5.2.4 *Terminal falling velocities*

In many process design calculations, it is necessary to know the terminal velocity of a sphere settling in a fluid under the influence of the gravitational field. When a spherical particle at rest is introduced into a liquid, it acceler-ates until the buoyant weight is exactly balanced by the fluid dynamic drag. Although the so-called terminal velocity is approached asymptotically, the effective transition period is generally of short duration for Newtonian and power-law fluids [Chhabra *et al*., 1998]. For instance, in the creeping flow regime, the terminal velocity is attained after the particle has traversed a path .of length equal to only a few diameters

For gravity settling of a sphere at its terminal velocity the drag force on it, F_D, is equal to the buoyant weight, i.e.

$$F_D = \frac{\pi d^3}{6}(\rho_s - \rho)g \tag{5.10}$$

Combining equations (5.4) and (5.10), the terminal velocity of a sphere in a power-law fluid (Re $<$ 1):

$$V = {\frac{gd^{n+1}(\rho_s - \rho)}{18mX}}^{(1/n)} \tag{5.11}$$

In shear-thinning power-law fluids, therefore, the terminal falling velocity shows a stronger dependence on sphere diameter and density difference than in a Newtonian fluid.

This method of calculation is satisfactory provided it is known a priori that the Reynolds number is small ($<$1). As the unknown velocity appears in both the Reynolds number and the drag coefficient, it is more satisfactory to work in terms of a new dimensionless group, Ar, the so-called Archimedes number defined by:

$$\mathrm{Ar} = C_D \mathrm{Re}^{2/(2-n)} = \frac{4}{3} g d^{(2+n)/(2-n)}(\rho_s - \rho)\rho^{n/(2-n)} m^{2/(n-2)} \tag{5.12}$$

For any given sphere and power-law liquid combination, the value of the Archimedes number can be evaluated using equation (5.12). The sphere Reynolds number can then be expressed in terms of Ar and n as follows:

$$\mathrm{Re} = a\mathrm{Ar}^b \tag{5.13}$$

$$a = 0.1 \quad \exp\left(\frac{0.51}{n}\right. \quad - 0.73n \tag{5.14}$$

$$b = \frac{0.954}{n} - 0.16 \tag{5.15}$$

The values calculated from equations (5.13) to (5.15) represent about 400 data points in visco-inelastic fluids ($0.4 \le n < 1$; $1 \le \mathrm{Re} \le 1000$; $10 \le \mathrm{Ar} \le 10^6$) with an average error of 14% and a maximum error of 21%. Finally, in view of the fact that non-Newtonian characteristics exert little influence on the drag, the use of predictive correlations for terminal falling velocities in Newtonian media yields only marginally larger errors for power-law fluids. Finally, attention is drawn to the fact that the estimation of terminal velocity in viscoplastic liquids requires an iterative solution, as illustrated in example 5.4.

Example 5.3

For spheres of equal terminal falling velocities, obtain the relationship between diameter and density difference between particle and fluid for creeping flow in power law fluids.

Solution

From equation (5.11), the terminal settling velocity of a sphere increases with both its density and size. For two spheres of different diameters d_A, d_B and densities, ρ_{SA} and ρ_{SB}, settling in the same fluid, the factor X is a function of n only (see Table 5.1) and

$$\frac{V_A}{V_B} = \left(\frac{d_A^{n+1}(\rho_{SA} - \rho)}{d_B^{n+1}(\rho_{SB} - \rho)}\right)^{(1/n)}$$

Thus for pseudoplastic fluids ($n < 1$), the terminal velocity is more sensitive to both sphere diameter and density difference than in a Newtonian fluid and it should, in principle, be easier to separate closely sized particles. For equal settling velocities,

$$\frac{d_B}{d_A} = \left(\frac{\rho_{SA} - \rho}{\rho_{SB} - \rho}\right)^{1/(n+1)}$$

For $n = 1$, this expression reduces to its Newtonian counterpart.

Example 5.4

Estimate the terminal settling velocity of a 3.18 mm steel sphere (density = 7780 kg/m^3) is a viscoplastic polymer solution of density 1000 kg/m^3. The flow curve for the polymer solution is approximated by the three parameter Herschel–Bulkley model as:

$$\tau = 3.3 + 3.69(\dot{\gamma})^{0.53}$$

The settling may be assumed to occur in creeping flow region.

Solution

In the creeping flow region, the drag coefficient is given by equation (5.9), i.e.

$$C_D = \frac{24}{\text{Re}}(1 + \text{Bi}^*) \tag{5.9}$$

The other dimensionless groups are:

$$C_D = \frac{4}{3}\frac{gd}{V^2}\left(\frac{\rho_s - \rho}{\rho}\right)$$

$$\text{Re} = \frac{\rho V^{2-n} d^n}{m}$$

$$\text{Bi}^* = \frac{\tau_0}{m(V/d)^n}$$

Trial and error solution is needed as the unknown velocity appears in all of these groups. The other values (in S.I. units) are:

$$\tau_0^H = 3.3\,\text{Pa}; \quad m = 3.69\,\text{Pa·s}^{0.53}; \quad n = 0.53; \quad d = 3.18 \times 10^{-3}\,\text{m}$$

$$\rho_s = 7780\,\text{kg/m}^3; \quad \rho = 1000\,\text{kg/m}^3; \quad g = 9.81\,\text{m/s}^2$$

Substituting these values:

$$C_D = \frac{0.2813}{V^2} \quad \text{or} \quad V = \sqrt{\frac{0.2813}{C_D}}$$

$$\text{Re} = \frac{(1000)(3.18 \times 10^{-3})^{0.53} V^{2-0.53}}{3.69} = 12.86 V^{1.47}$$

$$\text{Bi}^* = \frac{\tau_0^H}{m(V/d)^n} = \frac{3.3 \times (3.18 \times 10^{-3})^{0.53}}{3.69 \times V^{0.53}} = 0.0424 V^{-0.53}$$

Assume a value of $V = 15\,\text{mm/s} = 15 \times 10^{-3}\,\text{m/s}$.

$$\therefore\ \text{Bi}^* = 0.393; \quad \text{Re} = 12.86 \times (15 \times 10^{-3})^{1.47} = 0.0268$$

Now from equation (5.9), the value of C_D:

$$C_D = \frac{24}{\text{Re}}(1 + \text{Bi}^*) = \frac{24}{0.0268}(1 + 0.393)$$
$$= 1248$$

$$\therefore \text{ the velocity, } V = \sqrt{\frac{0.2813}{C_D}} = \sqrt{\frac{0.2813}{1248}} = 0.015\,\text{m/s} = 15\,\text{mm/s}$$

which matches with the assumed value. Also, in view of the small value of the Reynolds number ($\text{Re}_{\text{max}} \sim 70$, from equation 5.8), the assumption of the creeping flow is justified.