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Lecture 3

Contents

1. Types of damping

a) Viscus damping

b) Coulomb (dry friction) damping

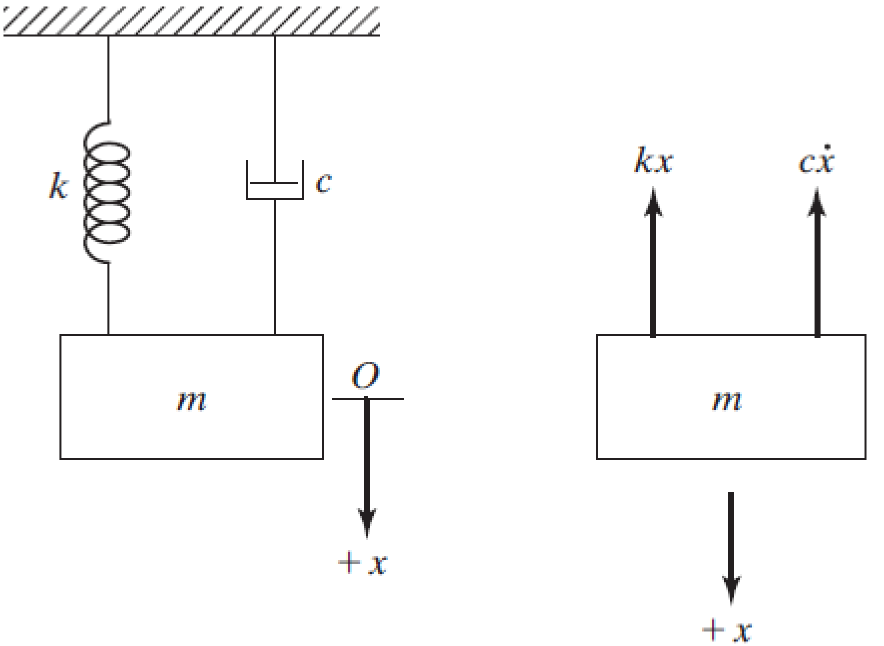
c) Structural damping

2. Free Vibration with Viscous Damping

The viscous damping force *F* is proportional to the velocity $\dot{x}$ or *v* and can be expressed as

$$F = -c\dot{x}$$

where c is the damping coefficient and $\dot{x}$ is the velocity and F is the damping force.



System
(a)

Free-body diagram
(b)

For a single-degree-of-freedom system with a viscous damper the equation of motion according to Newton's second law is as follow;

$$\sum F = m\ddot{x}$$

$$-kx - c\dot{x} = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad \text{[Equation of Motion]}$$

The solution for this equation is as follows;

$$x(t) = Ce^{st}$$

where *C* and *s* are undetermined constants,

$$ms^2 + cs + k = 0 \qquad \text{[Characteristic Equation]}$$

$$s_{1,2} = -\frac{c \mp \sqrt{c^2 - 4mk}}{2m}$$

Or $$s_{1,2} = \frac{-c}{2m} \mp \sqrt{\frac{c^2}{4m^2} - \frac{k}{m}}$$

$$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

$$x(t) = C_1 e^{\left[-\frac{c+\sqrt{c^2-4mk}}{2m}\right]t} + C_2 e^{\left[-\frac{c-\sqrt{c^2-4mk}}{2m}\right]t}$$

where C_1 and C_2 are arbitrary constants can be determined from the initial conditions of the system.

3. Critical Damping Constant and the Damping Ratio

The critical damping constant c_c can be calculated as follow;

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

$$\xi = \frac{c}{c_c}$$

$$\frac{c}{2m} = \xi\omega_n$$

$$c = 2\,m\,\xi\omega_n$$

The solution to the equation of motion as a function of ξ is as follows;

$$s_{1,2} = (-\xi \pm \sqrt{\xi^2 - 1})\,\omega_n$$

$$x(t) = C_1 e^{\left(-\xi + \sqrt{\xi^2-1}\right)\omega_n t} + C_2 e^{\left(-\xi - \sqrt{\xi^2-1}\right)\omega_n t}$$

The nature of the roots s_1 and s_2 and the behaviour of the solution for the equation of motion for damped free vibration, depends upon the magnitude of damping. It can be seen that the case $\xi = 0$ leads to the undamped vibrations discussed in previous lecture. Hence we assume that $\xi \neq 0$ and consider the following three cases.

Case 1) Underdamped vibration

when $\xi < 1$

$\frac{c^2}{4m^2} < \frac{k}{m}$ or c < c_c

c = 2 ξ mω_n

$$\omega_n = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad {\omega_n}^2 = \frac{k}{m}$$

$$\xi = \frac{c}{c_c}$$

$s_{1,2}$ =- $\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$; $j = \sqrt{-1}$

The solution for this equation of motion in this case has different forms and one of these forms is as follow;

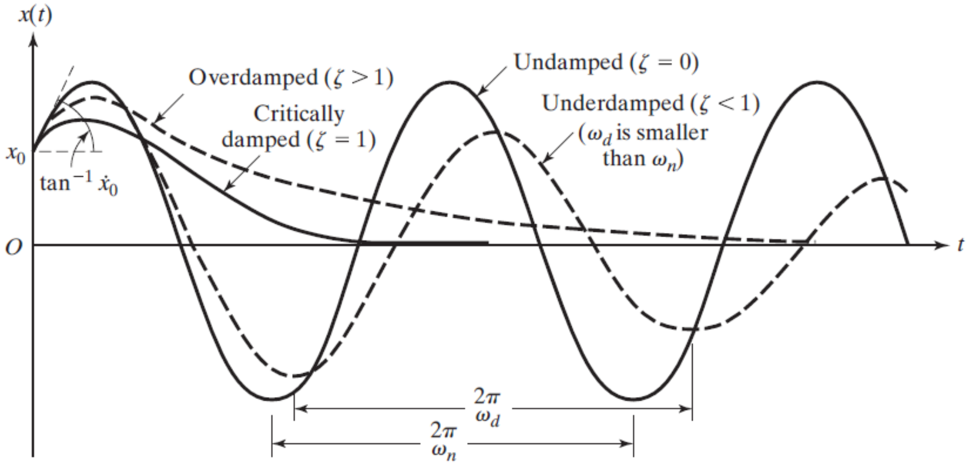
$$x(t) = C_1 e^{(-\xi + j\sqrt{1-\xi^2})\omega_n t} + C_2 e^{(-\xi - j\sqrt{1-\xi^2})\omega_n t}$$

As the amplitude of damped harmonic motion decreased exponentially with time, the quantity ω_d is called the frequency of damped vibration and can be calculated as follow;

$$\omega_d = \omega_n\sqrt{1 - \xi^2}$$

Also damped period τ_d for harmonic motion can be calculated based on damped frequency f_d or damped circular frequency of vibration ω_d as follow;

$$\tau_d = \frac{1}{f_d} = \frac{2\pi}{\omega_d}$$



Case 2) Critically damped vibration

when ξ = 1

$$\frac{c^2}{4m^2} = \frac{k}{m} = {\omega_n}^2$$

$$c = c_c = 2m\omega_n = 2\sqrt{mk}$$

$$s_{1,2} = -\ \xi\omega_n = -\ \omega_n$$

The solution for the equation of motion in this case is as follow;

$$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

$$x(t) = C_1 e^{-\omega_n t} + C_2 e^{-\omega_n t}$$

Case 3) Overdamped vibration

when $\xi > 1$

$\frac{c^2}{4m^2} > \frac{k}{m}$ or c > c_c

$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\ \xi^2\ - 1}$

The solution for the equation of motion in this case is as follow;

$$x(t) = C_1 e^{\left(-\,\xi + \sqrt{\xi^2\, - 1}\right)\omega_n t} + C_2 e^{\left(-\xi - \sqrt{\xi^2\, - 1}\right)\omega_n t}$$

For all these case we can apply the initial conditions in order to obtain the constants $C_1\ and\ C_2$

For example at time (t) =0

$x(t) = x_0$

$\dot{x}(t) = \dot{x}_0$