CHAPTER FIVE

CAPACITORS

5.1 INTRODUCTION

So far we have limited our study to resistive circuits. In this chapter, we shall introduce two new and important passive linear circuit elements: the capacitor and the inductor (the inductor is discussed in detail in **Chapter 7**). Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called storage elements. We begin by introducing capacitors and describing how to combine them in series or in parallel. Later, we do the same for inductors.

5.2 CAPACITORS

A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.

A capacitor consists of two conducting plates separated by an insulator (or dielectric).

In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.

The amount of charge stored, represented by q, is directly proportional to the applied voltage **v** so that

$$q = Cv \tag{5.1}$$

where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (**F**), in honor of the English physicist Michael Faraday (1791–1867). From **Eq. (5.1)**, we may derive the following definition.

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

Note from **Eq. (5.1)** that 1 farad = 1 coulomb/volt.

Although the capacitance **C** of a capacitor is the ratio of the charge **q** per plate to the applied voltage **v**, it does not depend on **q** or **v**. It depends on the physical dimensions of the capacitor. The capacitance is given by

$$C = \frac{\epsilon A}{d} \tag{5.2}$$

where **A** is the surface area of each plate, **d** is the distance between the plates, and **ε** is the permittivity of the dielectric material between the plates. Typically, capacitors have values in the **picofarad** (**pF**) to **microfarad** (**μF**) range. **Figure 5.1** shows the circuit symbols for fixed and variable capacitors.

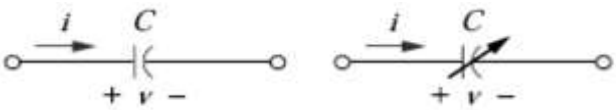

Figure 5.1 Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of **Eq. (5.1)**. Since

$$i = dq/dt \tag{5.3}$$

differentiating both sides of **Eq. (5.1)** gives

$$i = C\, dv/dt \tag{5.4}$$

The voltage-current relation of the capacitor can be obtained by integrating both sides of **Eq. (5.4)**. We get

$$v = \frac{1}{C} \int_{-\infty}^{t} i\, dt \tag{5.5}$$

or

$$v = \frac{1}{C} \int_{t_0}^{t} i\ dt + v(t_0) \tag{5.6}$$

where $v(t_0) = q(t_0)/C$ is the voltage across the capacitor at time t_0.

Eq. (5.6) shows that capacitor voltage depends on the past history of the capacitor current. Hence, the capacitor has memory—a property that is often exploited.

The instantaneous power delivered to the capacitor is

$$p = vi = Cv \frac{dv}{dt} \tag{5.7}$$

The energy stored in the capacitor is therefore

$$w = \frac{1}{2} C v^2 \quad \text{or} \quad w = \frac{q^2}{2C} \tag{5.8}$$

Eq. (5.8) represents the energy stored in the electric field that exists between the plates of the capacitor. This energy can be retrieved, since an ideal capacitor cannot dissipate energy

We should note the following important properties of a capacitor:

1. Note from **Eq. (5.4)** that when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus,

A capacitor is an open circuit to dc.

2. The voltage on the capacitor must be continuous.

The voltage on a capacitor cannot change abruptly.

The capacitor resists an abrupt change in the voltage across it.

3. The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

4. A real, nonideal capacitor has a parallel-model leakage resistance. The leakage resistance may be as high as 100 MΩ and can be neglected for most practical applications.

Example 5.1:

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.

(b) Find the energy stored in the capacitor.

Solution:

(a) Since **q = Cv**, $\mathbf{q = 3 \times 10^{-12} \times 20 = 60\ pC}$

(b) The energy stored is $w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600\ \text{pJ}$

Example 5.2:

The voltage across a 5-μF capacitor is **v(t) = 10 cos 6000t V** Calculate the current through it.

Solution:

By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10\cos 6000t)$$

$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t\ \text{A}$$

Practice problems:

1-What is the voltage across a 3-μF capacitor if the charge on one plate is **0.12 mC**? How much energy is stored?

Answer: 40 V, 2.4 mJ.

2-If a 10-µF capacitor is connected to a voltage source with **v(t) = 50 sin 2000t V** determine the current through the capacitor.

Answer: cos 2000t A.

5.3 SERIES AND PARALLEL CAPACITORS

We know from resistive circuits that series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered. We desire to replace these capacitors by a single equivalent capacitor **Ceq**.

First we obtain the equivalent capacitor $\mathbf{C_{eq}}$ of **N** capacitors in parallel,

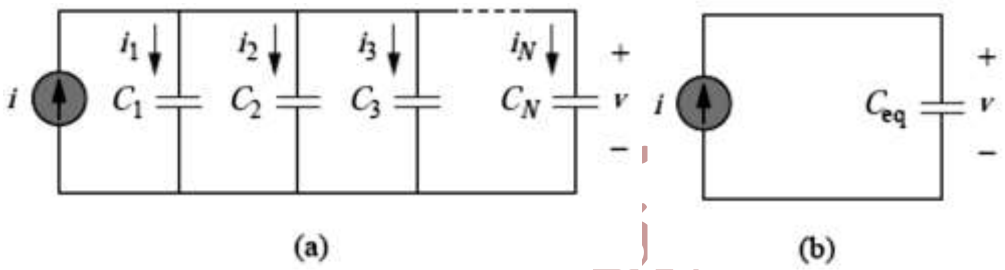

Figure 5.2 (a) Parallel-connected N capacitors, (b) equivalent circuit for the parallel capacitors.

$$\mathbf{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N} \qquad (5.9)$$

The equivalent capacitance of *N* parallel-connected capacitors is the sum of the individual capacitances.

We observe that capacitors in parallel combine in the same manner as resistors in series.

Now we will obtain $\mathbf{C_{eq}}$ of **N** capacitors connected in series

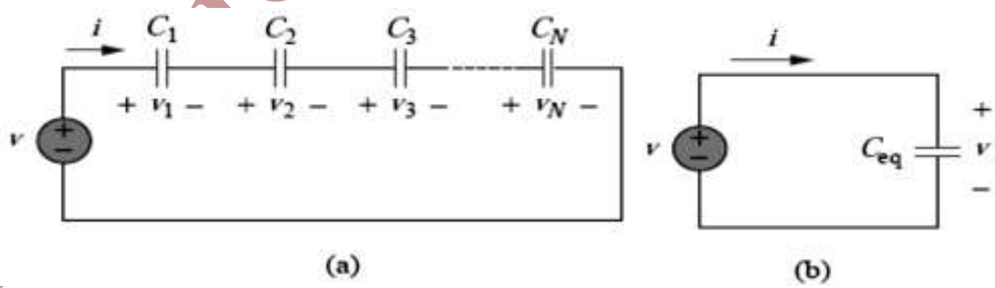

Figure 5.3 (a) Series-connected N capacitors, (b) equivalent circuit for the series capacitor.

Where
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N} \qquad (5.10)$$

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

Note that capacitors in series combine in the same manner as resistors in parallel. For **N = 2** (i.e., two capacitors in series), **Eq. (5.10)** becomes

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Or $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad (5.11)$$

Example 5.6:

Find the equivalent capacitance seen between terminals **a** and **b** of the circuit in **Fig. 5.4**.

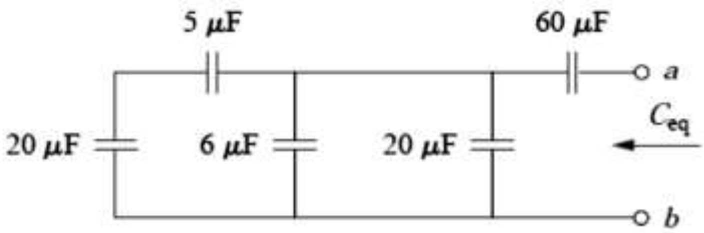

Figure 5.4 For Example 6.6.

Solution:

The 20-μF and 5-μF capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4\ \mu F$$

This 4-μF capacitor is in parallel with the 6-μF and 20-μF capacitors; their combined capacitance is

$$4 + 6 + 20 = 30\ \mu F$$

This 30-μF capacitor is in series with the 60-μF capacitor. Hence, the equivalent capacitance for the entire circuit is

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20\ \mu F$$

Practice problems:

1- Find the equivalent capacitance seen at the terminals of the circuit in Figure below.

Answer: 40 μF.