

Example 3.1

A gas initially at 1 MPa, 500°C is contained in a piston-cylinder arrangement of initial volume of 0.1 m³. The gas expanded isothermally to a final pressure of 100 kPa. Determine the work.

$$PV = \text{Constant}; V_2 = \frac{P_1 V_1}{P_2}$$

$$W = \int P dV = \int \frac{C}{V} dV$$

$$W = P_1 V_1 \ln \frac{V_2}{V_1}$$

Putting all relevant data, $W = 230.3 \text{ KJ}$

Example 3.2

Helium gas expands from 125 kPa, 350 K and 0.25 m³ to 100 kPa in a polytropic process with $\delta = 1.667$. How much work does it give out?

$$PV^\delta = \text{Constant}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{1/\delta} = 0.2852 \text{ m}^3$$

$$W_2 = \frac{P_2 V_2 - P_1 V_1}{1 - n} = 4.09 \text{ KJ}$$

Example 3.3

A chiller cools liquid water (Sp. Ht = 4.2 J/gmK) for air-conditioning purposes. Assume 2.5 kg/s water at 20°C and 100 kPa is cooled to 5°C in a chiller. How much heat transfer (kW) is needed?

Heat transferred needed = Q_{out}

$$\dot{Q}_{\text{out}} = \dot{m} \dot{q}_{\text{out}}$$

$$Q_{\text{out}} = \dot{m} C_p \Delta T = \dot{m} (H_i - H_e) = 156.75 \text{ KW}$$

Example 3.4

A piston-cylinder assembly contains 0.1 kg wet steam of quality 0.75 at 100 kPa. If 150 kJ energy is added as heat while the pressure of the steam is held constant determine the final state of steam.

From saturated steam tables one has the following data at 100kPa.

$$V^V = 1.694 \text{ m}^3 / \text{kg}; V^L = 10^{-3} \text{ m}^3 / \text{kg}; H^V = 2675 \text{ kJ} / \text{kg}; H^L = 417.5 \text{ kJ} / \text{kg}$$

$$\text{System molar volume: } V = V^L + x^V (V^V - V^L) = 1.27 \text{ m}^3 / \text{kg}$$

$$\text{System molar internal energy: } H = H^L + x^V (H^V - H^L) = 2110.0 \text{ kJ} / \text{kg}$$

$$\text{Total enthalpy at state 1} = H_1^t = mH_1$$

$$\text{By first law: } Q^t = H_2^t - H_1^t = 150 \text{ kJ}$$

$$\text{Thus } H_2 = (Q^t + H_1^t) / m = 3610.9 \text{ kJ} / \text{kg}$$

It may be seen that at 100kPa, the saturated steam enthalpy $H_2 > H^V \Rightarrow$ hence the state 2 is *superheated steam* at 100 kPa & $H = 3610.9 \text{ kJ/kg}$.

Example 3.5

An adiabatic compressor operating under steady-state conditions receives air (ideal gas) at 0.1 MPa and 300 K and discharges at 1 MPa. If the flow rate of air through the compressor is 2 mol/s, determine the power consumption of the compressor. Constant pressure specific heat for air = 1kJ/kg.

$$\text{For compressor } \dot{Q} = 0$$

Let 'i' and 'e' denote the inlet and exit streams respectively.

We assume that: $u_i = u_e = 0$ and that $z_i - z_e = 0$.

Thus, the 1st law becomes,

$$\dot{m}(H_e - H_i) = -\dot{W}_s \quad \dots(1)$$

$$\text{But, } H_e - H_i = C_p(T_e - T_i) \quad \dots(2)$$

$$C_p - C_v = R$$

$$C_p / C_v = \gamma$$

$$\text{Thus, } C_p = \frac{\gamma R}{\gamma - 1}$$

It follows that from the given data $\gamma = 1.4$

$$H_e - H_i = \frac{\gamma R}{\gamma - 1}(T_e - T_i) \quad \dots(3)$$

Also for adiabatic operation of the compressor:

$$T_e = T_i \left[\frac{P_e}{P_i} \right]^{\frac{\gamma-1}{\gamma}} \cong 580^0 \text{ K}$$

$$\dot{m} = 2 \text{ mol} / \text{s} = 0.058 \text{ kg} / \text{s}$$

$$\text{Substituting the relevant data: } -\dot{W}_S = \dot{m} \left[\frac{\gamma R}{\gamma - 1} \right] (T_e - T_i) = 16.25 \text{ kW} = 16.25$$

Example 3.6

An insulated piston-cylinder system has air at 400kPa & 600K. Through an inlet pipe to the cylinder air at certain temperature T(K) and pressure P (kPa) is supplied reversibly into the cylinder till the volume of the air in the cylinder is 4 times the initial volume. The expansion occurs isobarically at 400kPa. At the end of the process the air temperature inside the cylinder is 450K. Assume ideal gas behaviour compute the temperature of the air supplied through the inlet pipe.

Applying the first law ($\Delta PE = \Delta KE = 0$)

$$\frac{d(mU_{cv})}{dt} + \Delta(H \dot{m}) = \dot{Q} + \dot{W}$$

$$\frac{d(mU_{cv})}{dt} + \Delta \left(H \frac{dm}{dt} \right) = \frac{\delta Q}{dt} + \frac{\delta W}{dt}$$

$$\text{Or } d(mU_{cv}) + \Delta(Hdm) = \delta Q + \delta W \quad \dots(1)$$

For change of state from 1 – 2

$$\int_1^2 (mU_{cv}) + \Delta \int_1^2 Hdm = \int_1^2 \delta Q + \int_1^2 \delta W$$

$$\text{Or } \Delta U_1^{t2} + \int_1^2 H_e dm_e - \int_1^2 H_i dm_i = Q_{12} + W_{12}$$

$$\text{Here } Q_{12} = 0, \int_1^2 H_e dm_e = 0 \text{ as there is no exit}$$

Equation (1) may be expanded into:

$$(m_2 U_2 - m_1 U_1)_{cv} - H_i (m_2 - m_1) = -P (V_2^t - V_1^t) \dots\dots\dots(2)$$

$$\text{For ideal gas } m = PV^t / R_m T \dots\dots\dots (3);$$

$$R_m = R/\text{Mol. Wt.}$$

$$\text{Since incoming fluid and system fluid are same we can write (by ideal gas assumption): } H = C_p T; U = C_v T \text{ (ideal gas assumption) } \dots\dots\dots(4)$$

Putting relations (3) and (4) in (2) and simplifying we finally obtain

$$T_i = \frac{V_2^t}{\left\{ \frac{V_1^t}{T_1} + \frac{V_2^t - V_1^t}{T_2} \right\}}$$

Now $V_2^t = 4V_1^t$, $T_1 = 600$ K, $T_2 = 450$ K, to calculate T_i

Thus $T_i = 480$ K.