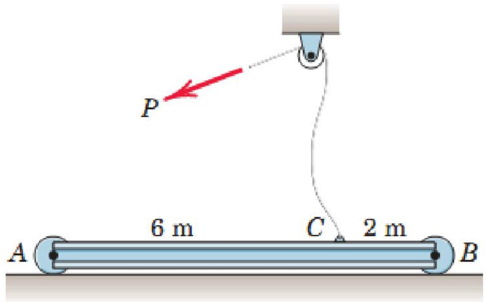
Equilibrium of the pulley in the x- and y-directions requires

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow T\cos 30 - F_x = 0 \Rightarrow 1.225\cos 30 - F_x = 0 \Rightarrow F_x = 1.06kN$$

$$+\uparrow \sum F_y = 0 \Rightarrow F_y + T\sin 30 - T_3 = 0 \Rightarrow F_y + 1.225\sin 30 - 1.225 = 0 \Rightarrow F_y = 0.6125kN$$

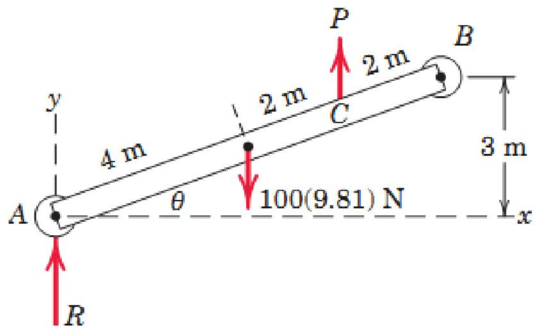
$$F = \sqrt{{F_x}^2 + {F_y}^2} = \sqrt{1.06^2 + 0.6125^2} = 1.225kN$$

SAMPLE PROBLEM 3/3



The uniform 100-kg I-beam is supported initially by its end rollers on the horizontal surface at A and B. By means of the cable at C it is desired to elevate end B to a position 3 m above end A. Determine the required tension P, the reaction at A, and the angle θ made by the beam with the horizontal in the elevated position.

Solution:



In constructing the free-body diagram, we note that the reaction on the roller at A and the weight are vertical forces. Consequently, in the absence of other horizontal forces, P must also be vertical. From Sample Problem 3/2 we see immediately that the tension P in the cable equals the tension P applied to the beam at C.

Moment equilibrium about A eliminates force R and gives

$[\Sigma M_A = 0]$ $\quad P(6\cos\theta) - 981(4\cos\theta) = 0 \qquad P = 654\ \text{N}$ *Ans.*

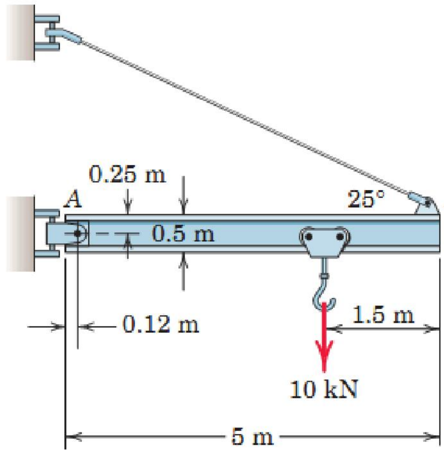
Equilibrium of vertical forces requires

$[\Sigma F_y = 0]$ $\quad 654 + R - 981 = 0 \qquad R = 327\ \text{N}$ *Ans.*

The angle θ depends only on the specified geometry and is

$$\sin\theta = 3/8 \qquad \theta = 22.0^\circ$$ *Ans.*

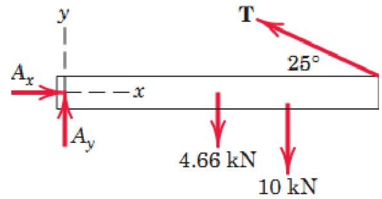
SAMPLE PROBLEM 3/4



Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.

Solution:

The free-body diagram of the beam is shown in the figure with the pin reaction at A represented in terms of its two rectangular components Ax and Ay. The weight of the beam is 95 (5)9.81(10^3)=4.66 kN and acts through its center. Note that there are three unknowns Ax, Ay, and T, which may be found from the three equations of equilibrium. We begin with a moment equation about A, which eliminates two(Ax, Ay) of the three unknowns from the equation. In applying the moment equation about A, it is simpler to consider the moments of the x- and y-components of T than it is to compute the perpendicular distance from T to A. Hence, with the counterclockwise sense as positive we write



$$[\Sigma M_A = 0] \qquad (T\cos 25^\circ)0.25 + (T\sin 25^\circ)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$$

from which $\qquad T = 19.61\ \text{kN}$ *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

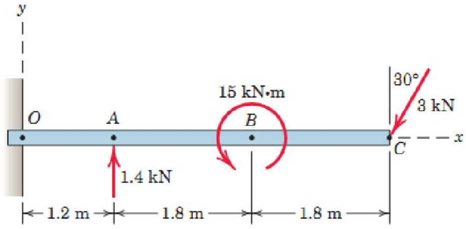
$[\Sigma F_x = 0]$ $\quad A_x - 19.61\cos 25^\circ = 0 \qquad A_x = 17.77\ \text{kN}$

$[\Sigma F_y = 0]$ $\quad A_y + 19.61\sin 25^\circ - 4.66 - 10 = 0 \qquad A_y = 6.37\ \text{kN}$

$\left[A = \sqrt{A_x^2 + A_y^2}\right]$ $\quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88\ \text{kN}$ *Ans.*

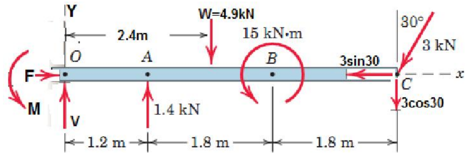
Prob. 3/5

The 500-kg uniform beam is subjected to the three external loads shown. Compute the reactions at the support point O. The x-y plane is vertical.



Solution:

Beam weight, W=500*9.81/1000=4.9 kN



$$\uparrow \sum F_y = 0 \Rightarrow V + 1.4 - 3\cos 30 - 4.9 = 0 \Rightarrow V = 6.1\ kN \uparrow$$

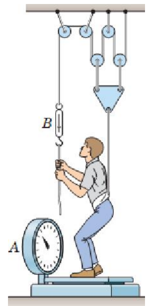
$$\rightarrow \sum F_x = 0 \Rightarrow F - 3\sin 30 = 0 \Rightarrow F = 1.5\ kN \rightarrow$$

$$\circlearrowright \sum M_o = 0$$

$$\Rightarrow (3\cos 30)4.8 + 4.9 * 2.4 - 1.4 * 1.2 - 15 - M = 0 \Rightarrow M = +7.55\ kN.m \circlearrowleft$$

Prob. 3/7

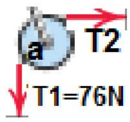
A former student of mechanics wishes to weigh him-self but has access only to a scale A with capacity limited to 400N and a small 80N spring dynamometer B. With the rig shown he discovers that when he exerts a pull on the rope so that B registers 76N, the scale A reads 286N. What is his correct weight?



Solution:

Pully a:

Let radius of the pully=r



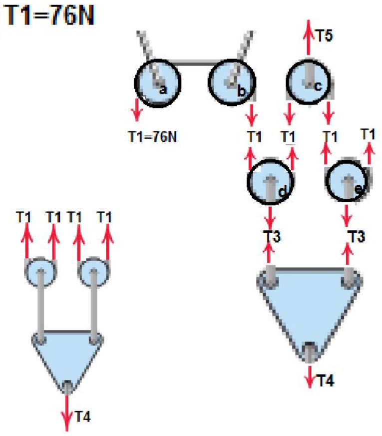
$$\circlearrowright \sum M_a = 0 \Rightarrow T_2 * r - T_1 * r = 0 \Rightarrow T_1 = T_2 = 76N$$

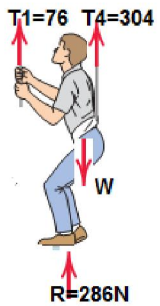
$$\uparrow \sum F_y = 0 \Rightarrow 2T_1 - T_3 = 0 \Rightarrow T_3 = 2T_1 = 2 * 76 = 152N \uparrow$$

$$\uparrow \sum F_y = 0 \Rightarrow 2T_3 - T_4 = 0 \Rightarrow T_4 = 2T_3 = 2 * 152 = 304N \downarrow$$

$$OR \uparrow \sum F_y = 0 \Rightarrow 4T_1 - T_4 = 0 \Rightarrow T_4 = 4T_1 = 4 * 76 = 304N \downarrow$$

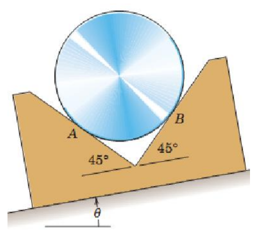
$$\uparrow \sum F_y = 0 \Rightarrow T_4 + T_1 + R - W = 0 \Rightarrow 304 + 76 + 286 - W = 0 \Rightarrow W = 666N \downarrow$$





Prob. 3/15

Find the angle of tilt θ with the horizontal so that the contact force at B will be one-half that at A for the smooth cylinder.



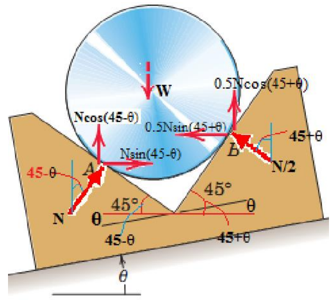
Solution:

$$\rightarrow \sum F_x = 0 \Rightarrow N\sin(45-\theta) - \frac{N}{2}\sin(45+\theta) = 0 \Big\} \div N$$

$$(\sin 45\cos\theta - \cos 45\sin\theta) - \frac{1}{2}(\sin 45\cos\theta + \cos 45\sin\theta) = 0$$

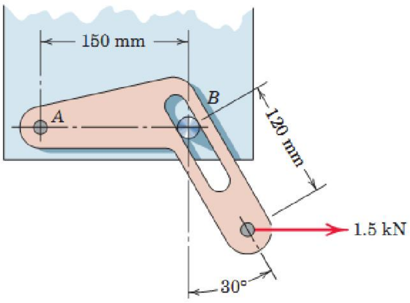
$$0.353\cos\theta - 1.06\sin\theta = 0$$

$$1.06\sin\theta = 0.353\cos\theta \Rightarrow \tan\theta = \frac{0.353}{1.06} \Rightarrow \theta = \tan^{-1} 0.333 = 18.42°\ \ Ans.$$



Prob. 3/38

Calculate the magnitude of the force supported by the pin at A under the action of the 1.5-kN load applied to the bracket. Neglect friction in the slot.



Solution:

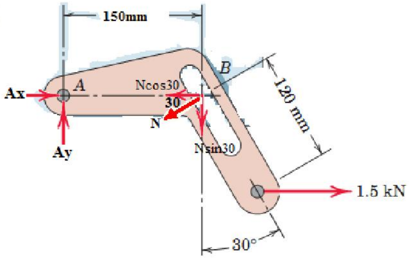
$$\circlearrowright \sum M_A = 0 \Rightarrow -1.5 * 0.12\cos 30 + (N\sin 30)0.15 = 0 \Rightarrow N = 2.08N$$

$\uparrow \sum F_y = 0 \Rightarrow A_y - N\sin 30 = 0 \Rightarrow A_y = 2.08\sin 30 = 1.04N \uparrow$

$$\rightarrow \sum F_x = 0 \Rightarrow A_x + 1.5 - N\cos 30 = 0 \Rightarrow A_x = N\cos 30 - 1.5$$

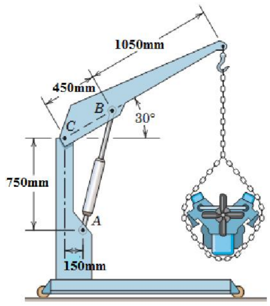
$A_x = 2.08\cos 30 - 1.5 = 0.3N \rightarrow$

$$A = \sqrt{{A_x}^2 + {A_y}^2} = \sqrt{0.3^2 + 1.04^2} = 1.083N$$

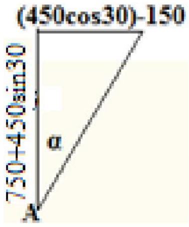


Prob. 3/44

The portable floor crane in the automotive shop is lifting a 100kg engine. For the position shown compute the magnitude of the force supported by the pin at C and the oil pressure p against the 80mm.-diameter piston of the hydraulic-cylinder unit AB.



Solution:



$$\tan\alpha = \frac{(450\cos 30) - 150}{750 + (450\sin 30)} = 13.8°$$

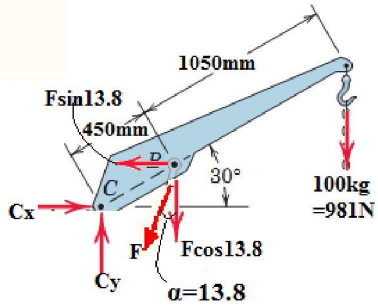
$\circlearrowright \sum M_C = 0 \Rightarrow 981(0.45 + 1.05)\cos 30 + (F\cos 13.8)0.45\cos 30 - (F\sin 13.8)\,0.45\sin 30 = 0$

$F = -3923\ N\ comp. \nearrow$

$$\uparrow \sum F_y = 0 \Rightarrow C_y + F\cos 13.8 - 981 = 0 \Rightarrow C_y = -3923\cos 13.8 + 981 = -2829N = 2829N \downarrow$$

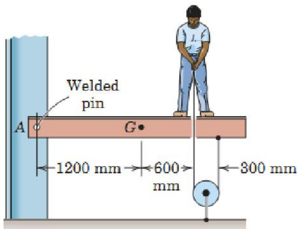
$$\rightarrow \sum F_x = 0 \Rightarrow C_x + F\sin 13.8 = 0 \Rightarrow C_x = -3923\sin 13.8 = -936N = 936N \leftarrow$$

$$R_C = \sqrt{{C_x}^2 + {C_y}^2} = \sqrt{936^2 + 2829^2} = 2980N \swarrow$$



Prob. 3/50

The pin A, which connects the 200-kg steel beam with center of gravity at G to the vertical column, is welded both to the beam and to the column. To test the weld, the 80-kg man loads the beam by exerting a 300-N force on the rope which passes through a hole in the beam as shown. Calculate the torque (couple) M supported by the pin.

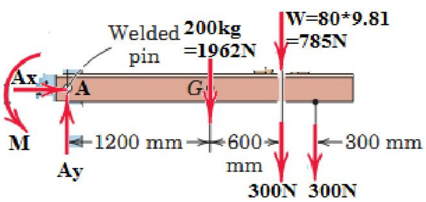


Solution:

$$\circlearrowright \sum M_A = 0 \Rightarrow -M + 1962 * 1.2 + (785 + 300)1.8 + 300 * 2.1 = 0 \Rightarrow M = 4937N.m \circlearrowleft$$

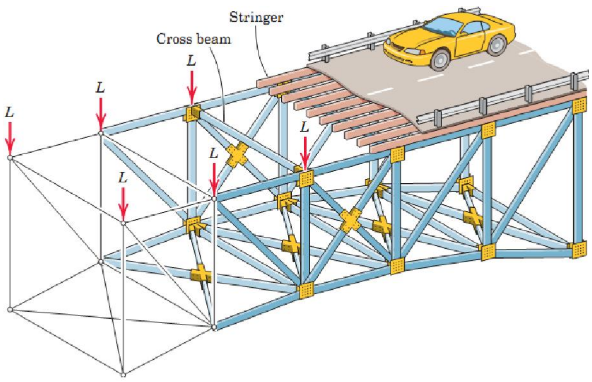
$$\uparrow \sum F_y = 0 \Rightarrow A_y - 1962 - 300 - 785 - 300 = 0 \Rightarrow A_y = 3347N \uparrow$$

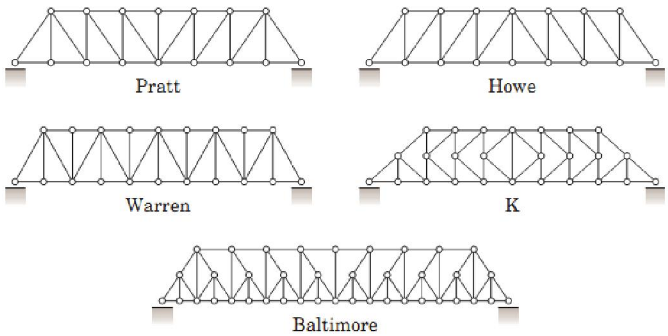
$$\Rightarrow \sum F_x = 0 \Rightarrow A_x = 0$$



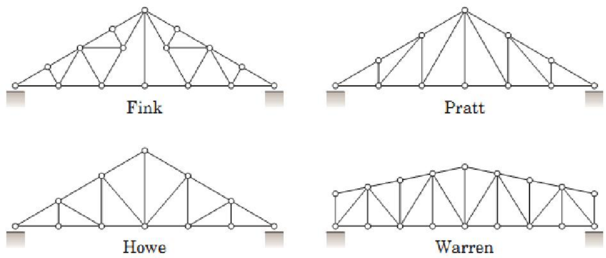
Truss:

A framework composed of members joined at their ends to form a rigid structure is called a truss.



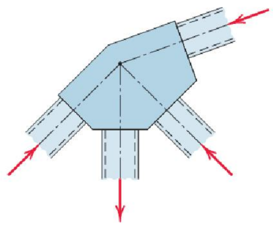


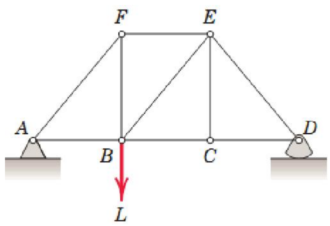
Commonly Used Bridge Trusses



Commonly Used Roof Trusses

- Common examples of trusses: Bridges, roof supports
- Structural members commonly used are I-beams, channels, angles
- *Ideal* truss members are fastened together at their ends by pins connections (the *centerlines* of the members are *concurrent* at the joint as in Fig). While *actual* truss members are fastened together at their ends by welding, riveted connections, or large bolts or pins.
- All external forces are applied at the pin connections (joints) as shown.
- For large trusses, a roller, rocker, or some kind of slip joint is used at one of the supports to provide for expansion and contraction due to temperature changes and for deformation from applied loads.



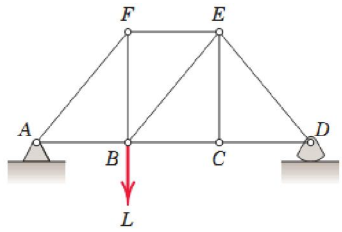


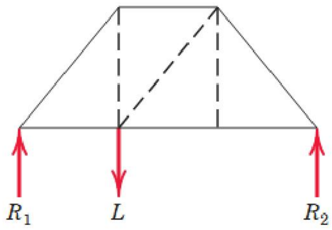
Method of Analysis: 1. Method of Joints, 2. Method of Sections.

Joint Method:

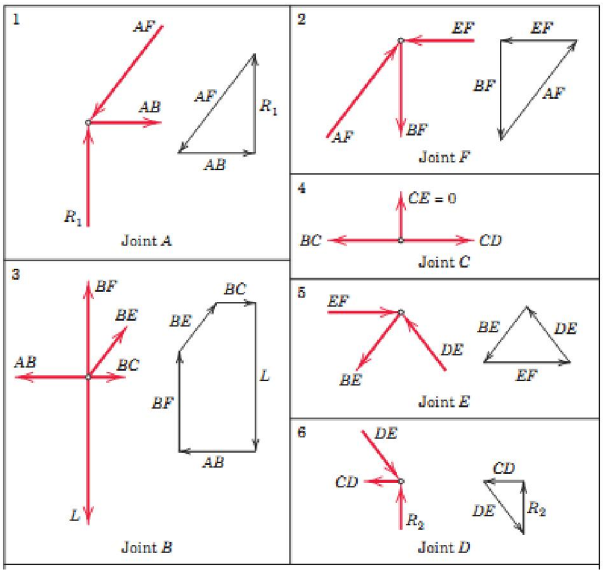
This method for finding the forces in the members of a truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint. The method therefore deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved.

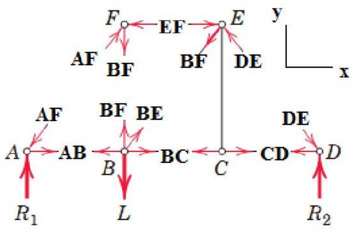
$\sum F_x = 0, \quad \sum F_y = 0,$ at each joint



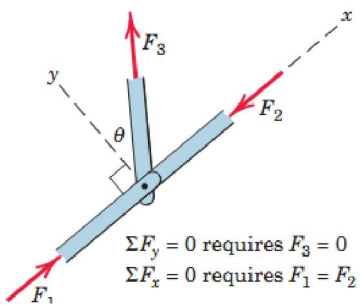


- Draw free-body diagram for the truss, then calculate reactions R1 and R2.(use $\sum M_{A\ or\ D} = 0,\ \ \sum F_y = 0$ for whole truss)
- Tension force, (such as AB) will always be indicated by an arrow *away* from the pin
- Compression force, (such as AF) will always be indicated by an arrow *toward* the pin.
- Draw free-body diagram for each joint.
- We begin the analysis with any joint where at least *one known load* exists and where *not* more than *two unknown forces* are present.
- Apply equilibirum for each joint $\sum F_x = 0$, $\sum F_y = 0$
- Solve equations of equilibirum to get unknowns.



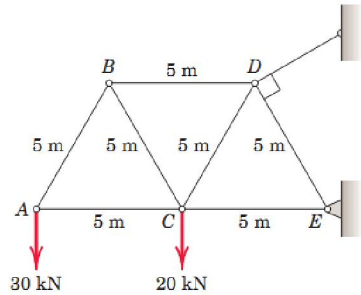


- It is often convenient to indicate the tension T and compression C of the various members
- Sometimes we cannot initially assign the correct direction of one or both of the unknown forces acting on a given pin. If so, we may make an arbitrary assignment. A negative computed force value indicates that the initially assumed direction is incorrect.
- When two collinear members are under compression, as indicated in Fig., it is necessary to add a third member to maintain alignment of the two members and prevent buckling. We see from a force summation in the y-direction that the force F3 in the third member must be zero and from the x-direction that F1=F2. This conclusion holds regardless of the angle θ and holds also if the collinear members are in tension. If an external force with a component in the y-direction were applied to the joint, then F3 would no longer be zero.



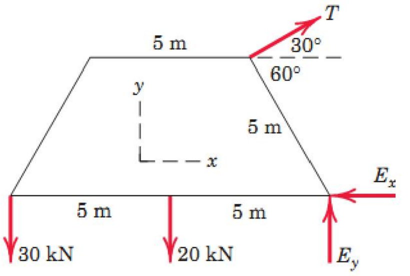
SAMPLE PROBLEM 4/1

Compute the force in each member of the loaded cantilever truss by the method of joints.



Solution.

If it were not desired to calculate the external reactions at D and E,the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the free-body diagram of the truss as a whole. The equations of equilibrium give

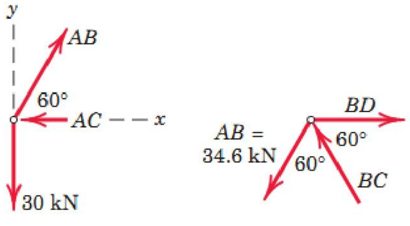


$$[\Sigma M_E = 0] \quad 5T - 20(5) - 30(10) = 0 \quad T = 80 \text{ kN}$$

$$[\Sigma F_x = 0] \quad 80 \cos 30° - E_x = 0 \quad E_x = 69.3 \text{ kN}$$

$$[\Sigma F_y = 0] \quad 80 \sin 30° + E_y - 20 - 30 = 0 \quad E_y = 10 \text{ kN}$$

draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence.



Joint A Joint B

Joint A:

ΣFy=0 AB sin 60- 30=0⇒AB=34.6 kN T

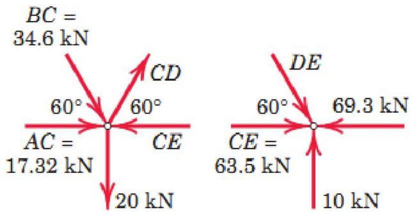
ΣFx=0 AC- (34.6)cos 60=0⇒AC=17.32 kN C

Joint B must be analyzed next, since there are more than two unknown forces on joint C.

Joint B

ΣFy=0 BCsin60 -ABsin60=0⇒BCsin60-(34.6)sin60=0⇒ BC= 34.6 kN C

ΣFx=0 BD- AB cos60 –BC cos60=0⇒ BD- 34.6 cos60 –34.6 cos60=0⇒BD=34.6 kN T



Joint C Joint E

Joint C

ΣFy=0 CDsin60-BCsin60-20=0⇒ CDsin60-34.6sin60-20=0⇒ CD=57.7 kN T

ΣFx=0⇒CE-AC-BCcos60-CDcos60=0⇒

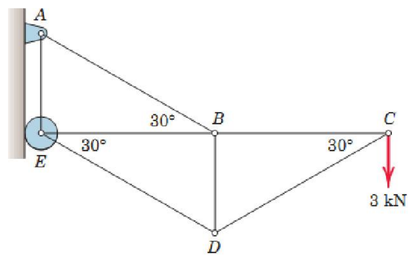
CE-17.32-(34.6)cos60-(57.7) cos60=0⇒CE= 63.5 kN C

Joint E

ΣFy=0 DEsin60-10=0⇒DE= 11.55 kN C

Prob. 4/9

Determine the force in each member of the loaded truss.



Solution:

Joint C

ΣFy=0 CDsin30-3=0⇒CD=6 kN C

ΣFx=0 CDcos30-CB=0⇒6cos30-CB=0⇒CB=5.2 kN T

Joint D

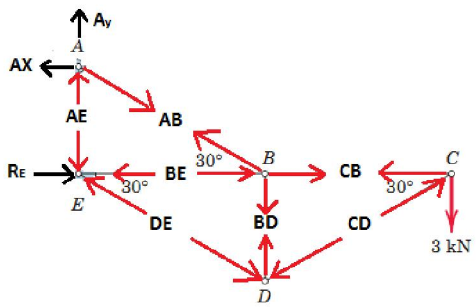
ΣFx=0 DEsin60-CDsin60=0⇒DEsin60-6sin60=0⇒DE=6 kN C

ΣFy=0 BD-CDcos60-DEcos60=0⇒BD-6cos60-6cos60=0⇒BD=6 kN T

Joint B

ΣFy=0 ABsin30-BD=0⇒ABsin30-6=0⇒AB=12 kN T

ΣFx=0 BE-ABcos30+CB=0⇒BE-12cos30+5.2=0⇒BE=5.2 kN C



Joint E

ΣFx=0 RE-BE-DEcos30=0⇒RE-5.2-6cos30=0⇒RE=10.4 kN →

ΣFy=0 DEsin30-AE=0⇒6sin30-AE=0⇒AE=3 kN C

Joint A

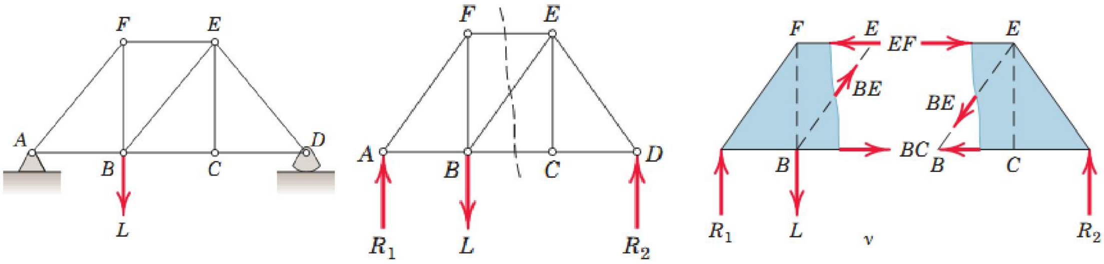
ΣFy=0 Ay+AE-ABcos60=0⇒Ay+3-12cos60=0⇒Ay=3 kN↑

ΣFx=0 -Ax+ABsin60=0⇒-Ax+12sin60=0⇒Ax=6kN←

Method of Sections

In choosing a section of the truss, in general, not more than three members whose forces are unknown should be cut, since there are only three available independent equilibrium relations.

Example: determine the force in the members BE,FE and BC.



- Draw free-body diagram for the truss, then calculate reactions R1 and R2.(use $\sum M_{A\ or\ D} = 0,\ \ \sum F_y = 0$ for whole truss)
- An imaginary section, indicated by the dashed line, is passed through the truss, cutting it into two parts,

Left-hand section

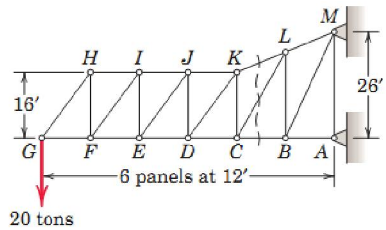
$\sum M_B = 0 \Rightarrow$ get force EF

$\sum F_y = 0 \Rightarrow$ get force BE

$\sum M_E = 0 \Rightarrow$ get force BC

SAMPLE PROBLEM 4/3

Calculate the forces induced in members KL, CL, and CB by the 20-ton load on the cantilever truss.



Solution:

BL=16+(26-16)/2=21 ft

↺$\Sigma M_L = 0$ 20(5)(12)-CB(21)=0⇒CB=57.1 tons C

$\theta=\tan^{-1}(5/12)=22.62°$, $\cos\theta=(12/13)$

↺$\Sigma M_C=0$ 20(4)(12)-(12/13)KL(16)=0⇒KL=65 tons T

PC/16=24/(26-16)⇒PC=38.4 ft

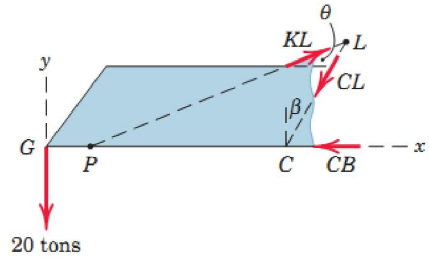
$\beta=\tan^{-1}(CB/BL)= \tan^{-1}(12/21)=29.7°$

$\Sigma M_p=0$ 20(48-38.4)- CL(cos29.7)(38.4)=0⇒CL= 5.76 tons C

OR

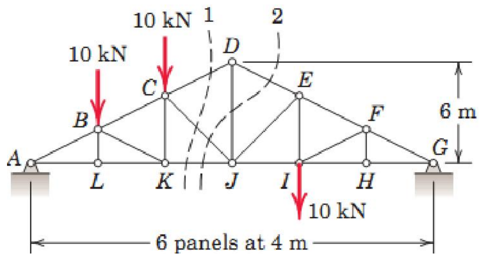
$\Sigma F_x=0$ -CB-CLsinβ +KLcosθ=0

-57.1-CLsin29.7+65cos22.62=0⇒CL= 5.76 tons C



SAMPLE PROBLEM 4/4

Calculate the force in member DJ of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.



Solution:

$\Sigma M_G=0$ A_y*6*4-10*2*4-10*4*4-10*5*4=0⇒ A_y=18.33 kN↑

$\Sigma F_y=0$ A_y + G_y-3*10=0⇒ G_y=11.67 kN↑

<u>section 1-left side</u>

$\theta=\tan^{-1}(4/4)=45°$

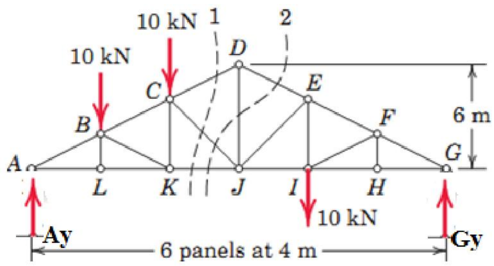
$\Sigma M_A=0$ CJ(cos 45)(2*4)+ CJ(sin 45)(4)+10(4)+10(8)=0⇒

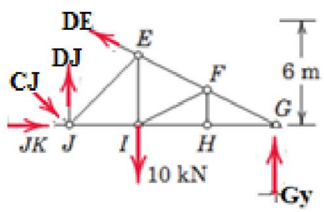
CJ=-14.14 kN =14.14 kN C

<u>section 2-right side</u>

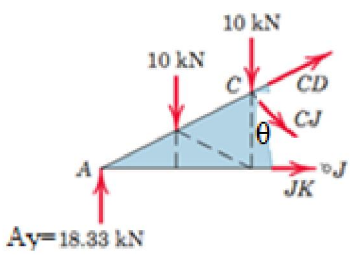
$\Sigma M_G=0$ -10*2*4 +DJ*3*4-CJ(cos45)3*4=0

DJ=16.674 kN T





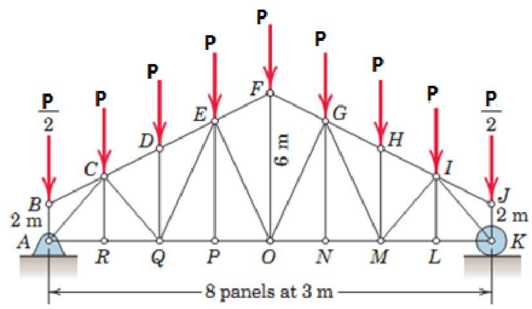
Section 2



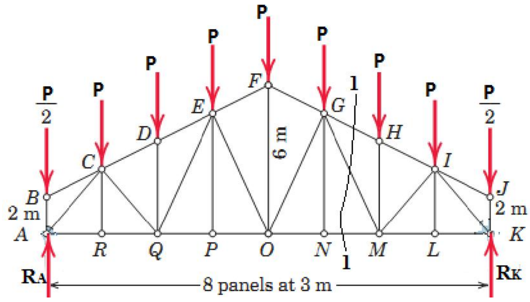
Section 1

Prob. 4/48

Compute the force in member GM of the loaded truss.



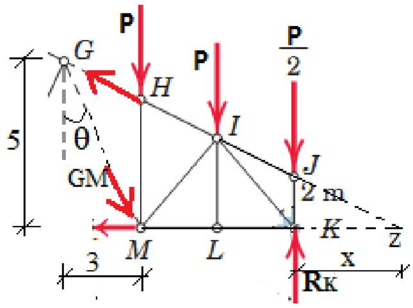
Solution:



$$\circlearrowright \sum M_A = 0 \Rightarrow -R_K * 24 + \frac{P}{2} * 24 + P(21 + 18 + 15 + 12 + 9 + 6 + 3) = 0 \Rightarrow R_K = 4P \uparrow$$

$$\uparrow \sum F_y = 0 \Rightarrow R_A + R_K - 8P = 0 \Rightarrow R_A = 4P \uparrow$$

Section 1-1 right hand part:

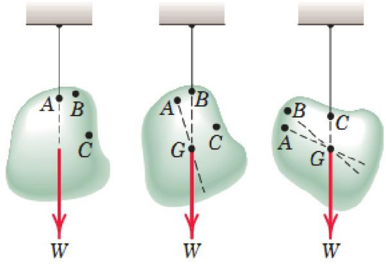


$$\frac{2}{x} = \frac{6-2}{4*3} \Rightarrow x = 6m$$

$$\circlearrowright \sum M_z = 0 \Rightarrow R_K * x - (GM * \cos\theta)12 - \frac{P}{2} * x - P(9 + 12) = 0$$

$$\Rightarrow 4P * 6 - \left(GM * \frac{5}{\sqrt{34}}\right)12 - \frac{P}{2} * 6 - P(9 + 12) = 0 \Rightarrow GM = 0$$

Distributed Forces



- The body will be in equilibrium under the action of the tension in the cord and the resultant W of the gravitational forces acting on all particles of the body. This resultant is clearly *collinear* with the cord.
- For all practical purposes these lines of action will be concurrent at a single point G, which is called the *center of gravity* of the body.

Determining the Center of Gravity

Apply the principle of moments to the parallel system of gravitational forces. The moment of the resultant gravitational force W about any axis equals the sum of the moments about the same axis of the gravitational forces dW acting on all particles treated as infinitesimal elements of the body. The resultant of the gravitational forces acting on all elements is the weight of the body and is given by the sum$W = \int dW$.

The moment about y-axis of the elemental weight=xdW

The sum of these moments for all elements of the body about y-axis = $\int xdW$.

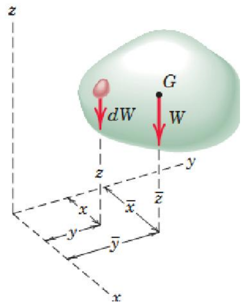
Moment of the sum =$\bar{x}W$

Moment of the sum must equal the sum of the moments, $\underbrace{\bar{x}W}_{\text{moment of sum}} = \underbrace{\int xdW}_{\text{sum of moments}} \rightarrow$

$$\bar{x} = \frac{\int xdW}{W}$$

Similar expressions for the other two coordinates,$\bar{y}, \bar{z}$ of the center of gravity G:

$$\left.\begin{aligned}\bar{x} &= \frac{\int xdW}{W}\\ \bar{y} &= \frac{\int ydW}{W}\\ \bar{z} &= \frac{\int zdW}{W}\end{aligned}\right\} \textit{center of gravity coordinate}$$



With the substitution of W= mg and dW= g dm, the expressions for the coordinates of the center of gravity become

$$\bar{x} = \frac{\int x\,g\,dm}{m\,g} \qquad \bar{y} = \frac{\int y\,g\,dm}{m\,g} \qquad \bar{z} = \frac{\int z\,g\,dm}{m\,g}$$

$$\left.\begin{aligned}\bar{x} &= \frac{\int x\,dm}{m}\\ \bar{y} &= \frac{\int y\,dm}{m}\\ \bar{z} &= \frac{\int z\,dm}{m}\end{aligned}\right\} \textit{center of mass coordinate}$$

Where, W: weight, m: mass, g: gravitational acceleration

If ρ, the density of a body is its mass per unit volume (V), then dm =ρdV

If ρ is not constant throughout the body

$$\left.\begin{aligned}\bar{x} &= \frac{\int x\,\rho dV}{\int \rho dV}\\ \bar{y} &= \frac{\int y\,\rho dV}{\int \rho dV}\\ \bar{z} &= \frac{\int z\,\rho dV}{\int \rho dV}\end{aligned}\right\} \textit{center of body coordinate}$$

Centroids of Lines, Areas, and Volumes

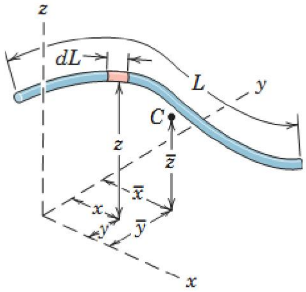
When the density of a body is uniform throughout, it will be a constant, then center of mass concise with geometrical center and termed *centroid*.

Centroid of Lines

L: length, A: cross-sectional area, ρ: density

If A and ρ are constant over the length of the rod, the coordinates of the center of mass also become the coordinates of the centroid C of the line segment

$$\left.\begin{aligned} \bar{x} &= \frac{\int x\,dL}{L} \\ \bar{y} &= \frac{\int y\,dL}{L} \\ \bar{z} &= \frac{\int z\,dL}{L} \end{aligned}\right\} \text{Centroid}$$



In general, the centroid C will not lie on the line. If the rod lies on a single plane, such as the x-y plane, only two coordinates need to be calculated.

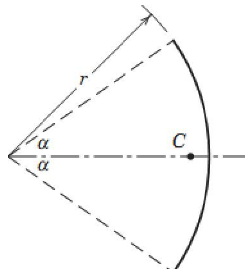
Remember,

$$_a^bL = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,.dx, \quad \text{if } y=f(x)$$

$$_c^dL = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,.dy\,, \text{ if } x=g(y)$$

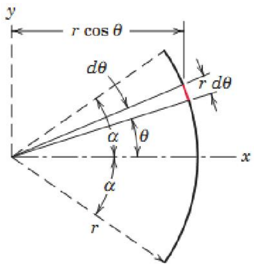
SAMPLE PROBLEM 5/1

Centroid of a *circular arc*. Locate the centroid of a circular arc as shown in the figure.



Solution.

Choosing the axis of symmetry as the x-axis makes $\bar{y} = 0$. A differential element of arc has the length dL=rdθ expressed in polar coordinates, and the x-coordinate of the element is r cosθ



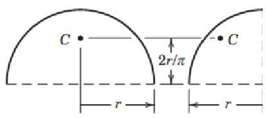
L= 2αr

$$\bar{x} = \frac{\int x\,dL}{L} \rightarrow = \frac{\int_{-\alpha}^{\alpha}(r\cos\theta)\ (rd\theta)}{2\alpha r} = \frac{r^2[\sin\theta]_{-\alpha}^{\alpha}}{2\alpha r} = \frac{r}{2\alpha}(\sin\alpha - \sin(-\alpha))$$

$$\bar{x} = \frac{r\sin\alpha}{\alpha}$$

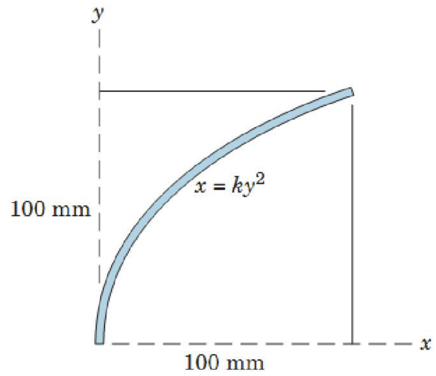
For a semicircular arc 2α=л, which gives $\bar{y} = \frac{r\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{2r}{\pi}$

For a quarter-circular arc 2α=л/2, which gives $\bar{x} = \bar{y} = \frac{2r}{\pi}$



Prob. 5/38

The homogeneous slender rod has a uniform cross section and is bent into the shape shown. Calculate the y-coordinate of the mass center of the rod.



Solution:

$$x = ky^2$$

$$@x = 100, y = 100 \rightarrow 100 = k100^2 \rightarrow k = \frac{1}{100}$$

$$dx/dy = 2ky$$

$$\bar{y} = \frac{\int y\, dL}{L}$$

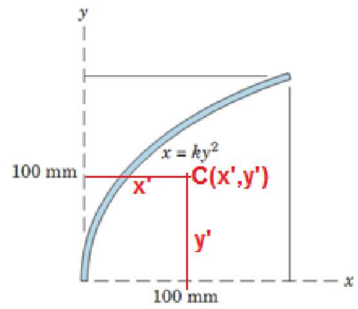
$$\int y\, dL = \int_0^{100} y\sqrt{1+(dx/dy)^2}.dy = \int_0^{100} y\sqrt{1+(2ky)^2}.dy = \int_0^{100} y\sqrt{1+4k^2y^2}.dy$$

$$\frac{8k^2}{8k^2}\int_0^{100} y\sqrt{1+4k^2y^2}.dy = \left[\frac{1}{8k^2}\frac{(1+4k^2y^2)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{100} = \left[\frac{1}{8(0.01)^2}\frac{(1+4k^2y^2)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{100}$$

$$= 8483\ mm^2$$

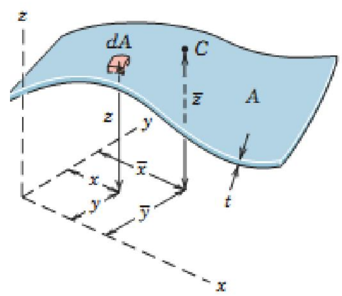
$$dL = \sqrt{1+(dx/dy)^2}.dy \rightarrow L = \int_0^{100}\sqrt{1+(dx/dy)^2}.dy = \int_0^{100}\sqrt{1+4k^2y^2}.dy = 148\ mm$$

$$\bar{y} = \frac{\int y\, dL}{L} = \frac{8483}{148} = 57.3\ mm$$



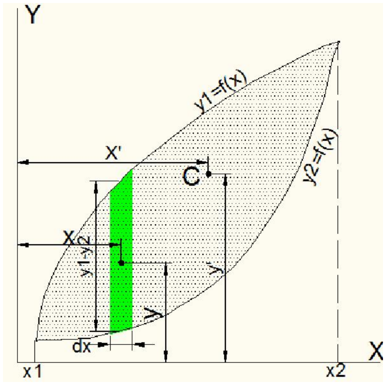
Centroids of Areas

When a body of density ρ has a small but constant thickness t, we can model it as a surface area A. The mass of an element becomes dm =ρ t dA. Again, if ρ and t are constant over the entire area, the coordinates of the center of mass of the body also become the coordinates of the centroid C of the surface area, and the coordinates may be written



$$\left.\begin{aligned} \bar{x} &= \frac{\overbrace{\int x\, dA}^{first\ moments\ of\ area}}{A} \\ \bar{y} &= \frac{\int y\, dA}{A} \\ \bar{z} &= \frac{\int z\, dA}{A} \end{aligned}\right\}\ \text{Centroid}$$

Choice of Element for Integration

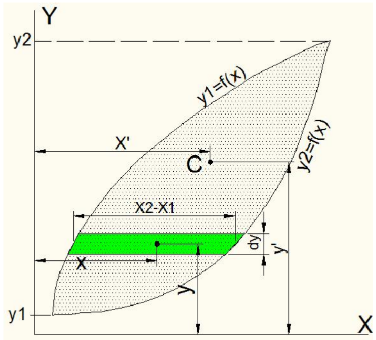


$$\bar{x} = \frac{\int x\, dA}{A} = \frac{\int_{x1}^{x2} x\,(y1 - y2)dx}{A}$$

$$A = \int_{x1}^{x2} (y1 - y2)dx$$

$$\bar{y} = \frac{\int y\, dA}{A} = \frac{\int_{x1}^{x2} y(y1 - y2)dx}{A}$$

OR



$$\bar{x} = \frac{\int x\, dA}{A} = \frac{\int_{y1}^{y2} x\,(x2 - x1)dy}{A}$$

$$A = \int_{y1}^{y2} (x2 - x1)dy$$

$$\bar{y} = \frac{\int y\, dA}{A} = \frac{\int_{y1}^{y2} y\,(x2 - x1)dy}{A}$$

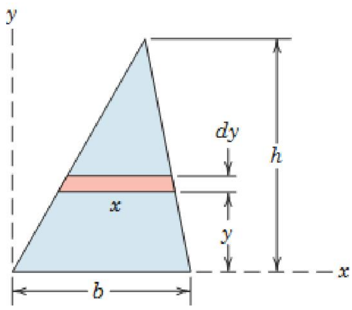
SAMPLE PROBLEM 5/2

Centroid of a triangular area. Determine the distance $\bar{h}$ from the base of a triangle of altitude h to the centroid of its area.

Solution:

By similar triangles x/(h-y)=b/h→x=(b/h)(h-y)

$$\bar{h} = \bar{y} = \frac{\int y\, dA}{A}$$



$$M_x = \int y\, dA = \int_0^h y\,(xdy) = \int_0^h y\left(\frac{b}{h}\right)(h - y)dy$$

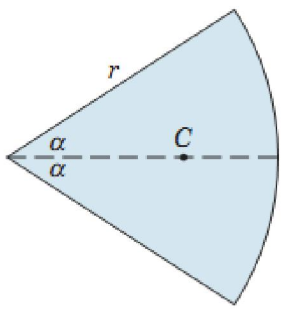
$$= \frac{b}{h}\int_0^h (hy - y^2)dy = \frac{b}{h}\left[\frac{hy^2}{2} - \frac{hy^3}{3}\right]_0^h = \frac{bh^2}{6}$$

$$A = \int_0^h xdy = \int_0^h \left(\frac{b}{h}\right)(h - y)dy = \frac{bh}{2}$$

$$\bar{h} = \frac{\int y\, dA}{A} = \frac{\frac{bh^2}{6}}{\frac{bh}{2}} = \frac{h}{3}$$

SAMPLE PROBLEM 5/3

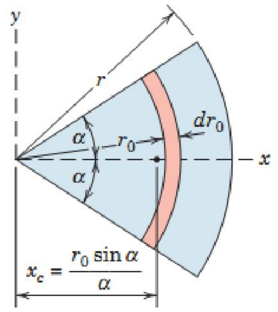
Centroid of the area of a *circular sector*. Locate the centroid of the area of a circular sector with respect to its vertex.



Solution I.

The x-axis is chosen as the axis of symmetry, and $\bar{y}$ is therefore automatically zero. We may cover the area by moving an element in the form of a partial circular ring, as shown in the figure, from the center to the outer periphery. The radius of the ring is r_0 and its thickness is dr_0, so that its area is

$dA=2r_0\alpha dr_0$.



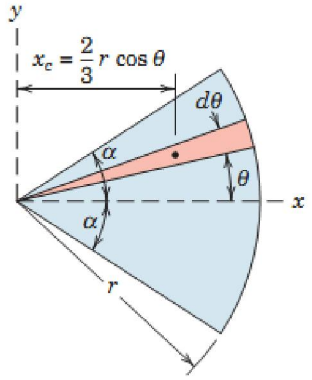
$$M_y = \int x\, dA = \int_0^r \left(\frac{r_0 \sin\alpha}{\alpha}\right)(2r_0\alpha dr_0) = 2\sin\alpha \int_0^r {r_0}^2 dr_0$$

$$= 2\sin\alpha \left[\frac{{r_0}^3}{3}\right]_0^r = \frac{2\sin\alpha\, r^3}{3}$$

$$A = r^2\pi \frac{2\alpha}{2\pi} = r^2\alpha$$

$$\bar{x} = \frac{\int x\, dA}{A} = \frac{\frac{2\sin\alpha\, r^3}{3}}{r^2\alpha} = \frac{2r\sin\alpha}{3\alpha}$$

Solution II. Triangle of differential area



$$M_y = \int x\, dA = \int_{-\alpha}^{\alpha} \left(\frac{2}{3} r\cos\theta\right)\left(\frac{r d\theta}{2} r\right) = \frac{r^3}{3}\int_{-\alpha}^{\alpha} \cos\theta\, d\theta =$$

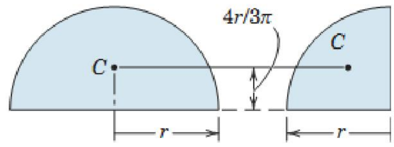
$$= \frac{r^3}{3}[\sin\theta]_{-\alpha}^{\alpha} = \frac{r^3}{3}[\sin\alpha - \sin -\alpha] = \frac{2}{3} r^3 \sin\alpha$$

$$A = r^2\pi \frac{2\alpha}{2\pi} = r^2\alpha$$

$$\bar{x} = \frac{\int x\, dA}{A} = \frac{\frac{2\sin\alpha\, r^3}{3}}{r^2\alpha} = \frac{2r\sin\alpha}{3\alpha}$$

For a *semicircular area* 2α=л, which gives $\bar{x} = \frac{2r\sin\alpha}{3\alpha} = \frac{2r\overbrace{\sin\frac{\pi}{2}}^{=1}}{3\frac{\pi}{2}} = \frac{4r}{3\pi}$

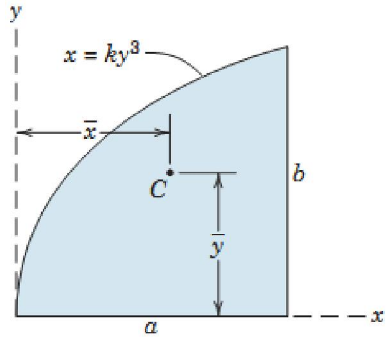
For a *quarter-circular area* 2α=л/2, which gives $\bar{x} = \bar{y} = \frac{4r}{3\pi}$



SAMPLE PROBLEM 5/4

Locate the centroid of the area under the curve $x=ky^3$ from x=0 to x=a.

Solution I. A vertical element

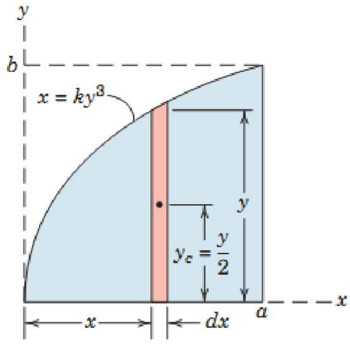


$$x = ky^3 \rightarrow a = kb^3 \rightarrow k = \frac{a}{b^3}$$

$$M_y = \int x\, dA = \int_0^{\alpha} x\,(ydx)$$

$$= \int_0^{\alpha} x\left(\sqrt[3]{\frac{x}{k}}dx\right) = \frac{1}{k^{\frac{1}{3}}}\int_0^{\alpha} x^{\frac{4}{3}}\, dx = \frac{1}{(\frac{a}{b^3})^{\frac{1}{3}}}\left[\frac{x^{\frac{7}{3}}}{\frac{7}{3}}\right]_0^{a} = \frac{3}{7}a^2 b$$

$$A = \int_0^a y\,dx = \int_0^a \sqrt[3]{\frac{x}{k}}\,dx = \frac{1}{k^{\frac{1}{3}}}\int_0^a x^{\frac{1}{3}}\,dx = \frac{1}{\left(\frac{a}{b^3}\right)^{\frac{1}{3}}}\left[\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\right]_0^a = \frac{3}{4}ab$$



$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\frac{3}{7}a^2 b}{\frac{3}{4}ab} = \frac{4}{7}a$$

$$M_x = \int \frac{y}{2}\,dA = \int_0^a \frac{y}{2}\,(ydx) = \int_0^a \frac{y^2}{2}\,dx = \frac{1}{2}\int_0^a \left[\left(\frac{x}{k}\right)^{\frac{1}{3}}\right]^2 dx$$

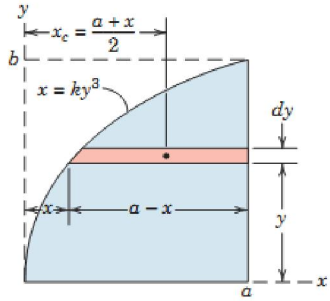
$$= \frac{1}{2(k)^{\frac{2}{3}}}\int_0^a x^{\frac{2}{3}}dx = \frac{1}{2\left(\frac{a}{b^3}\right)^{\frac{2}{3}}}\left[\frac{x^{\frac{5}{3}}}{\frac{5}{3}}\right]_0^a = \frac{3}{10}ab^2$$

$$\bar{y} = \frac{\int \frac{y}{2}\,dA}{A} = \frac{\frac{3}{10}ab^2}{\frac{3}{4}ab} = \frac{2}{5}b$$

Solution II. The horizontal element of area

$$M_y = \int x\,dA = \int_0^b \left(\frac{a+x}{2}\right)[(a-x)dy] = \int_0^b \left(\frac{a+ky^3}{2}\right)[(a-ky^3)dy] = \frac{3}{7}a^2 b$$

$$A = \int_0^b (a-x)dy = \int_0^b (a-ky^3)dy = \frac{3}{4}ab$$

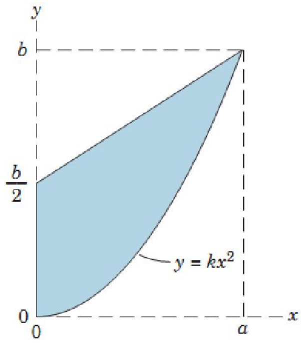


$$M_x = \int y\,dA = \int_0^b y\,[(a-x)dy] = \int_0^b y\,[(a-ky^3)dy] = \frac{3}{10}ab^2$$

Prob.5/18

Determine the coordinates of the centroid of the shaded area.

Solution:

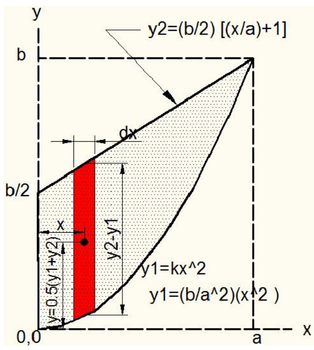


$$y_1 = k\,x^2 \rightarrow b = k\,a^2 \rightarrow \mathrm{k} = \frac{b}{a^2}$$

$$y_1 = \frac{b}{a^2}x^2$$

$$m = \frac{\Delta y}{\Delta x} = \frac{b-0.5b}{a-0} = \frac{b}{2a}$$

$$m = \frac{b}{2a} = \frac{y-0.5b}{x-0} \rightarrow y_2 = \frac{b}{2}\left(\frac{x}{a}+1\right)$$



$$A = \int_0^a (y2-y1)dx = \int_0^a \left(\left[\frac{b}{2}\left(\frac{x}{a}+1\right)\right] - \left[\frac{b}{a^2}\,x^2\right]\right)dx$$

$$= \left[\left[\frac{b}{2}\left(\frac{x^2}{2a}+x\right)\right] - \left[\frac{b}{a^2}\,\frac{x^3}{3}\right]\right]_0^a = \frac{5}{12}ba$$

$$M_y = \int x\,dA = \int_0^a x(y2-y1)dx = \int_0^a x\left(\left[\frac{b}{2}\left(\frac{x}{a}+1\right)\right] - \left[\frac{b}{a^2}\,x^2\right]\right)dx$$

$$= \int_0^a \left(\left[\frac{b}{2}\left(\frac{x^2}{a}+x\right)\right] - \left[\frac{b}{a^2}\,x^3\right]\right)dx = \left[\left[\frac{b}{2}\left(\frac{x^3}{3a}+\frac{x^2}{2}\right)\right] - \left[\frac{b}{a^2}\,\frac{x^4}{4}\right]\right]_0^a = \frac{ba^2}{6}$$

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\frac{ba^2}{6}}{\frac{5}{12}ba} = \frac{2}{5}a$$

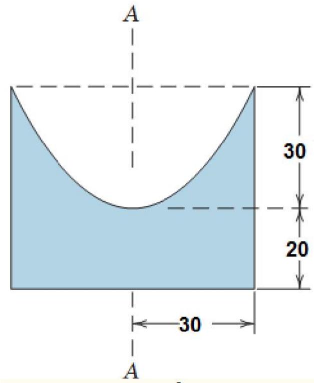
$$M_x = \int y\,dA = \int_0^a \left(\frac{y1+y2}{2}\right)(y2-y1)dx = \frac{1}{2}\int_0^a [{y_2}^2 - {y_1}^2]dx$$

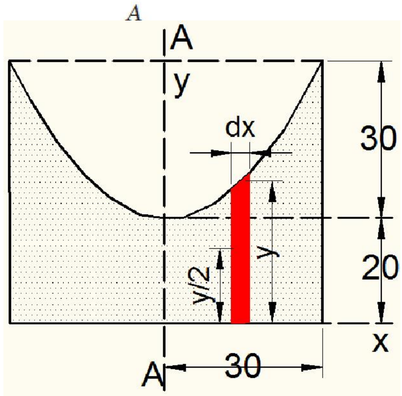
$$= \frac{1}{2}\int_0^a \left[\left(\frac{b}{2}\left(\frac{x}{a}+1\right)\right)^2 - \left(\frac{b}{a^2}x^2\right)^2\right]dx = \frac{23ab^2}{120}$$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{\frac{23ab^2}{120}}{\frac{5}{12}ba} = \frac{23}{50}b$$

Prob. 5/31

The figure represents a flat piece of sheet metal symmetrical about axis A-A and having a parabolic upper boundary. Choose your own coordinates and calculate the distance from the base to the center of gravity of the piece.





Solution:

$y = a_0 + a_1x + a_2x^2$

$\grave{y} = a_1 + 2a_2x$

$\grave{y}(0) = 0 = a_1 + 2a_2 * 0 \rightarrow a_1 = 0$

$y(0) = 20 = a_0 + a_2 * 0 \rightarrow a_0 = 20$

$y(30) = 50 = 20 + a_2 * 30^2 \rightarrow a_2 = \frac{1}{30}$

$$y = 20 + \frac{1}{30} * x^2$$

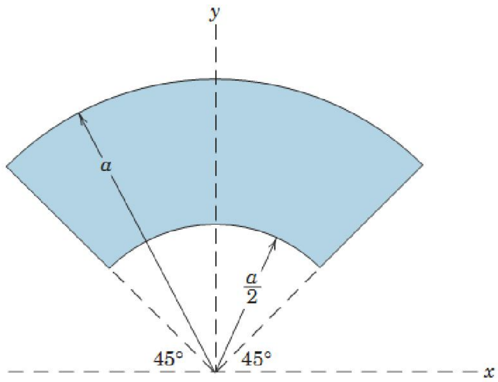
$$A = \int_{-30}^{30} ydx = 2\int_0^{30} ydx = 2\int_0^{30}\left(20 + \frac{x^2}{30}\right)dx = 1800\ mm^2$$

$$M_x = \int y\,dA = \int \frac{y}{2}\overbrace{ydx}^{dA} = 2\int_0^{30}\frac{y^2}{2}dx = \int_0^{30}\left(20 + \frac{x^2}{30}\right)^2 dx = 29400\ mm^3$$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{29400}{1800} = 16.33\ mm$$

Prob.5/29

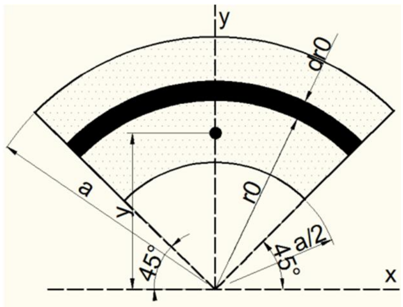
Determine the y-coordinate of the centroid of the shaded area.



Solution:

$$A = \int_{\frac{a}{2}}^{a} \frac{2\pi r_0}{4} dr_0 = \frac{\pi}{2}\left[\frac{{r_0}^2}{2}\right]_{\frac{a}{2}}^{a} = \frac{3}{16}\pi a^2$$

$$M_x = \int y\, dA = \int \overbrace{\frac{r_0 \sin\alpha}{\alpha}}^{y}\ \overbrace{\frac{2\pi r_0}{4} dr_0}^{dA} = \int_{\frac{a}{2}}^{a} \frac{r_0 \sin\frac{\pi}{4}}{\frac{\pi}{4}} * \frac{2\pi r_0}{4} dr_0$$

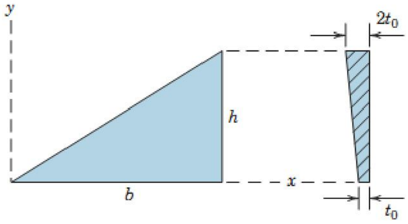


$$= 2\int_{\frac{a}{2}}^{a} \frac{1}{\sqrt{2}} * {r_0}^2 dr_0 = \frac{7}{12\sqrt{2}} a^3$$

$$\bar{y} = \frac{\int y\, dA}{A} = \frac{\frac{7}{12\sqrt{2}} a^3}{\frac{3}{16}\pi a^2} = \frac{28}{9\sqrt{2}}\frac{a}{\pi}$$

Prob. 5/36

The thickness of the triangular plate varies linearly with y from a value t_0 along its base y=0 to 2 t_0 at y=h. Determine the y-coordinate of the center of *mass* of the plate.



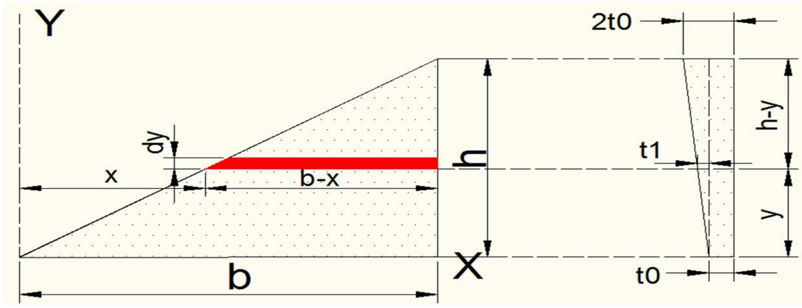
Solution:

$\frac{b-x}{h-y} = \frac{b}{h} \rightarrow x = \frac{b}{h} y$

$\frac{t_1}{y} = \frac{t_0}{h} \rightarrow t_1 = \frac{t_0}{h} y$

$t = t_0 + t_1 = t_0 + \frac{t_0}{h} y$

$= t_0 (1 + + \frac{y}{h})$



$$\bar{y} = \frac{\int y\, dm}{m} = \frac{\int y\, \rho dV}{\int \rho dV} = \frac{\int y\, \rho t dA}{\int \rho t dA}$$

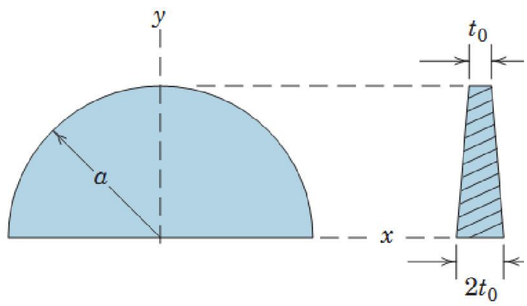
$$m = \int \rho t dA = \rho \int_0^h \left[t_0\left(1+\frac{y}{h}\right)\right](b-x)dy = t_0\rho \int_0^h \left(1+\frac{y}{h}\right)\left(b-\frac{b}{h}y\right) dy = \frac{2}{3} t_0 \rho b h$$

$$M_x = \int y\, \rho t dA = \rho \int y \left[t_0\left(1+\frac{y}{h}\right)\right](b-x)dy = t_0\rho \int y\left(1+\frac{y}{h}\right)\left(b-\frac{b}{h}y\right) dy = \frac{t_0 \rho b h^2}{4}$$

$$\bar{y} = \frac{\int y\, \rho t dA}{\int \rho t dA} = \frac{\frac{t_0\rho b h^2}{4}}{\frac{2}{3} t_0 \rho b h} = 0.375\, h$$ Compare with $\bar{y} = \frac{h}{3}$ for uniform thickness

Prob.5/42

The thickness of the semicircular plate varies linearly with y from a value $2t_0$ along its base y=0 to t_0 at y=a. Determine the y-coordinate of the *mass* center of the plate.

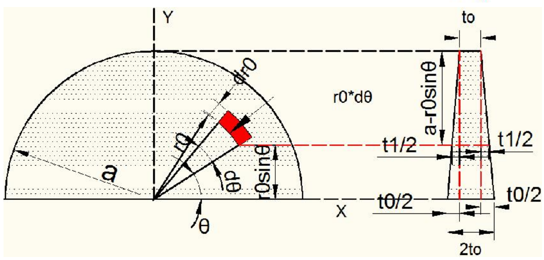


Solution:

$\frac{t_1/2}{a - r_0 \sin\theta} = \frac{t_0/2}{a} \rightarrow t_1 = t_0 (1 - \frac{r_0}{a} \sin\theta)$

$t(r,\theta) = t_0 + t_1 = t_0 + \left[t_0 (1 - \frac{r_0}{a} \sin\theta)\right]$

$= t_0 (2 - \frac{r_0}{a} \sin\theta)$



$$m=\int \rho t dA=\int_0^a\int_0^{\frac{\pi}{2}}\rho\,\overbrace{\boxed{t_0\left(2-\frac{r_0}{a}\sin\theta\right)}}^{t}\,\overbrace{r_0 d\theta\, dr_0}^{dA}=\rho t_0\int_0^a\int_0^{\frac{\pi}{2}}\left(2r_0-\frac{r_0{}^2}{a}\sin\theta\right)d\theta\, dr_0$$

$$=\rho t_0\int_0^{\frac{\pi}{2}}\left[\frac{2r_0{}^2}{2}-\frac{r_0{}^3}{3a}\sin\theta\right]_0^a d\theta\;=\rho t_0 a^2\int_0^{\frac{\pi}{2}}\left(1-\frac{\sin\theta}{3}\right)d\theta\;=\rho t_0 a^2\left[\theta+\frac{\cos\theta}{3}\right]_0^{\frac{\pi}{2}}$$

$$=\rho t_0 a^2\left(\frac{\pi}{2}-\frac{1}{3}\right)*\underbrace{2}_{two\ quarter-circular\ area}=2.475\,\rho t_0 a^2$$

$$M_x=\int y\,\rho t dA=\int_0^a\int_0^{\frac{\pi}{2}}\overbrace{r_0\sin\theta}^{y}\,\rho\,\overbrace{\boxed{t_0\left(2-\frac{r_0}{a}\sin\theta\right)}}^{t}\,\overbrace{r_0 d\theta\, dr_0}^{dA}$$

$$=\rho t_0\int_0^a\int_0^{\frac{\pi}{2}}\left(2r_0{}^2\sin\theta-\frac{r_0{}^3}{a}\sin^2\theta\right)d\theta\, dr_0$$

$$=\rho t_0\int_0^{\frac{\pi}{2}}\left[\frac{2r_0{}^3}{3}\sin\theta-\frac{r_0{}^4}{4a}\sin^2\theta\right]_0^a d\theta\;=\rho t_0\int_0^{\frac{\pi}{2}}\left(\frac{2a^3}{3}\sin\theta-\frac{a^3}{4}\overbrace{\sin^2\theta}^{1-\frac{\cos 2\theta}{2}}\right)d\theta$$

$$=\rho t_0\int_0^{\frac{\pi}{2}}\left[\frac{2a^3}{3}\sin\theta-\frac{a^3}{4}\left(1-\frac{\cos 2\theta}{2}\right)\right]d\theta$$

$$=\rho t_0 a^3\left[-\frac{2}{3}\cos\theta-\frac{1}{8}\left(\theta-\frac{\sin 2\theta}{2}\right)\right]_0^{\frac{\pi}{2}}=\rho t_0 a^3\left(-\frac{\pi}{16}+\frac{2}{8}\right)*2$$

$$=0.9406\,\rho t_0 a^3$$

$$\bar{y}=\frac{\int y\,\rho t dA}{\int \rho t dA}=\frac{0.9406\,\rho t_0 a^3}{2.475\,\rho t_0 a^2}=0.38\,a\ \text{Compare with}\ \bar{y}=\frac{4r}{3\pi}=0.424\,a\ \text{for uniform thickness}$$

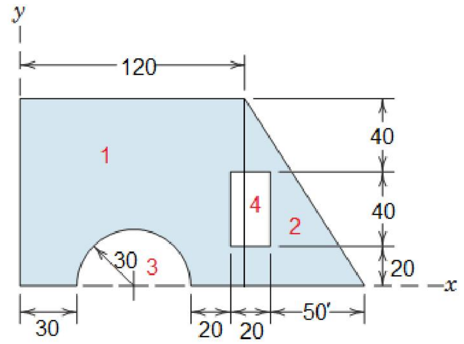
Centroid of Composite Figures

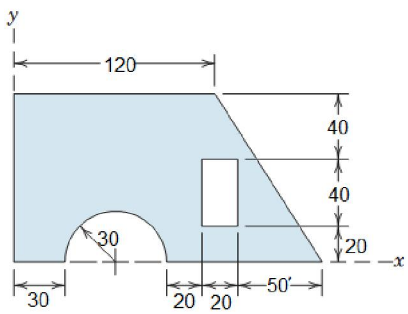
$$\bar{x}=\frac{\sum A*x}{\sum A}$$

$$\bar{y}=\frac{\sum A*y}{\sum A}$$

SAMPLE PROBLEM 5/6

Locate the centroid of the shaded area.





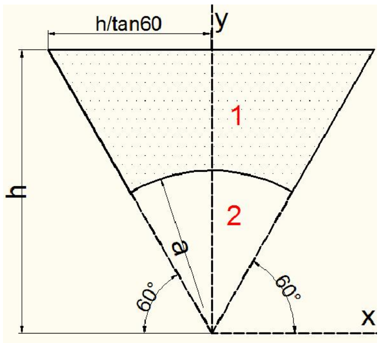
Part	A(mm²)	x(mm)	y(mm)	A*x	A*y
1 rect.	120*100=12000	120/2=60	100/2=50	12000*6=720000	12000*50=600000
2 tri.	(60*100)/2=3000	120+(60/3)=140	100/3=33.3	420000	100000
3 cir	$(30^2*\pi)/2$=-144	30+30=60	$\frac{2}{3}\frac{r\sin\alpha}{\alpha}=\frac{2}{3}\frac{30\sin\frac{\pi}{2}}{\frac{\pi}{2}}=12.73$	-84800	18000
4 rect	20*40=-800	30+30+30+20+(20/2)=120	20+(40/2)=40	-96000	-32000
Total	12790			959000	650000

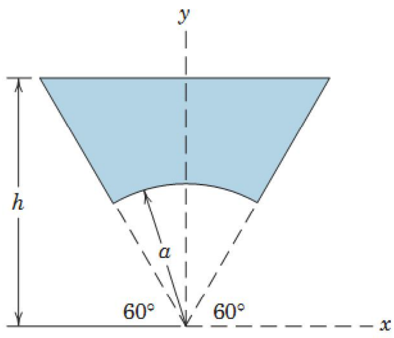
$$\bar{x} = \frac{\sum A * x}{\sum A} = \frac{959000}{12790} = 75\ mm$$

$$\bar{y} = \frac{\sum A * y}{\sum A} = \frac{650000}{12790} = 50.8\ mm$$

Prob.5/50

Determine the y-coordinate of the centroid of the shaded area.



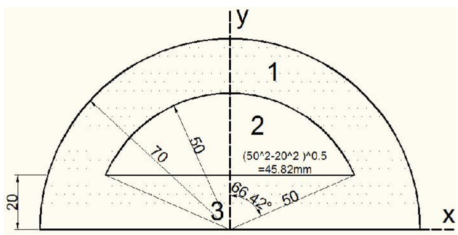


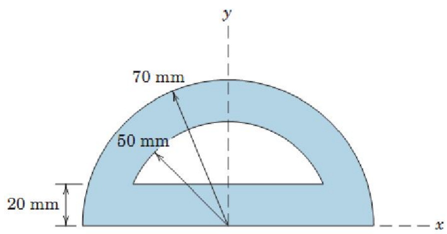
Part	A(mm²)	x(mm)	y(mm)	A*x	A*y
1 tri.	$\frac{h}{\tan 60} * h = \frac{h^2}{\tan 60}$	0	$\frac{2}{3}h$	0	$\frac{2}{3}\frac{h^3}{\tan 60}$
2 *circular sector*	$\frac{a^2\pi}{2\pi}\frac{\pi}{3} = -\frac{a^2\pi}{6}$	0	$\frac{2}{3}\frac{r\sin\alpha}{\alpha} = \frac{2}{3}\frac{a\sin\frac{\pi}{6}}{\frac{\pi}{6}} = \frac{2}{\pi}a$	0	$-\frac{a^3}{3}$
Total					

$$\bar{y} = \frac{\sum A * y}{\sum A} = \frac{\frac{2}{3}\frac{h^3}{\tan 60} - \frac{a^3}{3}}{\frac{h^2}{\tan 60} - \frac{a^2\pi}{6}} = \frac{4h^3 - 2\sqrt{3}\,a^3}{6h^2 - \sqrt{3}\pi a^2}$$

Prob.5/75

Determine the y-coordinate of the centroid of the shaded area.





Solution:

$$\theta = \cos^{-1}\frac{20}{50} = 66.42°$$

Part	A(mm²)	x(mm)	y(mm)	A*x	A*y
1 *semicircular*	$\frac{70^2\pi}{2} = 7697$	0	$\frac{4r}{3\pi} = \frac{4 * 70}{3\pi} = 29.71$	0	228670
2 *circular sector*	$\frac{50^2\pi}{360} * 132.84 = -2898$	0	$\frac{2}{3}\frac{r\sin\alpha}{\alpha} = \frac{2}{3}\frac{50\sin 66.42}{1.159} = 26.35$	0	−76365
3 Triangle	45.82*20=916.4	0	$\frac{2}{3}$*20=13.33	0	12218.7
Total	5715				164523

$$\bar{y} = \frac{\sum A * y}{\sum A} = \frac{164523}{5715} = 28.8\ \text{mm}$$

TABLE D/3 PROPERTIES OF PLANE FIGURES

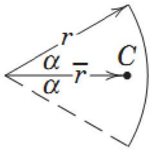
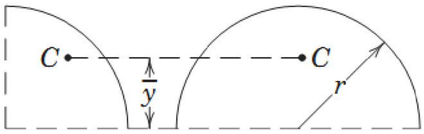
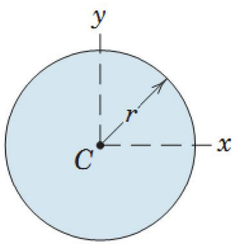
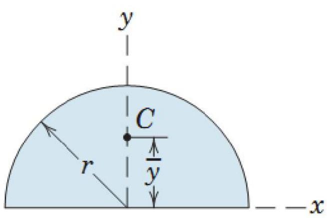
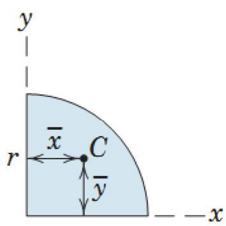
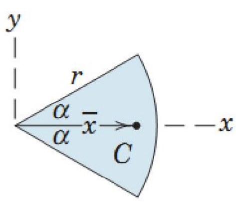
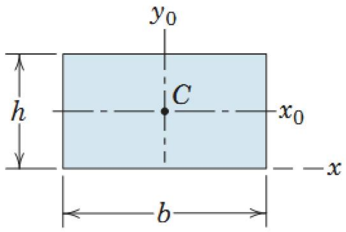
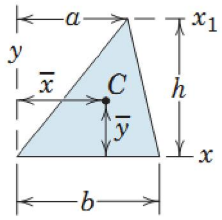
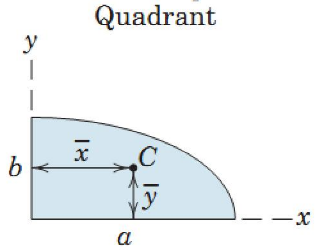
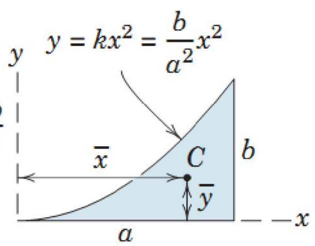
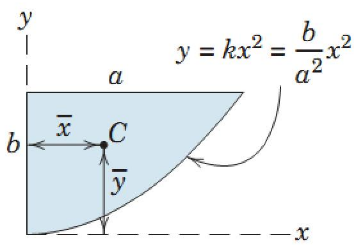
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Arc Segment 	$\bar{r} = \dfrac{r\sin\alpha}{\alpha}$	—
Quarter and Semicircular Arcs 	$\bar{y} = \dfrac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \dfrac{\pi r^4}{4}$ $I_z = \dfrac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{8}$ $\bar{I}_x = \left(\dfrac{\pi}{8} - \dfrac{8}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \dfrac{2}{3}\dfrac{r\sin\alpha}{\alpha}$	$I_x = \dfrac{r^4}{4}(\alpha - \dfrac{1}{2}\sin 2\alpha)$ $I_y = \dfrac{r^4}{4}(\alpha + \dfrac{1}{2}\sin 2\alpha)$ $I_z = \dfrac{1}{2} r^4 \alpha$

TABLE D/3 PROPERTIES OF PLANE FIGURES *Continued*

FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Rectangular Area 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
Area of Elliptical Quadrant 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}, \quad \bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ $I_y = \frac{\pi a^3 b}{16}, \quad \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3 b$ $I_z = \frac{\pi ab}{16}(a^2 + b^2)$
Subparabolic Area Area $A = \frac{ab}{3}$ 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$	$I_x = \frac{ab^3}{21}$ $I_y = \frac{a^3 b}{5}$ $I_z = ab\left(\frac{a^2}{5} + \frac{b^2}{21}\right)$
Parabolic Area Area $A = \frac{2ab}{3}$ 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$	$I_x = \frac{2ab^3}{7}$ $I_y = \frac{2a^3 b}{15}$ $I_z = 2ab\left(\frac{a^2}{15} + \frac{b^2}{7}\right)$

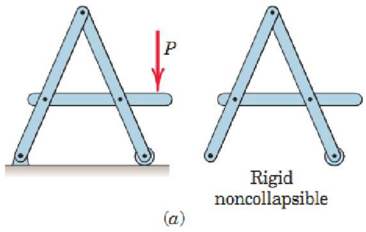
Frames and Machines

Frames: are structures which are designed to support applied loads and are usually fixed in position.

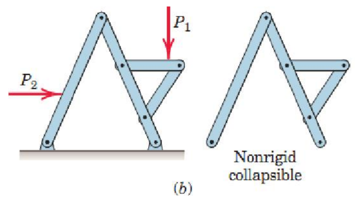
Machines: are structures which contain moving parts and are designed to transmit input forces or couples to output forces or couples.

The forces acting on each member of a connected system are found by isolating the member with a free-body diagram and applying the equations of equilibrium. The principle of action and reaction must be carefully observed when we represent the forces of interaction on the separate free-body diagrams.

If the frame or machine constitutes a *rigid* unit by itself when removed from its supports, like the A-frame in Fig. a, the analysis is best begun by establishing all the forces external to the structure treated as a single rigid body. We then dismember the structure and consider the equilibrium of each part separately. The equilibrium equations for the several parts will be related through the terms involving the forces of interaction.

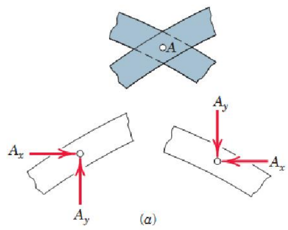


If the structure is *not a rigid* unit by itself but depends on its external supports for rigidity, as illustrated in Fig. b, then the calculation of the external support reactions cannot be completed until the structure is dismembered and the individual parts are analyzed.



It is not always possible to assign the proper sense to every force or its components when drawing the free-body diagrams and it becomes necessary to make an arbitrary assignment.

It is absolutely necessary that a force be consistently represented on the diagrams for interacting bodies which involve the force in question. Thus, for two bodies connected by the pin A, Fig. 4/15a, the force components must be *consistently* represented in *opposite directions* on the separate free-body diagrams.

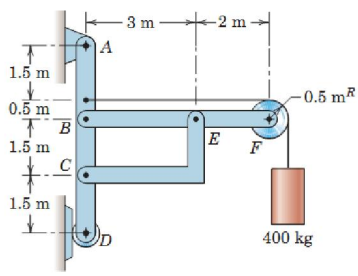


In order to separate the unknowns, need to solve two or more equations simultaneously

In most instances, can avoid simultaneous solutions by careful choice of the member or group of members for the free-body diagram and by a careful choice of moment axes which will eliminate undesired terms from the equations.

SAMPLE PROBLEM 4/6

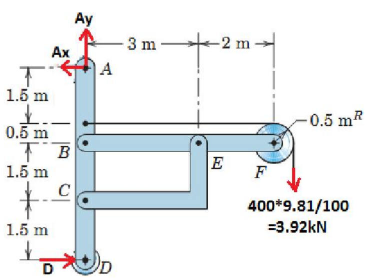
The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.



Solution:

Load, 400*9.81/1000=3.92kN

The three supporting members which constitute the frame form a *rigid* assembly that can be analyzed as a single unit. Also the arrangement of the external supports makes the frame statically determinate.



Determine the external reactions from the free-body diagram of the entire frame.

$$\circlearrowright \sum M_A = 0 \Rightarrow 3.92(3+2+0.5) - D(1.5+1.5+0.5+1.5) = 0$$

$$\Rightarrow D = 4.32kN \rightarrow$$

$$\rightarrow \sum F_x = 0 \Rightarrow D - Ax = 0 \Rightarrow Ax = D = 4.32kN \leftarrow$$

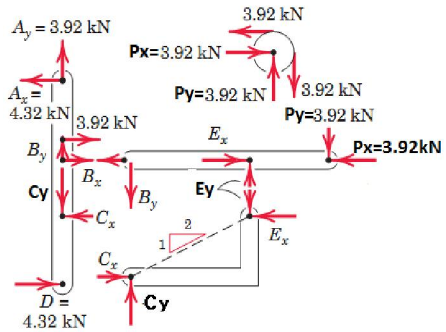
$\uparrow \sum F_y = 0 \Rightarrow Ay - 3.92 = 0 \Rightarrow Ay = 3.92kN \uparrow$

Dismember the frame and draw a separate free-body diagram of each member.

Pulley:

$\rightarrow \sum F_x = 0 \Rightarrow P_x - 3.92 = 0 \Rightarrow P_x = 3.92kN$

$\uparrow \sum F_y = 0 \Rightarrow P_y - 3.92 = 0 \Rightarrow P_y = 3.92kN$



Member CE (start with a two-force member)

$\circlearrowright \sum M_c = 0 \Rightarrow E_y * 3 - E_x * 1.5 = 0 \Rightarrow E_y = \frac{E_x}{2}$

$\circlearrowright \sum M_E = 0 \Rightarrow C_y * 3 - C_x * 1.5 = 0 \Rightarrow C_y = \frac{C_x}{2}$

$\rightarrow \sum F_x = 0 \Rightarrow C_x - E_x = 0 \Rightarrow C_x = E_x$

Member BF

$\circlearrowright \sum M_B = 0 \Rightarrow -E_y * 3 + 3.92 * 5 = 0$

$\Rightarrow -E_y * 3 + 3.92 * 5 = 0 \Rightarrow E_y = 6.53kN,$

$E_y = \frac{E_x}{2} \Rightarrow E_x = 2E_y = 2 * 6.53 \Rightarrow E_x = 13.08kN$

$C_x = E_x \Rightarrow C_x = 13.08kN$

$C_y = \frac{C_x}{2} \Rightarrow C_y = \frac{13.08}{2} = 6.54kN$

$\uparrow \sum F_y = 0 \Rightarrow -B_y + E_y - 3.92 = 0 \Rightarrow -B_y + 6.53 - 3.92 = 0 \Rightarrow B_y = 2.61kN$

$\rightarrow \sum F_x = 0 \Rightarrow -B_x + E_x - 3.92 = 0 \Rightarrow -B_x + 13.08 - 3.92 = 0 \Rightarrow B_x = 9.15kN$

Member AD for check only

$\circlearrowright \sum M_D = 0 \Rightarrow -A_x * 5 + 3.92 * 3.5 + B_x * 3 - C_x * 1.5$

$= -4.32 * 5 + 3.92 * 3.5 + 9.15 * 3 - 13.08 * 1.5 = 0 \;\; O.K$

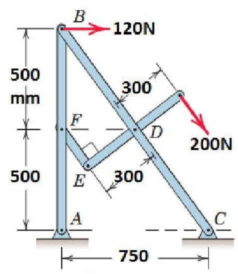
$\uparrow \sum F_y = 0 \Rightarrow A_y + B_y - \frac{C_x}{2} = 3.92 + 2.61 - \frac{13.08}{2} = 0 \;\; O.K$

$\rightarrow \sum F_x = 0 \Rightarrow -A_x + 3.92 + B_x - C_x + D = -4.32 + 3.92 + 9.15 - 13.08 + 4.32$

$= 0 \;\; O.K$

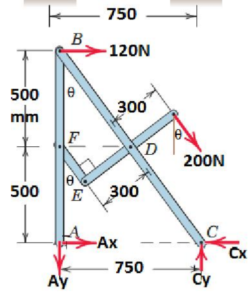
SAMPLE PROBLEM 4/7

Neglect the weight of the frame and compute the forces acting on all of its members.



Solution:

The frame is *not a rigid* unit when removed from its supports since BDEF is a movable quadrilateral and not a rigid triangle. Consequently the external reactions cannot be completely determined until the individual members are analyzed. However, the vertical components of the reactions at A and C can be determined from the free-body diagram of the frame as a whole. Thus,



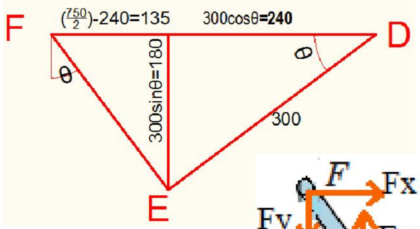
$\circlearrowright \sum M_c = 0 \Rightarrow -A_y * 0.75 + 120 * 1 + 200 * 0.3 = 0 \Rightarrow A_y = 240N \downarrow$

$\theta = \tan^{-1}\frac{750}{1000} = 36.87°$

$\uparrow \sum F_y = 0 \Rightarrow -A_y + C_y - 200\cos\theta = 0 \Rightarrow -240 + C_y - 200\cos 36.87° = 0 \Rightarrow C_y = 400N \downarrow$

Dismember the frame and draw the free-body diagram of each part:

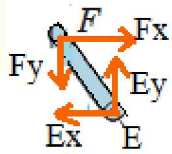
Member EF



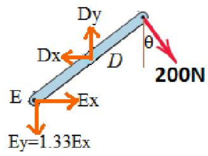
$\circlearrowright \sum M_F = 0 \Rightarrow E_x * 0.18 - E_y * 0.135 = 0 \Rightarrow E_y = 1.33E_x$

$\circlearrowright \sum M_E = 0 \Rightarrow F_x * 0.18 - F_y * 0.135 = 0 \Rightarrow F_y = 1.33F_x$

$\rightarrow \sum F_x = 0 \Rightarrow -E_x + F_x = 0 \Rightarrow F_x = E_x$



Member ED



$\circlearrowright \sum M_D = 0 \Rightarrow 200 * 0.3 - E_x * 0.18 - E_y * 0.24 = 0 \Rightarrow 200 * 0.3 - E_x * 0.18 - 1.33E_x * 0.24 = 0 \Rightarrow E_x = 120N \rightarrow$

$E_y = 1.33E_x \Rightarrow E_y = 1.33 * 120 \Rightarrow E_y = 160N \downarrow$

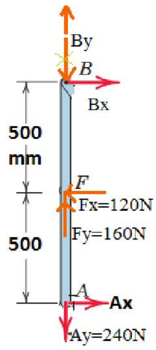
$F_x = E_x \Rightarrow F_x = 120N$

$F_y = 1.33F_x = 1.33 * 120 \Rightarrow F_y = 160N$

$\uparrow \sum F_y = 0 \Rightarrow D_y - E_y - 200\cos\theta = 0 \Rightarrow D_y - 160 - 200\cos 36.87 = 0 \Rightarrow D_y = 320N$
$\uparrow$

$\rightarrow \sum F_x = 0 \Rightarrow E_x - D_x + 200\sin 36.87 = 0 \Rightarrow 120 - D_x + 200\sin 36.87 = 0 \Rightarrow D_x = 240N \leftarrow$

Member AB

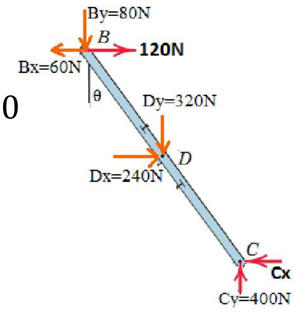


$\uparrow \sum F_y = 0 \Rightarrow -B_y + F_y - A_y = 0 \Rightarrow -B_y + 160 - 240 = 0 \Rightarrow B_y = -80N$
$= 80N \uparrow \textit{change dirction}$

$\circlearrowright \sum M_A = 0 \Rightarrow B_x * 1 - F_x * 0.5 = 0 \Rightarrow B_x * 1 - 120 * 0.5 = 0 \Rightarrow B_x = 60N \rightarrow$

$\rightarrow \sum F_x = 0 \Rightarrow B_x - F_x + A_x = 0 \Rightarrow 60 - 120 + A_x = 0 \Rightarrow A_x = 60N \rightarrow$

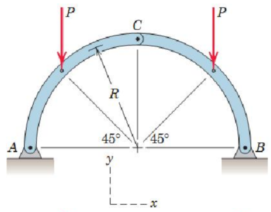
Member BC



$\rightarrow \sum F_x = 0 \Rightarrow -B_x + 120 + D_x + C_x = 0 \Rightarrow -60 + 120 + 240 + C_x = 0$
$\Rightarrow C_x = 300N \leftarrow$

Prob. 4/75

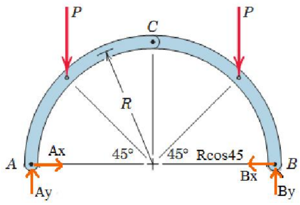
Determine the components of all forces acting on each member of the loaded frame.



Solution:

$\circlearrowright \sum M_B = 0 \Rightarrow A_y * 2r - P(r + r\cos 45) - P(r - r\cos 45) = 0 \Rightarrow 2A_y - 2P = 0 \Rightarrow A_y = P \uparrow$

$\uparrow \sum F_y = 0 \Rightarrow A_y + B_y - 2P = 0 \Rightarrow P + B_y - 2P = 0 \Rightarrow B_y = P \uparrow$

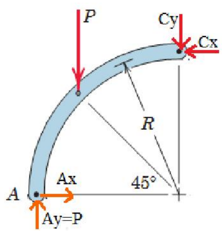


Member AC

$\circlearrowright \sum M_C = 0 \Rightarrow A_y * r - A_x * r - P(r\cos 45) = 0 \Rightarrow P * r - A_x * r - P(r\cos 45) = 0 \Rightarrow A_x = P(1 - \cos 45)$

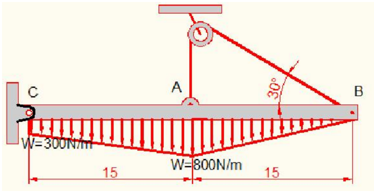
$\rightarrow \sum F_x = 0 \Rightarrow A_x - C_x = 0 \Rightarrow A_x = C_x \Rightarrow C_x = P(1 - \cos 45)$

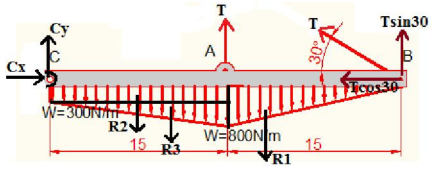
$\uparrow \sum F_y = 0 \Rightarrow A_y - P - C_y = 0 \Rightarrow C_y = 0$



Problem 4-145(Hibbeler)

Determine support reactions.





Solution:

$$R_1 = \frac{800 * 15}{2} = 6000N$$

$$R_2 = 300 * 15 = 4500N$$

$$R_3 = \frac{(800 - 300) * 15}{2} = 3750N$$

$$\circlearrowright \sum M_C = 0 \Rightarrow -(T \sin 30)30 - 15T + R_1\left(15 + \tfrac{15}{3}\right) + R_2\left(\tfrac{15}{2}\right) + R_3\left(\tfrac{2}{3}*15\right) = 0$$

$$\Rightarrow -(T \sin 30)30 - 15T + 6000\left(15 + \tfrac{15}{3}\right) + 4500\left(\tfrac{15}{2}\right) + 3750\left(\tfrac{2}{3}*15\right) = 0 \Rightarrow T = 6375N$$

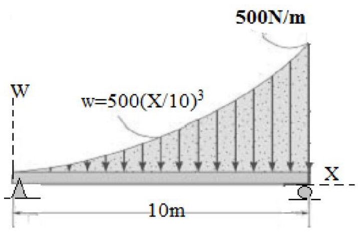
$$\uparrow \sum F_y = 0 \Rightarrow C_y + T + T \sin 30 - R_1 - R_2 - R_3 = 0$$

$$\Rightarrow C_y + 6375 + 6375 \sin 30 - 6000 - 4500 - 3750 = 0 \Rightarrow C_y = 4687N \uparrow$$

$$\rightarrow \sum F_x = 0 \Rightarrow C_x - T \cos 30 = 0 \Rightarrow C_x - 6375 \cos 30 = 0 \Rightarrow C_x = 5521N \rightarrow$$

Problem 4-156 (Hibbeler)

Wind has blown sand over a platform such that the intensity of the load can be approximated by the function $w = 500\left(\frac{x}{10}\right)^3$. Determine support reactions

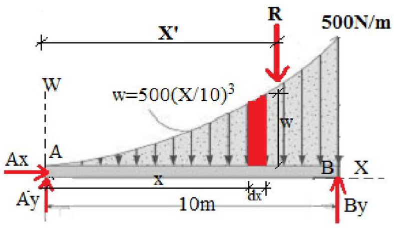


Solution:

Area under the curve=load, R $R = \int_0^{10} w dx = \int_0^{l} 500\left(\frac{x}{10}\right)^3 dx = 1250N \downarrow$

$$M_w = \int_0^{10} x w dx = \int_0^{10} x 500\left(\frac{x}{10}\right)^3 dx = 10000N.m$$

$$\bar{x} = \frac{M_w}{R} = \frac{10000}{1250} = 8m$$



$$\circlearrowright \sum M_B = 0 \Rightarrow A_y * 10 - R * 2 = 0 \Rightarrow A_y * 10 - 1250 * 2 = 0$$

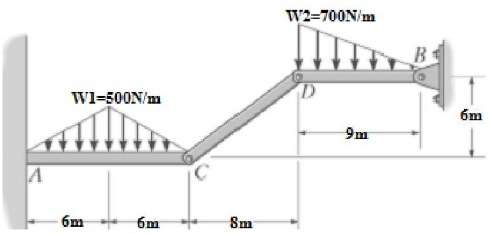
$$\Rightarrow A_y = 250N \uparrow$$

$$\uparrow \sum F_y = 0 \Rightarrow A_y + B_y - R = 0 \Rightarrow 250 + B_y - 1250 = 0 \Rightarrow B_y = 1000N \uparrow$$

$$\rightarrow \sum F_x = 0 \Rightarrow A_x = 0$$

Problem 6-77(Hibbeler)

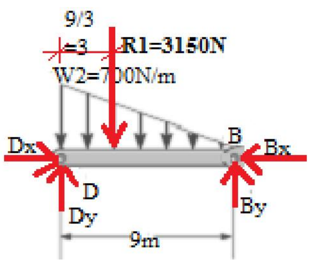
Determine the reactions at supports A and B.



Solution:

Member DB:

$$R_1 = \frac{700 * 9}{2} = 3150N$$

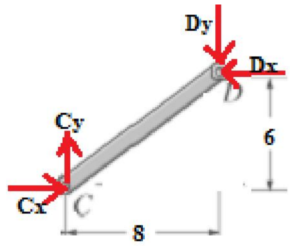


$\circlearrowright \sum M_B = 0 \Rightarrow 9D_y - R_1 * 6 = 0 \Rightarrow 9D_y - 3150 * 6 = 0 \Rightarrow D_y = 2100N \uparrow$

$$\uparrow \sum F_y = 0 \Rightarrow D_y + B_y - R_1 = 0 \Rightarrow 2100 + B_y - 3150 = 0 \Rightarrow B_y = 1050N \uparrow$$

$$\rightarrow \sum F_x = 0 \Rightarrow D_x - B_x = 0 \Rightarrow D_x = B_x$$

Member CD:



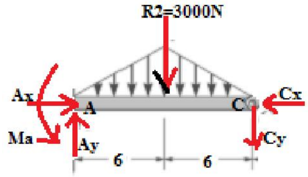
$$\uparrow \sum F_y = 0 \Rightarrow C_y - D_y = 0 \Rightarrow C_y - 2100 = 0 \Rightarrow C_y = 2100N \uparrow$$

$$\circlearrowright \sum M_C = 0 \Rightarrow 8D_y - 6D_x = 0 \Rightarrow D_x = \frac{8*2100}{6} \Rightarrow D_x = 2800N \leftarrow$$

$$D_x = B_x \Rightarrow B_x = 2800N$$

$$\rightarrow \sum F_x = 0 \Rightarrow C_x - D_x = 0 \Rightarrow C_x - 2800 = 0 \Rightarrow C_x = 2800N \rightarrow$$

Member AC:

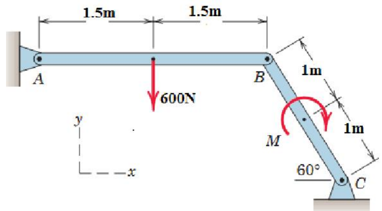


$$R_2 = \frac{500*12}{2} = 3000N$$

$$\uparrow \sum F_y = 0 \Rightarrow A_y - C_y - R_2 = 0 \Rightarrow A_y - 2100 - 3000 = 0 \Rightarrow A_y = 5100N \uparrow$$

$$\circlearrowright \sum M_C = 0 \Rightarrow 12A_y - 6R_2 - M_a = 0 \Rightarrow 12*5100 - 6*3000 - M_a = 0 \Rightarrow M_a = 43200N.mm \circlearrowleft$$

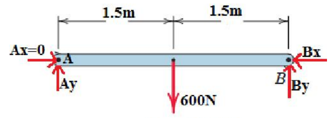
Prob. 4/74



For what value M of the clockwise couple will the horizontal component $\mathbf{A_x}$ of the pin reaction at A be zero? If a couple of that same magnitude M were applied in a counterclockwise direction, what would be the value of $\mathbf{A_x}$?

Solution:

Member AB:

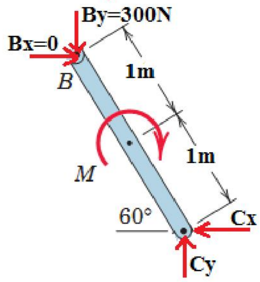


$$\circlearrowright \sum M_B = 0 \Rightarrow 3A_y - 600*1.5 = 0 \Rightarrow A_y = 300N \uparrow$$

$$\uparrow \sum F_y = 0 \Rightarrow A_y - 600 - B_y = 0 \Rightarrow 300 - 600 - B_y = 0 \Rightarrow B_y = 300N \uparrow$$

$$\rightarrow \sum F_x = 0 \Rightarrow A_x - B_x = 0 \Rightarrow A_x = B_x = 0$$

Member **BC**:



$$\circlearrowright \sum M_C = 0 \Rightarrow M - B_y(2\cos 60) = 0 \Rightarrow M - 300(2\cos 60) = 0 \Rightarrow M = 300N.m \circlearrowright$$

If $M = 300N.m \circlearrowleft$, find $\mathbf{A_x}$

Member **BC**:

$$\circlearrowright \sum M_C = 0 \Rightarrow -M - B_y(2\cos 60) + B_x(2\sin 60) = 0$$

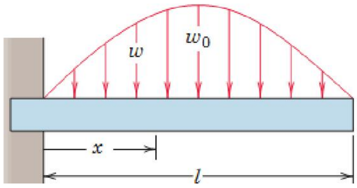
$$\Rightarrow -300 - B_y(2\cos 60) + B_x(2\sin 60) = 0 \Rightarrow B_y = -300 + 1.73B_x$$

Member AB:

$$\circlearrowright \sum M_A = 0 \Rightarrow -3B_y + 600*1.5 = 0 \Rightarrow -3(= -300 + 1.73B_x) + 600*1.5 = 0 \Rightarrow B_x = 347N \leftarrow$$

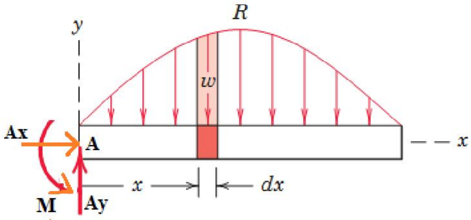
OR, $\rightarrow \sum F_x = 0 \Rightarrow A_x - B_x = 0 \Rightarrow A_x = B_x = 347N \rightarrow$

SAMPLE PROBLEM 5/14



The cantilever beam is subjected to the load intensity (force per unit length) which varies as $w = w_0 \sin(\frac{\pi x}{l})$. Determine the support reactions as functions of the ratio x/l.

Solution:



Area under the curve=load,R $R = \int_0^l w dx = \int_0^l w_0 \sin(\frac{\pi x}{l})\, dx =$

$w_0 \frac{l}{\pi}\left[-\cos\left(\frac{\pi x}{l}\right)\right]_0^l = w_0 \frac{l}{\pi}\left[-\cos\left(\frac{\pi l}{l}\right) + \cos\left(\frac{\pi 0}{l}\right)\right] =$

$w_0 \frac{l}{\pi}[-(-1) + (1)] = \frac{2w_0 l}{\pi}$

$\uparrow \sum F_y 0 \Rightarrow A_y - R = 0 \Rightarrow A_y - \frac{2w_0 l}{\pi} = 0 \Rightarrow A_y = \frac{2w_0 l}{\pi} \uparrow$

$\rightarrow \sum F_x = 0 \Rightarrow A_x = 0$

Centroid of the load, R:

$$M_y = \int_0^l xwdx = \int_0^l xw_0 \sin(\frac{\pi x}{l})\, dx = w_0 \int_0^l \underbrace{x}_{u} \underbrace{\sin(\frac{\pi x}{l})\, dx}_{dv}$$

Let u=x→du=dx

$$dv = \sin(\frac{\pi x}{l})\, dx \Rightarrow v = \frac{-l}{\pi}\cos(\frac{\pi x}{l})$$

$$\int u.dv = u.v - \int v.du \Rightarrow$$

$$\int x.\sin(\frac{\pi x}{l})dx = \underbrace{x}_{u}\underbrace{\frac{-l}{\pi}\cos(\frac{\pi x}{l})}_{v} - \int_0^l \underbrace{\frac{-l}{\pi}\cos(\frac{\pi x}{l})}_{v}\underbrace{dx}_{du} = \left[x\underbrace{\frac{-l}{\pi}\cos(\frac{\pi x}{l})}_{v} + \frac{l}{\pi}\left\{\frac{l}{\pi}\sin(\frac{\pi x}{l})\right\}\right]_0^l =$$

$$\left[l\underbrace{\frac{-l}{\pi}\cos(\frac{\pi l}{l})}_{v} + \frac{l}{\pi}\left\{\frac{l}{\pi}\sin(\frac{\pi l}{l})\right\}\right] - \left[0\underbrace{\frac{-l}{\pi}\cos(\frac{\pi 0}{l})}_{v} + \frac{l}{\pi}\left\{\frac{l}{\pi}\sin(\frac{\pi 0}{l})\right\}\right] = \left[\frac{l^2}{\pi}\right] - [0] = \frac{l^2}{\pi}$$

$$M_y = \int_0^l xwdx = w_0 \frac{l^2}{\pi}$$

$\bar{x} = \frac{M_y}{R} = \frac{w_0 \frac{l^2}{\pi}}{\frac{2w_0 l}{\pi}} = \frac{l}{2}$ OR due to symmetry, $\bar{x} = \frac{l}{2}$

$$\circlearrowright \sum M_A = 0 \Rightarrow R\bar{x} - M = 0 \Rightarrow M = \frac{2w_0 l}{\pi}\frac{l}{2} \Rightarrow M = \frac{w_0 l^2}{\pi}$$

Friction

In the preceding lectures, the forces of action and reaction between contacting surfaces are assumed that act normal to the surfaces. This assumption characterizes the interaction between smooth surfaces. Although this ideal assumption often involves only a relatively small error, there are many problems in which we must consider the ability of contacting surfaces to support tangential as well as normal forces. Tangential forces generated between contacting surfaces are called *friction forces* and occur to some degree in the interaction between all real surfaces. Whenever a tendency exists for one contacting surface to slide along another surface, the friction forces developed are always in a direction to oppose this tendency.

In some types of machines and processes we want to *minimize* the retarding effect of friction forces. Examples are bearings of all types, power screws, gears, the flow of fluids in pipes, and the propulsion of aircraft and missiles through the atmosphere. In other situations we wish to *maximize* the effects of friction, as in brakes, clutches, belt drives, and wedges. Wheeled vehicles depend on friction for both starting and stopping, and ordinary walking depends on friction between the shoe and the ground.

Types of Friction

(a) Dry Friction.

Dry friction occurs when the *unlubricated* surfaces of two solids are in contact under a condition of sliding or a tendency to slide. A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place. The direction of this friction force always *opposes* the motion or impending motion.

(b) Fluid Friction.

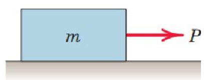
Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities. This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers. When there is no relative velocity, there is no fluid friction. Fluid friction depends not only on the velocity gradients within the fluid but also on the viscosity of the fluid, which is a measure of its resistance to shearing action between fluid layers.

(c) Internal Friction.

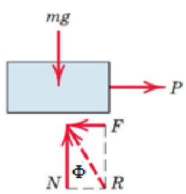
Internal friction occurs in all solid materials which are subjected to cyclical loading. For highly elastic materials the recovery from deformation occurs with very little loss of energy due to internal friction. For materials which have low limits of elasticity and which undergo appreciable plastic deformation during loading, a considerable amount of internal friction may accompany this deformation. The mechanism of internal friction is associated with the action of shear deformation, which is discussed in references on materials science.

Dry Friction

Consider a solid block of mass m resting on a horizontal surface, as shown in Fig. a. the contacting surfaces are assumed have some roughness. The *experiment* involves the application of a horizontal force P which continuously increases from zero to a value sufficient to move the block and give it an appreciable velocity. The free-body diagram of the block for any value of P is shown in Fig. b, where the tangential friction force exerted by the plane on the block is labeled F. This friction force acting on the body will always be in a direction to *oppose* motion or the tendency toward motion of the body. There is also a normal force N which in this case equals mg, and the total force R exerted by the supporting surface on the block is the resultant of N and F.



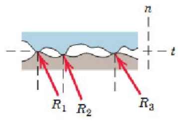
(a)



(b)

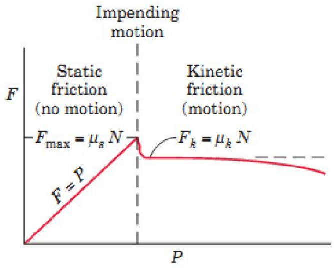
N: normal force
F: tangential friction force
R: the resultant of N and F
Φ: angle of internal friction

A magnified view of the irregularities of the mating surfaces, Fig. c, helps us to visualize the mechanical action of friction. Support is necessarily intermittent and exists at the mating humps. The direction of each of the reactions on the block, R1, R2, R3, etc. depends not only on the geometric profile of the irregularities but also on the extent of local deformation at each contact point. The total normal force N is the sum of the n-components of the R's, and the total frictional force F is the sum of the t-components of the R's. When the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the t-components of the R's are smaller than when the surfaces are at rest relative to one another. This observation helps to explain the well-known fact that the force P necessary to maintain motion is generally less than that required to start the block when the irregularities are more nearly in mesh.



(c)

If we perform the experiment and record the friction force F as a function of P, we obtain the relation shown in Fig. d. When P is zero, equilibrium requires that there be no friction force. As P is increased, the friction force must be equal and opposite to P as long as the block does not slip. During this period the block is in equilibrium, and all forces acting on the block must satisfy the equilibrium equations. Finally, we reach a value of P which causes the block to slip and to move in the direction of the applied force. At this same time the friction force decreases slightly and abruptly. It then remains essentially constant for a time but then decreases still more as the velocity increases.



(d)

Static Friction

The region in Fig. d up to the point of slippage or impending motion is called the range of *static friction*, and in this range the value of the friction force is determined by the equations of equilibrium. This friction force F may have any value from zero up to and including the maximum value F_{max}. For a given pair of mating surfaces the experiment shows that this maximum value of static friction F_{max} is proportional to the normal force N. Thus, we may write

$F_{max} = \mu_s N$ Applies only to cases where motion is impending with the friction force at its peak value.

F_{max}: maximum frictional force.
μ_s: Coefficient of static friction, $\mu_s = \tan \emptyset_s = \frac{F}{N}$
N: normal force.

For a condition of *static equilibrium* when motion is *not* impending, the static friction force is

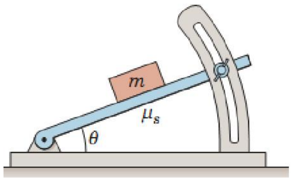
$F < \mu_s N$

Kinetic Friction

After slippage occurs, a condition of *kinetic friction* accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. The kinetic friction force F_k is also proportional to the normal force. Thus,
$F_k = \mu_k N$
μ_k: Coefficient of kinetic friction, $\mu_k = \tan \emptyset_k = \frac{F}{N}$, ($\mu_k$ is generally less than μ_s)
F_k: Kinetic friction force

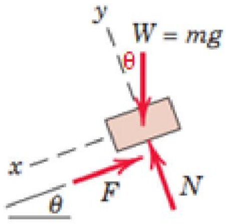
SAMPLE PROBLEM 6/1

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.



Solution.

The free-body diagram of the block shows its weight W=mg, the normal force N, and the friction force F exerted by the incline on the block. The friction force acts in the direction to oppose the slipping which would occur if no friction were present.



$$\nearrow \sum F_x = 0 \Rightarrow F - mg\sin\theta = 0 \Rightarrow F = mg\sin\theta$$

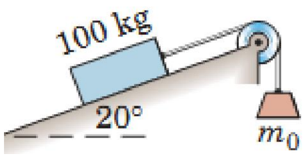
$$\nwarrow \sum F_y = 0 \Rightarrow N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$$

$$F_{max} = \mu_s N = \mu_s mg\cos\theta$$

Max. Angle θ occurs ⇒impending motion⇒ $F = F_{max} \Rightarrow mg\sin\theta = \mu_s mg\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \mu_s \Rightarrow \tan\theta = \mu_s \Rightarrow \theta = \tan^{-1}\mu_s$

SAMPLE PROBLEM 6/2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



Solution.

The maximum value of m_0 will be given by the requirement for motion impending *up* the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure.

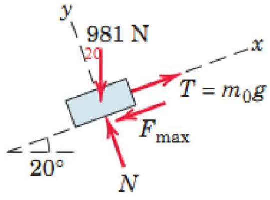
$$W = mg = 100 * 9.81 = 981N$$

$$\nwarrow \sum F_y = 0 \Rightarrow N - W\cos 20 = 0 \Rightarrow N - 981\cos 20 = 0 \Rightarrow N = 922N$$

$$F_{max} = \mu_s N = 0.3 * 922 = 277N$$

$$\nearrow \sum F_x = 0 \Rightarrow m_0 g - F_{max} - W\sin 20 = 0$$

$$\Rightarrow m_0 * 9.81 - 277 - 981\sin 20 = 0 \Rightarrow m_0 = 62.4kg \quad \text{maximum value}$$



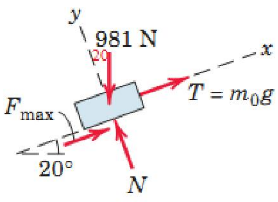
The minimum value of m_0 is determined when motion is impending *down* the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II.

$$\nwarrow \sum F_y = 0 \Rightarrow N - W\cos 20 = 0 \Rightarrow N - 981\cos 20 = 0 \Rightarrow N = 922N$$

$$F_{max} = \mu_s N = 0.3 * 922 = 277N$$

$$\nearrow \sum F_x = 0 \Rightarrow m_0 g + F_{max} - W\sin 20 = 0$$

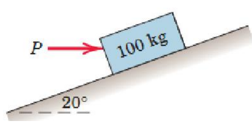
$$\Rightarrow m_0 * 9.81 + 277 - 981\sin 20 = 0 \Rightarrow m_0 = 6kg \quad \text{minimum value}$$



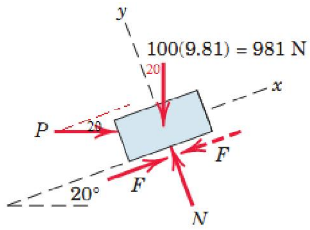
Thus, m_0 may have any value from 6 to 62.4 kg, and the block will remain at rest.

SAMPLE PROBLEM 6/3

Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, P= 500 N and, second, P=100 N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



Solution.



There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P. It is therefore necessary that we make an assumption, so we will take the *friction force to be up the plane*, as shown by the solid arrow. From the free-body diagram a balance of forces in both x- and y-directions gives

$W = mg = 100 * 9.81 = 981N$

$\nearrow \sum F_x = 0 \Rightarrow P\cos 20 + F - W\sin 20 = 0 \Rightarrow P\cos 20 + F - 981\sin 20 = 0 \ldots..1$

$\nwarrow \sum F_y = 0 \Rightarrow N - P\sin 20 - W\cos 20 = 0 \Rightarrow N - P\sin 20 - 981\cos 20 = 0 \ldots..2$

Case I. P= 500 N

$P\cos 20 + F - 981\sin 20 = 0 \ldots..1 \Rightarrow 500\cos 20 + F - 981\sin 20 = 0 \Rightarrow F = -134N$

The negative sign tells us that *if* the block is in equilibrium, the friction force acting on it is in the direction opposite to that assumed and therefore is down the plane, as represented by the dashed arrow.

$N - P\sin 20 - 981\cos 20 = 0 \ldots..2 \Rightarrow N - 500\sin 20 - 981\cos 20 = 0 \Rightarrow N = 1093N$

$F_{max} = \mu_s N = 0.2 * 1093 = 219N$

Since $F_{max} = 219N > F = 134N$(required for equilibrium), we conclude that the assumption of static equilibrium was correct.

Case II. P= 100 N

$P\cos 20 + F - 981\sin 20 = 0 \ldots..1 \Rightarrow 100\cos 20 + F - 981\sin 20 = 0 \Rightarrow F = +242N$ *friction force to be up the plane is correct assumption.*

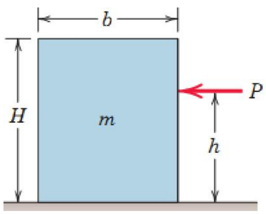
$N - P\sin 20 - 981\cos 20 = 0 \ldots..2 \Rightarrow N - 100\sin 20 - 981\cos 20 = 0 \Rightarrow N = 956N$

$F_{max} = \mu_s N = 0.2 * 956 = 191N$

Since $F = +242N > F_{max} = 191N \Rightarrow$Therefore, static equilibrium cannot exist, and we obtain the correct value of the friction force by using the kinetic coefficient of friction accompanying the motion down the plane.

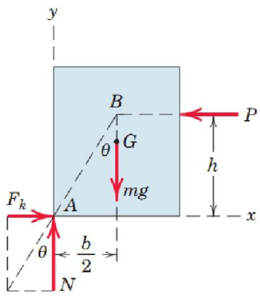
$F_k = \mu_k N = 0.17 * 956 = 163N \textit{up the plane}$

SAMPLE PROBLEM 6/4



The homogeneous rectangular block of *mass m*, *width b*, and *height H* is placed on the horizontal surface and subjected to a horizontal force *P* which moves the block along the surface with a constant velocity. The coefficient of kinetic friction between the block and the surface is μ_k. Determine (a) the greatest value which *h* may have so that the block will slide without tipping over and (b) the location of a point C on the bottom face of the block through which the resultant of the friction and normal forces acts if h=H/2.

Solution.



(a) With the block on the verge of tipping, we see that the entire reaction between the plane and the block will necessarily be at A. The free-body diagram of the block shows this condition. Since slipping occurs, the friction force is the limiting value $F_k = \mu_k N$, and the angle θ becomes $\theta = \tan^{-1}\mu_k$. The resultant of F_k and N passes through a point B through which P must also pass, since three coplanar forces in equilibrium are concurrent. Hence, from the geometry of the block

$$\tan\theta = \mu_k = \frac{b/2}{h} \Rightarrow h = \frac{b}{2\mu_k} \quad ans.$$

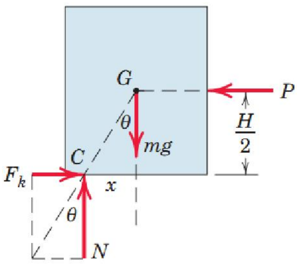
If $h > \frac{b}{2\mu_k}$, moment equilibrium about A would not be satisfied, and the block would tip over.

OR

$$\uparrow \sum F_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg$$

$$\rightarrow \sum F_x = 0 \Rightarrow F_k - P = 0 \Rightarrow P = F_k = \mu_k N \Rightarrow P = \mu_k mg$$

$$\circlearrowright \sum M_A = 0 \Rightarrow mg\frac{b}{2} - Ph = 0 \Rightarrow h = \frac{mgb}{2P} \Rightarrow h = \frac{mgb}{2\mu_k mg} \Rightarrow h = \frac{b}{2\mu_k} \quad ans.$$



(b) With h=H/2 we see from the free-body diagram for case (b) that the resultant of F_k and N passes through a point C which is a distance x to the left of the vertical centerline through G. The angle θ is still $\theta = \emptyset = \tan^{-1}\mu_k$ as long as the block is slipping. Thus, from the geometry of the figure

$$\tan\theta = \mu_k = \frac{x}{H/2} \Rightarrow x = \frac{\mu_k H}{2} \quad ans.$$

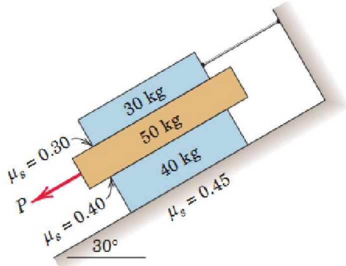
OR

$$\uparrow \sum F_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg$$

$$\rightarrow \sum F_x = 0 \Rightarrow F_k - P = 0 \Rightarrow P = F_k = \mu_k N \Rightarrow P = \mu_k mg$$

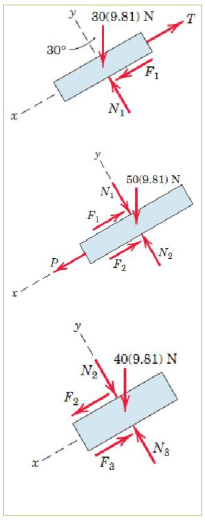
$$\circlearrowright \sum M_C = 0 \Rightarrow mgx - P\frac{H}{2} = 0 \Rightarrow x = \frac{PH}{2mg} \Rightarrow x = \frac{\mu_k mgH}{2mg} \Rightarrow x = \frac{\mu_k H}{2} \quad ans.$$

SAMPLE PROBLEM 6/5



The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the *maximum* value which P may have before any slipping takes place.

Solution:



$$\nwarrow \sum F_y = 0$$

$$30kg\ block \Rightarrow N_1 - mg\cos 30 = 0 \Rightarrow N_1 - 30 * 9.81\cos 30 = 0 \Rightarrow N_1 = 255N$$

$$50kg\ block \Rightarrow N_2 - N_1 - mg\cos 30 = 0 \Rightarrow N_2 - 255 - 50 * 9.81\cos 30 = 0$$
$$\Rightarrow N_2 = 680N$$

$$40kg\ block \Rightarrow N_3 - N_2 - mg\cos 30 = 0 \Rightarrow N_3 - N_2 - 40 * 9.81\cos 30 = 0$$
$$\Rightarrow N_3 = 1019N$$

There are two possible conditions for *impending* motion. Either the 50-kg block slips and the 40-kg block remains in place, or the 50-and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.

1. The 50-kg block slips, so that the 40-kg block remains in place (i.e. $F_1 = F_{1max}, F_2 = F_{2max}, F_3 < F_{3max}$).

For *impending* slippage at both surfaces of the 50-kg block,

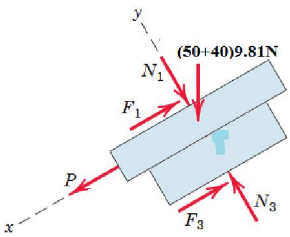
$F_{max} = \mu_s N$

$F_{1max} = \mu_s N_1 = 0.3 * 255 \Rightarrow F_{1max} = 76.5N$

$F_{2max} = \mu_s N_2 = 0.4 * 680 \Rightarrow F_{2max} = 272N$

$$\swarrow \sum F_{x})_{5\ 0kg} = 0 \Rightarrow P - F_1 - F_2 + mg\sin 30 = 0 \Rightarrow P - 76.5 - 272 + 50 * 9.81\sin 30 = 0 \Rightarrow P = 103.5N$$

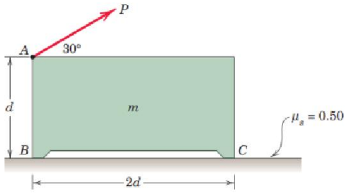
2. The 50-and 40-kg blocks move together with *impending* slippage occurring between the (50-and 40-kg blocks) and the (incline and 30-kg block)(i.e. $F_1 = F_{1max}, F_3 = F_{3max}, F_2 < F_{2max}$)



$$\nwarrow \sum F_{y)_{(4\ 0+5\ 0)kg}} = 0 \Rightarrow N_3 - N_1 - (50+40) * 9.81 \cos 30 = 0$$
$$\Rightarrow N_3 - 255 - (50+40) * 9.81 \cos 30 = 0 \Rightarrow N_3 = 1019N$$
$F_{3max} = \mu_s N_3 = 0.45 * 1019 \Rightarrow F_{3max} = 459N$
$$\swarrow \sum F_{x)_{(4\ 0+5\ 0)kg}} = 0 \Rightarrow -F_3 - F_1 + P + (50+40)9.81 \sin 30 = 0$$
$$\Rightarrow -459 - 76.5 + P + (50+40)9.81 \sin 30 = 0 \Rightarrow P = 94N$$
Choose minimum value of $P(103.5, 94)N$,
$\therefore P = 94N \quad ans.$

Prob. 6/5

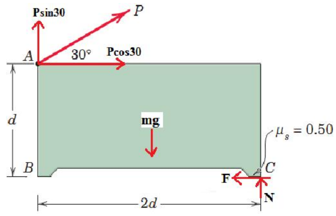
The magnitude of force **P** is slowly increased. Does the homogeneous box of mass ***m*** slip or tip first? State the value of **P** which would cause each occurrence. Neglect any effect of the size of the small feet.



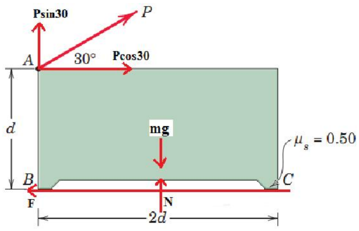
Solution:

1. Assume tipping occur first about point c.

$$\circlearrowright \sum M_C = 0 \Rightarrow P \sin 30 * 2d + P \cos 30 * d - mg\frac{2d}{2} = 0 \Rightarrow P_{\text{tipping}} = 0.536mg$$



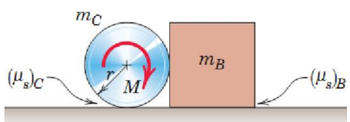
2. Assume sliding occur first.



$$\uparrow \sum F_y = 0 \Rightarrow N + P \sin 30 - mg = 0 \Rightarrow N = mg - P \sin 30$$
$$\leftarrow \sum F_x = 0 \Rightarrow F - P \cos 30 = 0 \Rightarrow F = P \cos 30$$
$F_{max} = \mu_s N = 0.5(mg - P\ sin\ 30)$
For slip beginning, $F = F_{max} \Rightarrow$
$P\ cos\ 30 = 0.5(mg - P\ sin\ 30) \Rightarrow P_{slipping} = 0.448mg$
Since $P_{slipping} = 0.448mg < P_{tipping} = 0.536mg \Rightarrow \therefore slipping\ occur\ first\ \ Ans.$

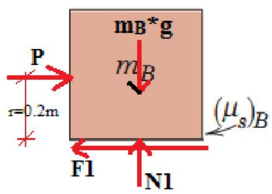
Prob. 6/24

A clockwise couple **M** is applied to the circular cylinder as shown. Determine the value of **M** required to initiate motion for the conditions **m_B=3kg, m_C=6kg, $(\mu_s)_B$=0.5, $(\mu_s)_C$=0.4, and r = 0.2 m**. Friction between the cylinder C and the block B is negligible.



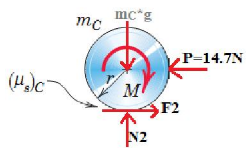
Solution:

Body B:



$$\uparrow \sum F_y = 0 \Rightarrow N_1 - m_B g = 0 \Rightarrow N_1 - 3 * 9.81 = 0 \Rightarrow N_1 = 29.43N$$
$F_{1max} = \mu_{B)s} N_1 = 0.5 * 29.43 = 14.7N$
For slipping of body B, $F_1 = F_{1max} = 14.7N$
$$\leftarrow \sum F_x = 0 \Rightarrow F_1 - P = 0 \Rightarrow P = F_1 = 14.7N$$

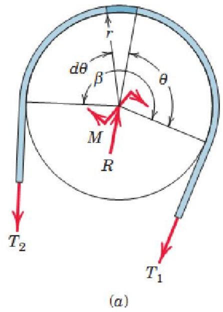
Body C:



$$\uparrow \sum F_y = 0 \Rightarrow N_2 - m_C g = 0 \Rightarrow N_2 - 6 * 9.81 = 0 \Rightarrow N_2 = 59N$$
$$\rightarrow \sum F_x = 0 \Rightarrow F_2 - P = 0 \Rightarrow F_2 - 14.7 = 0 \Rightarrow F_2 = 14.7N$$
$F_{2max} = \mu_{C)s} N_2 = 0.4 * 59 = 23.5N$
$F_2 = 14.7N < F_{2max} = 23.5N$ O.K (for motion, not slipping)
$$\circlearrowright \sum M_{Center} = 0 \Rightarrow M - F_2 * r = 0 \Rightarrow M - 14.7 * 0.2 = 0 \Rightarrow M = 2.94N.m \circlearrowright$$

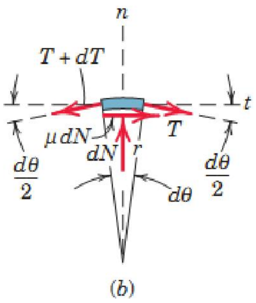
Flexible Belts

The impending slippage of flexible cables, belts, and ropes over sheaves and drums is important in the design of belt drives of all types, band brakes, and hoisting rigs.



(a)

Figure a, shows a drum subjected to the two belt tensions T1 and T2, the torque M necessary to prevent rotation, and a bearing reaction R. With M in the direction shown, T2 is greater than T1. The free-body diagram of an element of the belt of length rdθ is shown in figure b. We analyze the forces acting on this differential element analyzeed by establishing the equilibrium of the element, in a manner similar to that used for other variable-force problems. The tension increases from T at the angle θ to T+dT at the angle θ+dθ. The normal force is a differential dN, since it acts on a differential element of area. Likewise the friction force, which must act on the belt in a direction to oppose slipping, is a differential and is μdN for impending motion.



(b)

Equilibrium in the t-direction gives

$$\sum F_t = 0 \Rightarrow T \underbrace{\cos\frac{d\theta}{2}}_{=1} + \mu dN = (T + dT)\underbrace{\cos\frac{d\theta}{2}}_{=1} \Rightarrow T + \mu dN = T + dT \Rightarrow$$

$\mu dN = dT \ldots 1$

Equilibrium in the n-direction requires that

$$\sum F_n = 0 \Rightarrow dN = (T + dT)\underbrace{\sin\frac{d\theta}{2}}_{=\frac{d\theta}{2}} + T\underbrace{\sin\frac{d\theta}{2}}_{=\frac{d\theta}{2}} \Rightarrow dN = T\frac{d\theta}{2} + \underbrace{dT\frac{d\theta}{2}}_{\text{ignored}} + T\frac{d\theta}{2} \Rightarrow$$

$dN = Td\theta \ldots 2$

Combining the two equilibrium relations gives

$$\frac{dT}{T} = \mu d\theta$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu d\theta \Rightarrow [\ln T]_{T_1}^{T_2} = [\mu\theta]_0^{\beta} \Rightarrow \ln T_2 - \ln T_1 = \mu\beta \Rightarrow \ln\frac{T_2}{T_1} = \mu\beta \Rightarrow e^{\ln\frac{T_2}{T_1}} = e^{\mu\beta} \Rightarrow$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \ldots 3$$

T2: greater force

T1: smaller force

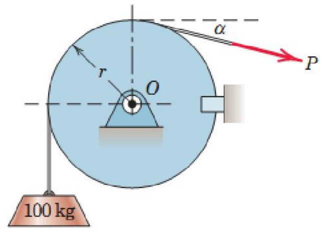
μ: coefficient of static friction.

β: total angle of belt contact and must be expressed in radians. If a rope were wrapped around drum n times, the angle β would be $2\pi n$ radians

Equation 3 holds equally well for a noncircular section where the total angle of contact is β. This conclusion is evident from the fact that the radius r of the circular drum in Fig. does not enter into the equations for the equilibrium of the differential element of the belt.

SAMPLE PROBLEM 6/9

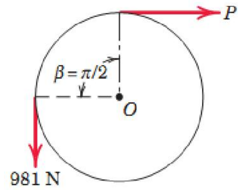
A flexible cable which supports the 100-kg load is passed over a fixed circular drum and subjected to a force P to maintain equilibrium. The coefficient of static friction μ between the cable and the fixed drum is 0.30. (a) For α=0, determine the maximum and minimum values which P may have in order not to raise or lower the load. (b) For P =500 N, determine the minimum value which the angle α may have fore the load begins to slip.



Solution:

(a)

W=100*9.81=981N



(a) $\alpha = 0$

With α=0, $\beta = 90° = \frac{\pi}{2}\ rad.$

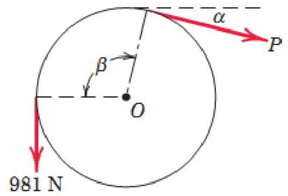
For impending upward motion of the load, $T_2 = P_{max}$, $T_1 = W = 981N$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{P_{max}}{981} = e^{0.3*\frac{\pi}{2}} \Rightarrow P_{max} = 1572\ N$$

For impending downward motion of the load, $T_2 = 981N$ and $T_1 = P_{min}$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{981}{P_{min}} = e^{0.3*\frac{\pi}{2}} \Rightarrow P_{min} = 612\ N$$

(b)



(b) $P = 500$ N

$$T_2 = 981N\ ,\ T_1 = P = 500\ N$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{981}{500} = e^{0.3\beta} \Rightarrow 1.962 = e^{0.3\beta} \Rightarrow \ln 1.962 = \ln e^{0.3\beta}$$

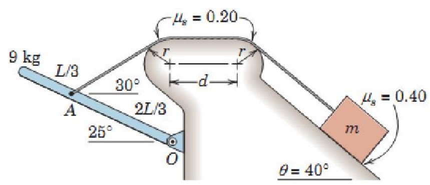
$$\Rightarrow \ln 1.962 = 0.3\beta \Rightarrow \beta = \frac{\ln 1.962}{0.3} = 2.25\,rad.$$

$$\beta_{degree} = 2.25 * \frac{180}{\pi} = 128.7°$$

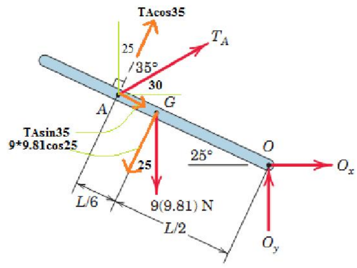
$$\beta = 90° + \alpha \Rightarrow 128.7° = 90° + \alpha \Rightarrow \alpha = 38.7°$$

SAMPLE PROBLEM 6/10

Determine the range of mass *m* over which the system is in static equilibrium. The coefficient of static friction between the cord and the upper curved surface is 0.20, while that between the block and the incline is 0.40. Neglect friction at the pivot O.



Solution:

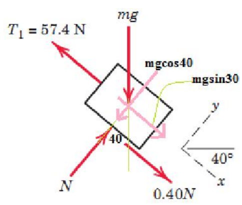


$$\circlearrowright \sum M_o = 0 \Rightarrow T_A \cos 35 * \frac{2}{3}L - 9 * 9.81 \cos 25 * \frac{L}{2} = 0 \Rightarrow T_A = 73.3\ N$$

<u>I. Motion of m impends up the incline.</u>

$$\therefore T_A = P_{max} = 73.3\ N$$

$$\beta = (40+30)\frac{\pi}{180} = 1.22\,rad.$$



Case I

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{73.3}{T_1} = e^{0.2*1.22} \Rightarrow T_1 = 57.4\ N$$

$$\nearrow \sum F_y = 0 \Rightarrow N - mg\cos 40 = 0 \Rightarrow N - mg\cos 40 = 0 \Rightarrow N = 0.76mg$$

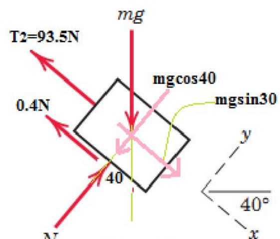
$$\nwarrow \sum F_x = 0 \Rightarrow T_1 - mg\sin 40 - \mu N = 0$$

$$\Rightarrow 57.4 - mg\sin 40 - 0.4 * 0.76mg = 0 \Rightarrow m = 6.16\ kg$$

<u>II. Motion of m impends down the incline.</u>

$$\therefore T_A = P_{min} = 73.3\ N$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{T_2}{73.3} = e^{0.2*1.22} \Rightarrow T_2 = 93.5\ N$$



Case II

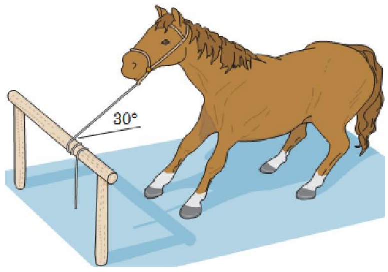
$$\nearrow \sum F_y = 0 \Rightarrow N = 0.76mg$$

$$\nwarrow \sum F_x = 0 \Rightarrow T_2 - mg\sin 40 + \mu N = 0$$

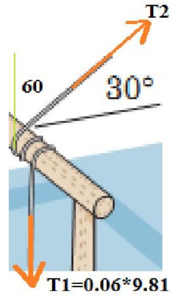
$$\Rightarrow 93.5 - mg\sin 40 + 0.4 * 0.76mg = 0 \Rightarrow m = 28.3kg$$

So the requested range is $6.16 \le m \le 28.3\ kg$.

Prob. 6/104



In western movies, cowboys are frequently observed hitching their horses by casually winding a few turns of the reins around a horizontal pole and letting the end hang free as shown—no knots! If the freely hanging length of rein weighs 0.6kg and the number of turns is as shown, what tension T does the horse have to produce in the direction shown in order to gain freedom? The coefficient of friction between the reins and wooden pole is 0.70.



Solution:

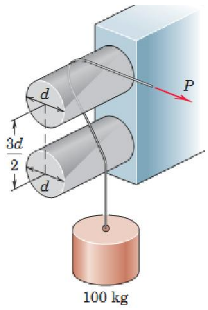
$T_1 = 0.06 * 9.81 = 0.5886\, N$

$\beta = 2 * 2\pi + 60 * \frac{\pi}{180} = 13.61\, rad.$

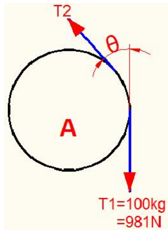
$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{T_2}{0.5886} = e^{0.7*13.61} \Rightarrow T_2 = T = 8079N \cong 8.1\, kN$

Prob. 6/105

Calculate the horizontal force P required to raise the 100-kg load. The coefficient of friction between the rope and the fixed bars is 0.40.



Solution:

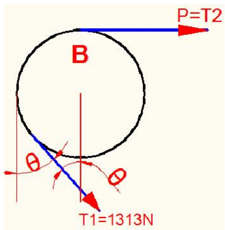


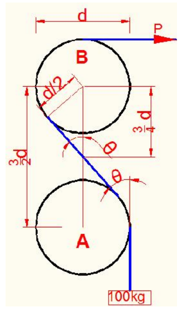
$\theta = \sin^{-1}\frac{d/2}{\frac{3}{4}d} = 41.8°$

Pulley A

$\beta = \theta = 41.8 * \frac{\pi}{180} = 0.7295\, rad.$

$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{T_2}{100 * 9.81} = e^{0.4*0.7295} \Rightarrow T_2 = 1313\, N$



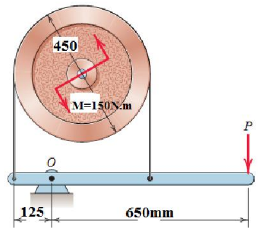


Pulley B

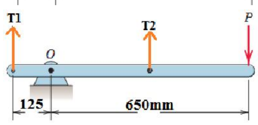
$\beta = 90 + \theta = (90 + 41.8) * \frac{\pi}{180} = 2.3\, rad.$

$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{P}{1313} = e^{0.4*2.3} \Rightarrow P = 3.3\, kN$

Prob. 6/111



A counterclockwise moment 150 N.m is applied to the flywheel. If the coefficient of friction between the band and the wheel is 0.20, compute the minimum force P necessary to prevent the wheel from rotating.



Solution:

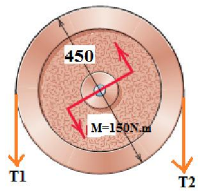
$\circlearrowright \sum M_o = 0 \Rightarrow 650P + 125T_1 - (450 - 125)T_2 = 0 \Rightarrow$

$650P + 125T_1 - 325T_2 = 0 \ldots\ldots 1$

$\beta = \pi$

$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{T_2}{T_1} = e^{0.2\pi} \Rightarrow \frac{T_2}{T_1} = 1.874 \ldots\ldots 2$

Applied Torque= $150 * 10^3 N.mm$



Resisting Torque = $(T_2 - T_1)r \ldots 3 \Rightarrow \left(T_2 - \frac{T_2}{1.874}\right)\frac{450}{2} \ldots\ldots 3 + 2$

$M_{applied} = M_{resisting}$

$150 * 10^3 = \left(T_2 - \frac{T_2}{1.874}\right)\frac{450}{2} \Rightarrow T_2 = 1429N$

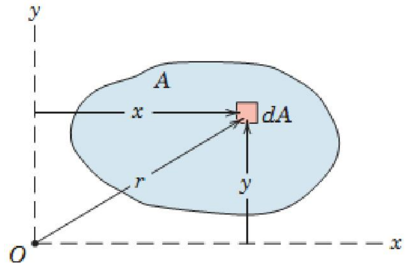
$T_1 = \frac{T_2}{1.874} = \frac{1429}{1.874} \Rightarrow T_1 = 762N$

$650P + 125T_1 - 325T_2 = 0 \Rightarrow 650P + 125 * 762 - 325 * 1429 = 0 \Rightarrow P = 568N$

Area Moments of Inertia

$$I_x = \int y^2 dA$$

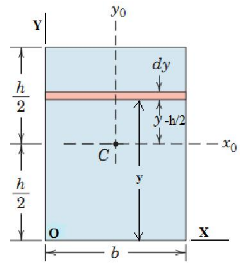
$$I_y = \int x^2 dA$$



SAMPLE PROBLEM A/1

Determine the moments of inertia of the rectangular area about the centroidal x_0- and y_0-axes.

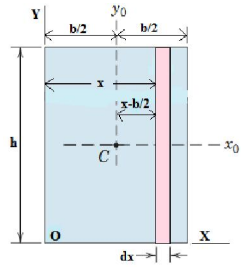
Solution:



$$I_{x0} = \int_0^h (y - \tfrac{h}{2})^2 (bdy) = b\int_0^h (y^2 - hy + \tfrac{h^2}{4})dy = b\left[\frac{y^3}{3} - \frac{hy^2}{2} + \frac{h^2}{4}y\right]_0^h$$

$$= b\left[\left(\frac{h^3}{3} - \frac{hh^2}{2} + \frac{h^2}{4}h\right) - 0\right] = \frac{bh^3}{12}\, about\ centroidal\ axis$$

$$I_x = \int_0^h y^2(bdy) = b\left[\frac{y^3}{3}\right]_0^h = \frac{bh^3}{3}\, about\ base\ axis$$



$$I_{y0} = \int_0^b (x - \tfrac{b}{2})^2 (hdx) = h\int_0^b (x^2 - bx + \tfrac{b^2}{4})dx = h\left[\frac{x^3}{3} - \frac{bx^2}{2} + \frac{b^2}{4}x\right]_0^b$$

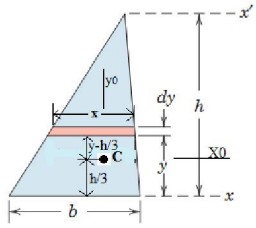
$$= b\left[\left(\frac{b^3}{3} - \frac{bb^2}{2} + \frac{b^2}{4}b\right) - 0\right] = \frac{hb^3}{12}\, about\ centroidal\ axis$$

$$I_y = \int_0^b x^2(hdx) = h\left[\frac{x^3}{3}\right]_0^b = \frac{hb^3}{3}\, about\ base\ axis$$

SAMPLE PROBLEM A/2

Determine the moments of inertia of the triangular area about its base and about parallel axes through its centroid and vertex

Solution:



$$\frac{x}{h-y} = \frac{b}{h} \Rightarrow x = \frac{b}{h}(h-y)$$

$$I_{x0} = \int_0^h \left(y - \tfrac{h}{3}\right)^2 (xdy) = \int_0^h \left(y^2 - \tfrac{2}{3}hy + \tfrac{h^2}{9}\right)\frac{b}{h}(h-y)dy$$

$$= \frac{b}{h}\int_0^h (hy^2 - \tfrac{2}{3}h^2y + \tfrac{h^3}{9} - y^3 + \tfrac{2}{3}hy^2 - \tfrac{h^2y}{9})dy = \frac{b}{h}\left[\frac{hy^3}{3} - \frac{2}{3}h^2\frac{y^2}{2} + \frac{h^3}{9}y - \frac{y^4}{4} + \frac{2}{3}h\frac{y^3}{3} - \frac{h^2y^2}{9*2}\right]_0^h$$

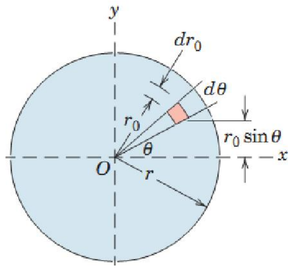
$$= \frac{bh^3}{36}\, about\ centroidal\ axis$$

$$I_x = \int_0^h y^2(xdy) = \int_0^h y^2\frac{b}{h}(h-y)dy = \frac{b}{h}\int_0^h (y^2h - y^3)dy = \frac{b}{h}\left[\frac{y^3}{3}h - \frac{y^4}{4}\right]_0^h$$

$$= \frac{bh^3}{12}\, about\ base\ axis$$

SAMPLE PROBLEM A/3

Calculate the moments of inertia of the area of a circle about a diametral axis



Solution:

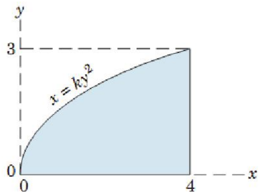
$$I_x = \int y^2(dA) = \int_0^{2\pi}\int_0^{r} (r_0 \sin\theta)^2 (r_0 d\theta dr_0) = \int_0^{2\pi}\int_0^{r} {r_0}^3 (\sin\theta)^2 d\theta dr_0$$

$$= \int_0^{2\pi} \left[\frac{{r_0}^4}{4}\right]_0^r (\sin\theta)^2 d\theta = \int_0^{2\pi} \frac{r^4}{4}(\sin\theta)^2 d\theta = \frac{r^4}{4}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta = \frac{r^4}{8}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} = \frac{\pi r^4}{4}$$

$$I_x = I_y = \frac{\pi r^4}{4}$$

SAMPLE PROBLEM A/4

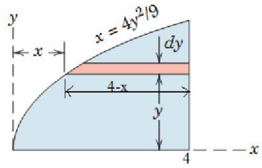
Determine the moment of inertia of the area under the parabola about the x-axis. Solve by using (a) a horizontal strip of area and (b) a vertical strip of area.



Solution:

$$x = ky^2 \Rightarrow 4 = k3^2 \Rightarrow k = \frac{4}{9}$$

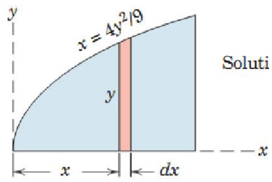
$$x = \tfrac{4}{9}y^2$$



1. Horizontal Strip

$$I_x = \int_0^3 y^2(4-x)dy = \int_0^3 y^2\left(4-\tfrac{4}{9}y^2\right)dy = \int_0^3\left(4y^2 - \tfrac{4}{9}y^4\right)dy = \left[\frac{4y^3}{3} - \frac{4}{9*5}y^5\right]_0^3 = 14.4\ (units)^4$$

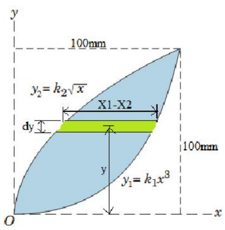
2. Vertical Strip



$$I_x = \int_0^4 \frac{y^3}{3}dx = \frac{1}{3}\int_0^4 \left[\left(\frac{9x}{4}\right)^{\frac{1}{2}}\right]^3 dx = \frac{27}{24}\left[\frac{2x^{\frac{5}{2}}}{5}\right]_0^4 = 14.4\ (units)^4$$

Prob. A/32

Calculate the moments of inertia of the shaded area about the x- and y-axes, and find the polar moment of inertia about point O.



Solution:

$$y_2 = k_2\sqrt{x} \Rightarrow 100 = k_2\sqrt{100} \Rightarrow k_2 = 10$$

$$y_1 = k_1x^3 \Rightarrow 100 = k_1 100^3 \Rightarrow k_1 = \frac{1}{10000}$$

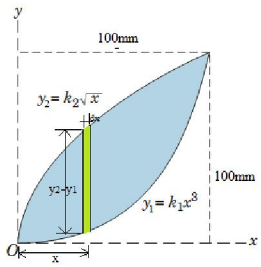
$$y_1 = \frac{1}{10000}x^3,\ y_2 = 100\sqrt{x}$$

$$I_x = \int_0^{100} y^2(x_1 - x_2)dy = \int_0^{100} y^2\left(\sqrt[3]{\frac{y}{k_1}} - \frac{y^2}{{k_2}^2}\right)dy = \left[\frac{3y^{\frac{10}{3}}}{10\sqrt[3]{k_1}} - \frac{y^5}{5{k_2}^2}\right]_0^{100} = 1*10^7 mm^4$$

$$I_y = \int_0^{100}\frac{{x_1}^3 dy}{3} - \int_0^{100}\frac{{x_2}^3 dy}{3} = \frac{1}{3}\int_0^{100}({x_1}^3 - {x_2}^3)dy =$$

$$\frac{1}{3}\int_0^{100}\left(\left(\sqrt[3]{\frac{y}{k_1}}\right)^3 - \left(\frac{y^2}{{k_2}^2}\right)^3\right)dy = \frac{1}{3}\int_0^{100}\left(\frac{y}{k_1} - \frac{y^6}{{k_2}^6}\right)dy = 11.9 * 10^6 mm^4$$

OR



$$I_y = \int_0^{100} x^2(y_2 - y_1)dx = \int_0^{100} x^2(k_2\sqrt{x} - k_1x^3)dx = 11.9 * 10^6 mm^4$$

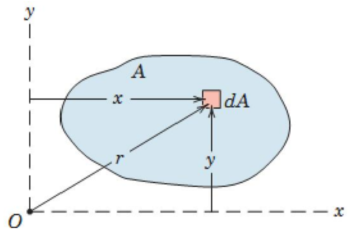
Polar Moments of Inertia:

The moment of inertia of dA about the pole O (z-axis) is

$$I_z = \int r^2 dA = \int \underbrace{(x^2 + y^2)}_{r^2} dA = \int x^2 dA + \int y^2 dA = I_y + I_x$$

Radius of Gyration,k

Is a measure of the distribution of the area from the axis in question.

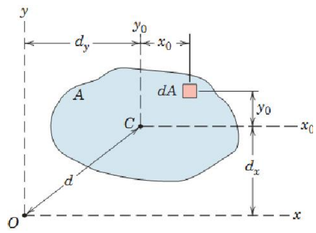


$$k = \sqrt{\frac{I}{A}}$$

$$k_x = \sqrt{\frac{I_x}{A}}, \quad k_y = \sqrt{\frac{I_y}{A}}, \quad k_z = \sqrt{\frac{I_z}{A}}$$

$$I_z = I_x + I_y \Rightarrow {k_z}^2 A = {k_x}^2 A + {k_y}^2 A \Rightarrow {k_z}^2 = {k_x}^2 + {k_y}^2$$

Transfer of Axes(parallel-axis theorems)



C: the centroid of the area.

$$I_x = \int (y_0 + d_x)^2 dA = \int ({y_0}^2 + 2y_0 d_x + {d_x}^2) dA$$

$$= \underbrace{\int ({y_0}^2) dA}_{I_{x0}} + \underbrace{\int (2y_0 d_x) dA}_{zero} + \underbrace{{d_x}^2 \int dA}_{{d_x}^2 A}$$

The second integral is zero, since $\int (y_0) dA = A\bar{y}_0$ and $\bar{y}_0$is automatically zero with the centroid on the x_0-axis.

$$I_x = I_{x0} + {d_x}^2 A$$

$$I_y = I_{y0} + {d_y}^2 A$$

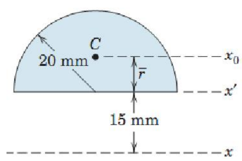
$$I_z = I_x + I_y = I_{x0} + {d_x}^2 A + I_{y0} + {d_y}^2 A = \underbrace{I_{x0} + I_{y0}}_{I_{z0}} + A \underbrace{({d_x}^2 + {d_y}^2)}_{=d^2} = I_{z0} + Ad^2$$

Two points in particular should be noted.

1. The axes between which the transfer is made must be parallel.
2. One of the axes must pass through the centroid of the area.

SAMPLE PROBLEM A/5

Find the moment of inertia about the x-axis of the semicircular area.



Solution.

$$\bar{r} = \frac{4r}{3\pi} = \frac{4*20}{3\pi} = \frac{80}{3\pi}$$

$$I_{\bar{x}} = \frac{1}{2}\left(\frac{\pi r^4}{4}\right) = \frac{1}{2}\left(\frac{\pi 20^4}{4}\right) = 62832\ mm^4$$

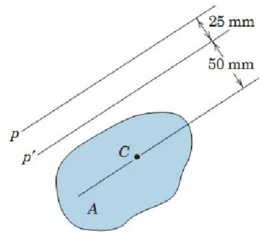
$$I_{\bar{x}} = I_{x_0} + A * (\bar{r})^2 \Rightarrow 62832 = I_{x_0} + \frac{\pi 20^2}{2} * \left(\frac{80}{3\pi}\right)^2 \Rightarrow I_{x_0} = 17561\ mm^4$$

$$\text{OR } I_{x_0} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) 20^4 = 17561\ mm^4$$

$$I_x = I_{x_0} + A * (\bar{r} + 15)^2 = 17561 + \frac{\pi 20^2}{2} * \left(\frac{80}{3\pi} + 15\right)^2 = 36.4 * 10^4\ mm^4$$

Prob. A/5

The moments of inertia of the area A about the parallel p- and p' -axes differ by $15*10^6$ mm^2 . Compute the area A, which has its centroid at C.



Solution:

$$I_{\bar{p}} = I_0 + A * d^2 = I_0 + A * 50^2$$

$$I_p = I_0 + A * d^2 = I_0 + A * 75^2$$

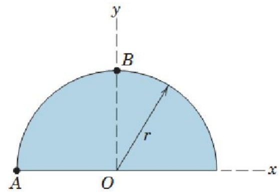
$$I_p - I_{\bar{p}} = (I_0 + A * 75^2) - (I_0 + A * 50^2) = 75^2 A - 50^2 A$$

$$75^2 A - 50^2 A = 15 * 10^6 \Rightarrow A = 4800\ mm^2$$

Prob. A/7

Determine the polar moments of inertia of the semi-circular area about points A and B.

Solution:



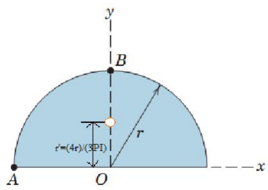
$$I_x = I_y = \frac{1}{2}\left(\frac{\pi r^4}{4}\right) = \frac{\pi r^4}{8}$$

$$I_x = I_{x_0} + Ad^2 \Rightarrow \frac{\pi r^4}{8} = I_{x_0} + \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2 \Rightarrow I_{x_0} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

Point B

$$I_{\bar{x}} = I_{x_0} + Ad^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 + \frac{\pi r^2}{2}\left(r - \frac{4r}{3\pi}\right)^2 = \left(\frac{5}{8}\pi - \frac{4}{3}\right) r^4$$

$$I_z = I_{\bar{x}} + I_y = \left(\frac{5}{8}\pi - \frac{4}{3}\right) r^4 + \frac{\pi r^4}{8} = \left(\frac{3}{4}\pi - \frac{4}{3}\right) r^4$$



Point A

$$I_{\bar{y}} = I_{y_0} + Ad^2 = \frac{\pi r^4}{8} + \frac{\pi r^2}{2} r^2 = \frac{5}{8}\pi r^4$$

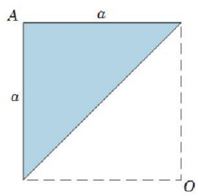
$$I_z = I_x + I_{\bar{y}} = \frac{\pi r^4}{8} + \frac{5}{8}\pi r^4 = \frac{3}{4}\pi r^4$$

Prob. A/9

Determine the polar radii of gyration of the triangular area about points O and A.

Solution:

Point A

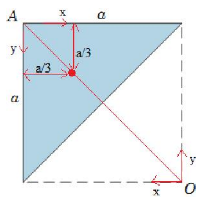


$$I_x = I_y = \frac{aa^3}{12}$$

$$I_z = I_x + I_y = \frac{a^4}{12} + \frac{a^4}{12} = \frac{a^4}{6}$$

$$A = \frac{a}{2} * a = \frac{a^2}{2}$$

$$k_A = \sqrt{\frac{I}{A}} = \sqrt{\frac{a^4/6}{a^2/2}} = \frac{a}{\sqrt{3}}$$



Point O

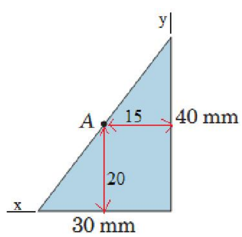
$$I_x = I_y = \frac{aa^3}{3} - \frac{aa^3}{12} = \frac{a^4}{4}$$

$$I_z = I_x + I_y = \frac{a^4}{4} + \frac{a^4}{4} = \frac{a^4}{2}$$

$$k_O = \sqrt{\frac{I}{A}} = \sqrt{\frac{a^4/2}{a^2/2}} = a$$

Prob. A/13

Determine the radius of gyration about a polar axis through the midpoint A of the hypotenuse of the right-triangular area. (Hint: Simplify your calculation by observing the results for a 30*40-mm rectangular area.)



Solution:

$$I_{x0} = \frac{30*40^3}{36} = 53333\ mm^4, I_{y0} = \frac{40*30^3}{36} = 30000\ mm^4,\ A = \frac{40*30}{2} = 600mm^2$$

Poi nt A

$$I_x = I_{x_0} + Ad^2 = 53333 + 600 * (20 - \frac{40}{3})^2 = 80000mm^4$$

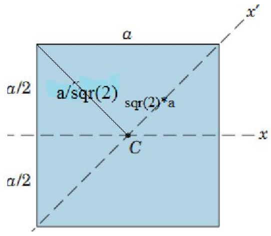
$$I_y = I_{y_0} + Ad^2 = 30000 + 600 * (15 - \frac{30}{3})^2 = 45000mm^4$$

$$I_z = I_x + I_y = 80000 + 45000 = 125000mm^4$$

$$k_A = \sqrt{\frac{I}{A}} = \sqrt{\frac{125000}{600}} = 14.43mm$$

Prob. A/17

In two different ways show that the moments of inertia of the square area about the x- and x'-axes are the same.



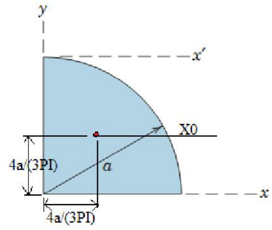
Solution:

$$I_x = \frac{aa^3}{12} = \frac{a^4}{12}$$

$$I_{\bar{x}} = \left[\frac{\sqrt{2}a(\frac{a}{\sqrt{2}})^3}{12}\right] = \frac{a^4}{12}$$

Prob. A/23

Determine the moment of inertia of the quarter-circular area about the tangent x' -axis.



Solution:

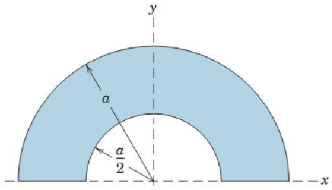
$$I_x = \frac{\pi r^4}{16} = \frac{\pi a^4}{16}$$

$$I_x = I_{x_0} + Ad^2 \Rightarrow \frac{\pi a^4}{16} = I_{x_0} + \frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right)^2 \Rightarrow I_{x_0} = a^4(\frac{\pi}{16} - \frac{4}{9\pi})$$

$$I_{\bar{x}} = I_{x_0} + Ad^2 = a^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right) + \frac{\pi a^2}{4}\left(a - \frac{4a}{3\pi}\right)^2 = 0.315a^4$$

Prob. A/33

By the methods of this article, determine the rectangular and polar radii of gyration of the shaded area about the axes shown.



Solution:

For circular area, $I_x = I_y = \frac{\pi r^4}{4}$

For half-circular area, $I_x = I_y = \frac{\pi r^4}{8} = \frac{\pi[a^4 - (\frac{a}{2})^4]}{8} = \frac{15}{128}\pi a^4$

$$I_z = I_x + I_y = \frac{15}{128}\pi a^4 * 2 = \frac{15}{64}\pi a^4$$

$$k_z = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{15}{64}\pi a^4}{\frac{\pi[a^2 - (\frac{a}{2})^2]}{2}}} = 0.79a$$

TABLE D/3 PROPERTIES OF PLANE FIGURES

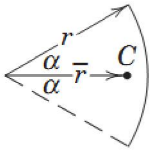
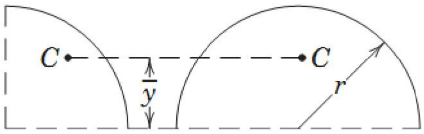
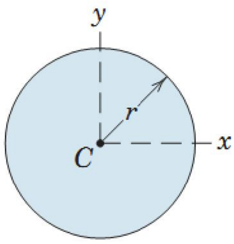
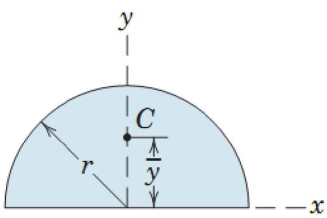
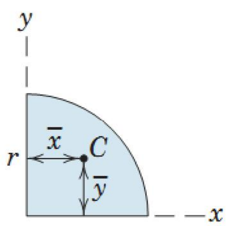
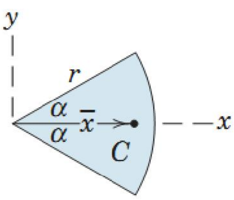
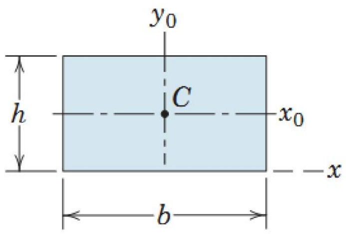
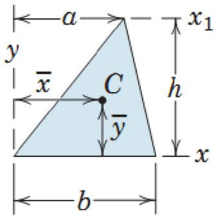
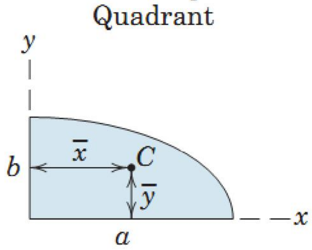
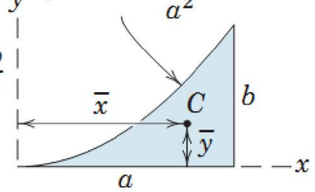
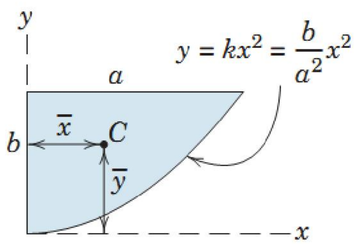
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Arc Segment 	$\bar{r} = \dfrac{r\sin\alpha}{\alpha}$	—
Quarter and Semicircular Arcs 	$\bar{y} = \dfrac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \dfrac{\pi r^4}{4}$ $I_z = \dfrac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{8}$ $\bar{I}_x = \left(\dfrac{\pi}{8} - \dfrac{8}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \dfrac{2}{3}\,\dfrac{r\sin\alpha}{\alpha}$	$I_x = \dfrac{r^4}{4}\left(\alpha - \dfrac{1}{2}\sin 2\alpha\right)$ $I_y = \dfrac{r^4}{4}\left(\alpha + \dfrac{1}{2}\sin 2\alpha\right)$ $I_z = \dfrac{1}{2} r^4 \alpha$

TABLE D/3 PROPERTIES OF PLANE FIGURES *Continued*

FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Rectangular Area 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
Area of Elliptical Quadrant 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}, \quad \bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ $I_y = \frac{\pi a^3 b}{16}, \quad \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3 b$ $I_z = \frac{\pi ab}{16}(a^2 + b^2)$
Subparabolic Area $y = kx^2 = \frac{b}{a^2}x^2$ Area $A = \frac{ab}{3}$ 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$	$I_x = \frac{ab^3}{21}$ $I_y = \frac{a^3 b}{5}$ $I_z = ab\left(\frac{a^3}{5} + \frac{b^2}{21}\right)$
Parabolic Area $y = kx^2 = \frac{b}{a^2}x^2$ Area $A = \frac{2ab}{3}$ 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$	$I_x = \frac{2ab^3}{7}$ $I_y = \frac{2a^3 b}{15}$ $I_z = 2ab\left(\frac{a^2}{15} + \frac{b^2}{7}\right)$

Moment of Inertia of Composite Areas

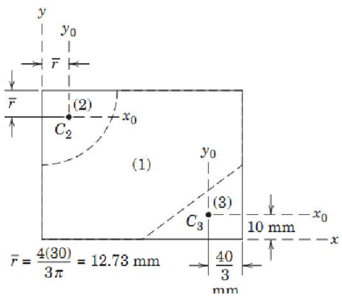
Part	Area, A	d_x	d_y	$Ad_x{}^2$	$Ad_y{}^2$	$\bar{I}_x or\ I_{x0}$	$\bar{I}_y or\ I_{y0}$
1							
2							
3							
Sums	$\sum A$			$\sum Ad_x{}^2$	$\sum Ad_y{}^2$	$\sum \bar{I}_x$	$\sum \bar{I}_y$

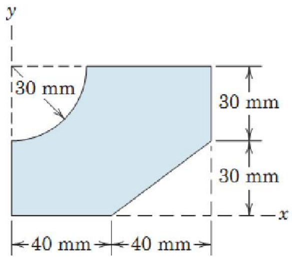
$$I_x = \sum \bar{I}_x + \sum Ad_x{}^2$$
$$I_y = \sum \bar{I}_y + \sum Ad_y{}^2$$
$$k_x = \sqrt{\frac{I_x}{A}},\quad k_y = \sqrt{\frac{I_y}{A}},\quad k_z = \sqrt{\frac{I_z}{A}}$$

SAMPLE PROBLEM A/7

Determine the moments of inertia about the x- and y-axes for the shaded area. Make direct use of the expressions given in Table D/3 for the centroidal moments of inertia of the constituent parts.

Solution:



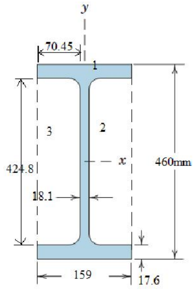


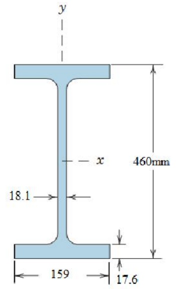
Part	A, mm^2	d_x, mm	d_y, mm	$Ad_x{}^2, mm^4$	$Ad_y{}^2, mm^4$	$\bar{I}_x or\ I_{x0}, mm^4$	$\bar{I}_y or\ I_{y0}, mm^4$
1.Rct.	80*60=4800	$\frac{60}{2} = 30$	$\frac{80}{2} = 40$	$4800 * 30^2 = 4.32 * 10^6$	$7.68 * 10^6$	$\frac{80*60^3}{12} = 1.44 * 10^6$	$\frac{60*80^3}{12} = 2.56 * 10^6$
2. quarter-circular	$-\frac{\pi 30^2}{4} = -707$	$60 - \frac{4*30}{3\pi} = 47.27$	$\frac{4*30}{3\pi} = 12.73$	$-1.58 * 10^6$	$-0.1146 * 10^6$	$-\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)30^4 = -0.044 * 10^6$	$-\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)30^4 = -0.044 * 10^6$
3. triangular	$-\frac{40*30}{2} = -600$	$\frac{30}{3} = 10$	$80 - \frac{40}{3} = 66.67$	$-0.06 * 10^6$	$-2.67 * 10^6$	$\frac{40 * 30^3}{36} = -0.03 * 10^6$	$\frac{30*40^3}{36} = -0.0533 * 10^6$
Sums	3493			$2.68 * 10^6$	$4.9 * 10^6$	$1.366 * 10^6$	$2.462 * 10^6$

$$I_x = \sum I_{x0} + \sum Ad_x{}^2 = 1.366 * 10^6 + 2.68 * 10^6 = 4.046 * 10^6 mm^4$$
$$I_y = \sum I_{y0} + \sum Ad_y{}^2 = 2.462 * 10^6 + 4.9 * 10^6 = 7.36 * 10^6 mm^4$$
$$I_z = I_x + I_y = 4.046 * 10^6 + 7.36 * 10^6 = 11.406 * 10^6 mm^4$$
$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.046*10^6}{3493}} = 34mm$$
$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{7.36*10^6}{3493}} = 46mm$$
$$k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{11.406*10^6}{3493}} = 57mm\ OR\ k_z{}^2 = k_x{}^2 + k_y{}^2 = 34^2 + 46^2 \Rightarrow k_z = 57mm$$

Prob. A/40

The cross-sectional area of a wide-flange I-beam has the dimensions shown. Obtain a close approximation to the handbook value of by treating the section as being composed of three rectangles.



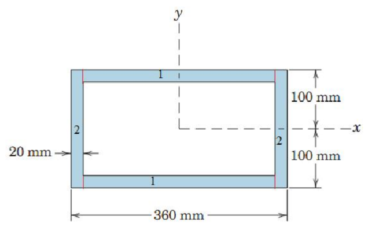


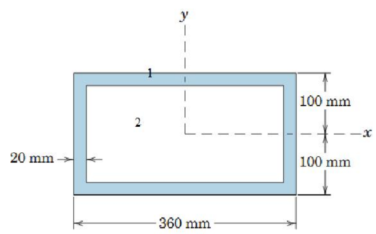
Part	A, mm^2	d_x, mm	d_y, mm	$Ad_x{}^2, m$	$Ad_y{}^2, mm^4$	$\bar{I}_x or\ I_{x0}, mm^4$	$\bar{I}_y or\ I_{y0}, mm^4$
1.Rct.	159*460 =73140	0	0	0	0	$\frac{159*460^3}{12} = 1290*10^6$	$\frac{460*159^3}{12} = 154.1*10^6$
2. Rct.	70.45*424.8 = -29927	0	$\frac{70.45}{2}+\frac{18.1}{2} = 35.22$	0	$-37.12*10^6$	$\frac{70.45*424.8^3}{12} = -450*10^6$	$\frac{424.8*70.45^3}{12} = -12.38*10^6$
3. Rct.	70.45*424.8 = -29927	0	$\frac{70.45}{2}+\frac{18.1}{2} = 35.22$	0	$-37.12*10^6$	$\frac{70.45*424.8^3}{12} = -450*10^6$	$\frac{424.8*70.45^3}{12} = -12.38*10^6$
Sums	13286			0	$-74.24*10^6$	$390*10^6$	$129.34*10^6$

$$I_x = \sum I_{x0} + \sum Ad_x{}^2 = 390*10^6 + 0 = 390*10^6 mm^4$$
$$I_y = \sum I_{y0} + \sum Ad_y{}^2 = 129.34*10^6 - 74.24*10^6 = 55.1*10^6 mm^4$$
$$I_z = I_x + I_y = 390*10^6 + 55.1*10^6 = 445.1*10^6 mm^4$$
$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{390*10^6}{13286}} = 171mm$$
$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{55.1*10^6}{13286}} = 64mm$$
$$k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{445.1*10^6}{13286}} = 183mm\ OR\ k_z{}^2 = k_x{}^2 + k_y{}^2 = 171^2 + 64^2 \Rightarrow k_z = 183mm$$

Prob. A/41

Determine the moment of inertia of the shaded area about the x-axis in two ways. The wall thickness is 20 mm on all four sides of the rectangle.





Solution: way1

Part	A, mm^2	d_x, mm	d_y, mm	$Ad_x{}^2, mm^4$	$Ad_y{}^2, mm^4$	$\bar{I}_x or\ I_{x0}, mm^4$	$\bar{I}_y or\ I_{y0}, mm^4$
1.Rct.	200*360 =72000	0	0	0	0	$\frac{360*200^3}{12} = 240*10^6$	$\frac{200*360^3}{12} = 777*10^6$
2. Rct.	160*320 = -51200	0	0	0	0	$\frac{320*160^3}{12} = -109*10^6$	$\frac{160*320^3}{12} = -437*10^6$
Sums	20800			0	0	$131*10^6$	$340*10^6$

$$I_x = \sum I_{x0} + \sum Ad_x{}^2 = 131*10^6 + 0 = 131*10^6 mm^4$$
$$I_y = \sum I_{y0} + \sum Ad_y{}^2 = 340*10^6 - 0 = 340*10^6 mm^4$$

way2

Part	**A**, mm^2	d_x, mm	d_y, mm	$Ad_x{}^2, mm^4$	$Ad_y{}^2, mm^4$	$\bar{I}_x or\ I_{x0}, mm^4$	$\bar{I}_y or\ I_{y0}, mm^4$
1.Rct.	(320*20)*2 =12800	90	0	$103.7 * 10^6$	0	$\left(\frac{320*20^3}{12}\right)*2 = 0.427*10^6$	$\left(\frac{20*320^3}{12}\right)*2 = 109.2*10^6$
2. Rct.	(200*20)*2 =8000	0	170	0	$231.2*10^6$	$\left(\frac{20*200^3}{12}\right)*2 = 26.67*10^6$	$\left(\frac{200*20^3}{12}\right)*2 = 0.267*10^6$
Sums	20800			$103.7 * 10^6$	$231.2*10^6$	$27.1*10^6$	$109.47*10^6$

$$I_x = \sum I_{x0} + \sum Ad_x{}^2 = 27.1*10^6 + 103.7*10^6 = 131*10^6 mm^4$$
$$I_y = \sum I_{y0} + \sum Ad_y{}^2 = 109.47*10^6 - 231.2*10^6 = 340*10^6 mm^4$$