Lecture #4

- **Mohr's Circle in 3-D**
- **Octahedral planes and stresses**

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References:

1- Hosford W.F, (2005), Mechanical Behavior of Materials, Cambridge

2- Meyers M.A. and Chawla K.K., (2009). Mechanical Behavior of Materials, Prentice- Hall.

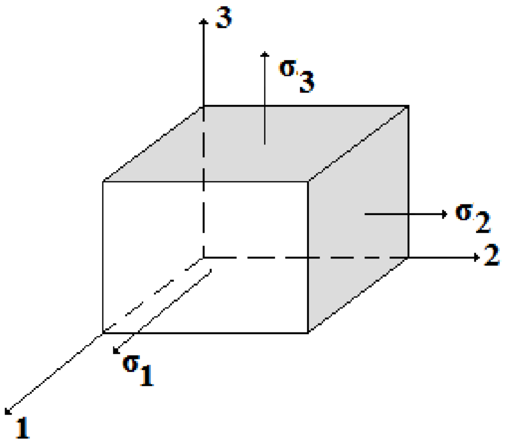
3- Dieter G.E., (1986), Mechanical Metallurgy, McGraw-Hill Cambridge university press.

4- Hearn E.j., 1977, Mechanics of Materials, Vol.1&2., Pergamon Press, London.

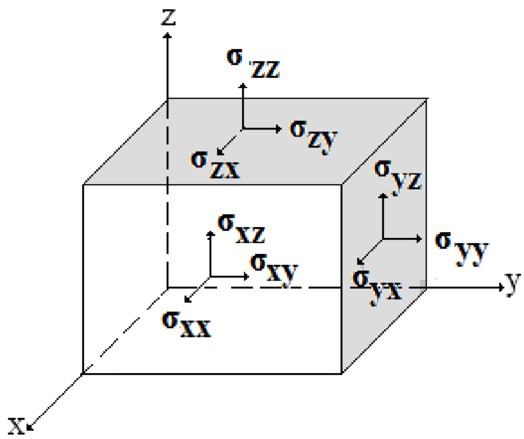
Lecture# 4:

Mohr’s Circle in 3-D

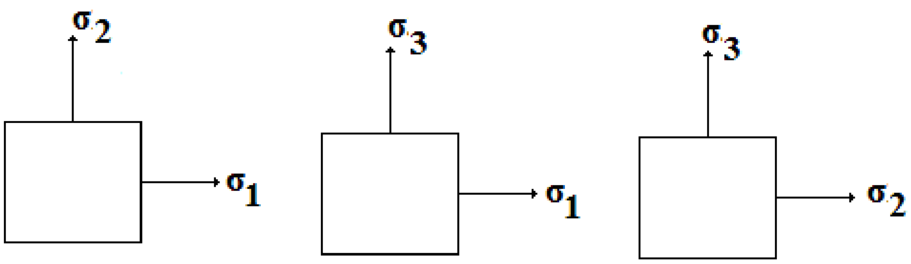
We can use Mohr’s circle to visualize the 3-dimensional state of stress and to ascertain principal stresses. You can construct the 3-D Mohr’s circle by considering the combination of stresses that act paired faces of our elemental cubic object (e.g., *x-y* faces, *x-z* faces, *y-z* faces). This yields 3 intersecting Mohr’s circles.



Principal element

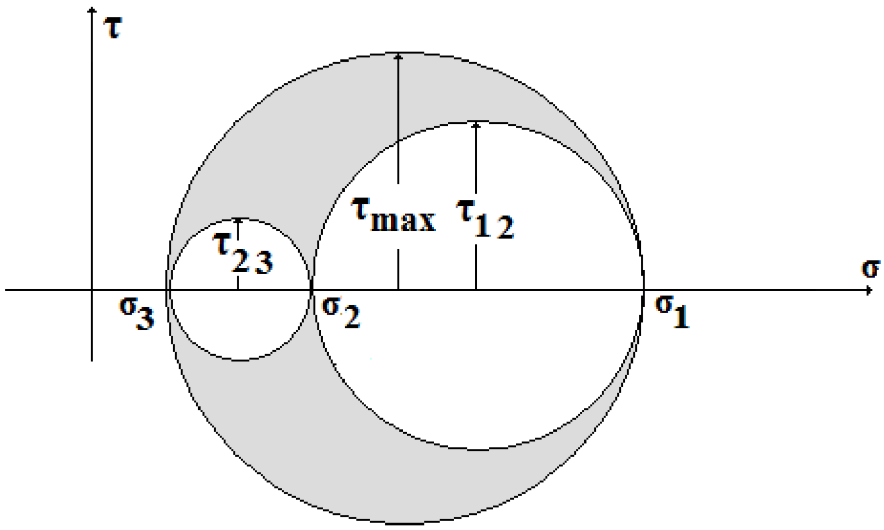


General ***three-dimensional*** state of stress at any point in a body



Views on the various faces of the principal element

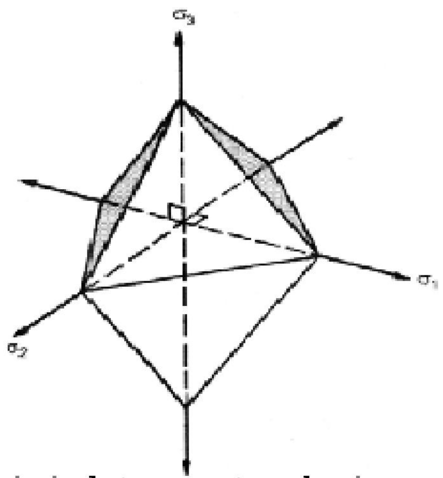
For each two-dimensional stress condition a Mohr’s circle may be drawn. These can then be combined to produce the complete three-dimensional Mohr’s circle representation.



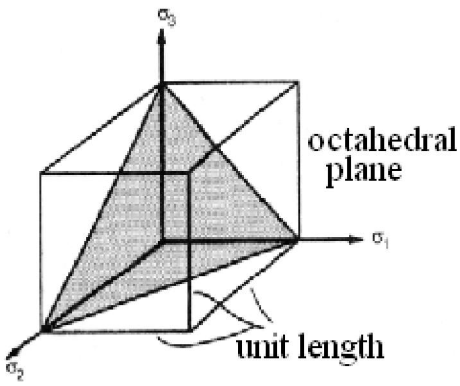
The **maximum shear stress** occurs when: $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$, where $\sigma_1 > \sigma_2 > \sigma_3$

Octahedral planes and stresses:

Any complex three-dimensional stress system produces three mutually perpendicular principal stresses**.** Associated with this stress state are so-called **octahedral planes** each of which cuts across the corners of a principal element such as that shown in the fig. to produce the octahedron (8-sided figure) shown.



principal stress system showing the 8-octahedral planes forming an octahedron



The stresses acting on the octahedral planes have particular significance. The normal stresses acting on each of the octahedral planes are equal in value and tend to compress or enlarge the octahedron without distorting its shape. They are thus said to be **hydrostatic** stresses and have values given by:

$$\sigma_{normal} = \sigma_{xx} l^2 + \sigma_{yy} m^2 + \sigma_{zz} n^2 \quad \text{(page 6 Lec.3 put shear=0)}$$

$$\sigma_{normal} = \sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

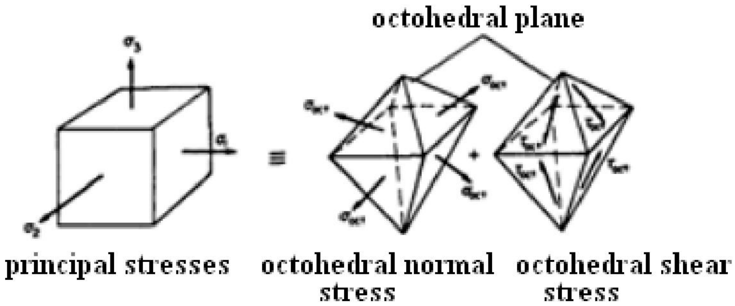
where $l = m = n = \frac{1}{\sqrt{3}}$, $\theta = 54^0 44'$ equivalent to (111) plane in FCC.

Similarly, the shear stresses acting on each of the octahedral planes are also identical and tend to distort the octahedron without changing its volume.

(put shear=0 in the equations of S, S_x, S_y, S_z – Lect.#3)

$$\tau^2 = S^2 - \sigma^2_{normal} = \sigma^2_{xx} l^2 + \sigma^2_{yy} m^2 + \sigma^2_{zz} n^2 - \left(\sigma_{xx} l^2 + \sigma_{yy} m^2 + \sigma_{zz} n^2\right)^2$$

$$\Rightarrow \tau_{oct} = \frac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$



Since octahedral stress is hydrostatic stress, it can not produce yielding in solid materials, so octahedral shear stress is the component of stress responsible for plastic deformation.

For failure $$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_{yieldpoint}$$