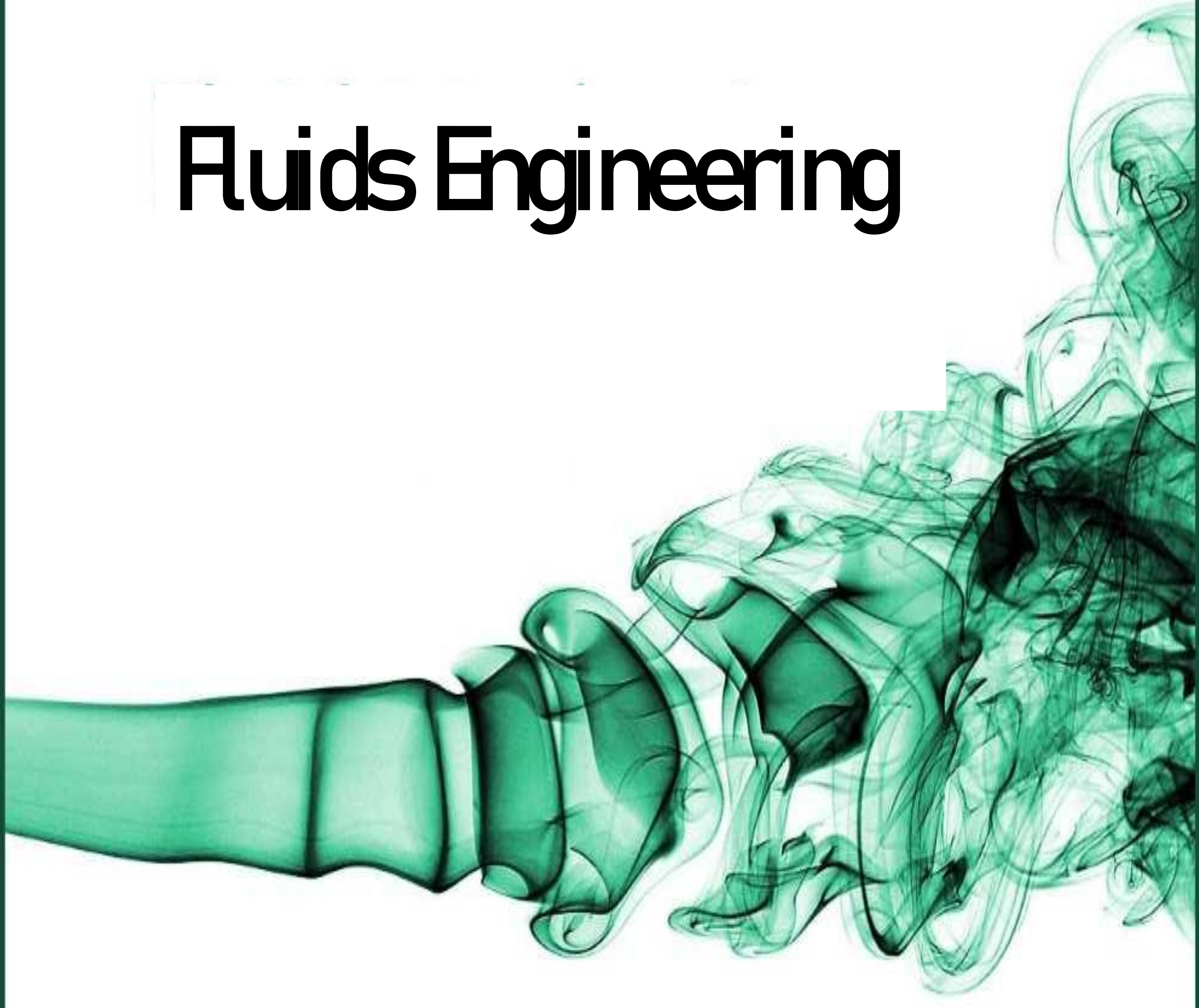
Fluids Engineering



Scope of Fluid Mechanics

As the name implies, fluid mechanics is the study of fluids at rest or in motion. It has traditionally been applied in such areas as the design of canal, levee, and dam systems; the design of pumps, compressors, and piping and ducting used in the water and air conditioning systems of homes and businesses, as well as the piping systems needed in chemical plants; the aerodynamics of automobiles and sub- and supersonic airplanes; and the development of many different flow measurement devices such as gas pump meters.

Definition of a Fluid

We already have a common-sense idea of when we are working with a fluid, as opposed to a solid: Fluids tend to flow when we interact with them (e.g., when you stir your morning coffee); solids tend to deform or bend (e.g., when you type on a keyboard, the springs under the keys compress). Engineers need a more formal and precise definition of a fluid: A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be. Because the fluid motion continues under the application of a shear stress, we can also define a fluid as any substance that cannot sustain a shear stress when at rest. Hence liquids and gases (or vapors) are the forms, or phases, that fluids can take. We wish to distinguish these phases from the solid phase of matter. We can see the difference between solid and fluid behavior in Fig. 1.1. If we place a specimen of either substance between two plates (Fig. 1.1a) and then apply a shearing force F, each will initially deform (Fig. 1.1b); however, whereas a solid will then be at rest (assuming the force is not large enough to go beyond its elastic limit), a fluid will continue to deform (Fig. 1.1c, Fig. 1.1d, etc) as long as the force is applied. Note that a fluid in contact with a solid surface does not slip—it has the same velocity as that surface because of the no-slip condition, an experimental fact.

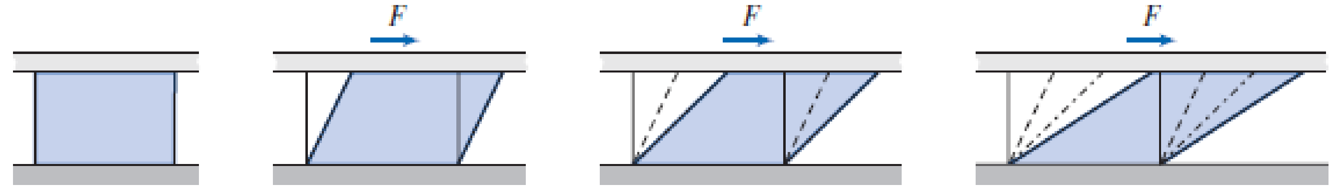


Fig. 1.1 Difference in behavior of a solid and a fluid due to a shear force.

The amount of deformation of the solid depends on the solid's modulus of rigidity G; in Chapter 2 we will learn that the rate of deformation of the fluid depends on the fluid's viscosity μ. We refer to solids as being elastic and fluids as being viscous. More informally, we say that solids exhibit "springiness." For example, when you drive over a pothole, the car bounces up and down due to the car suspension's metal coil springs compressing and expanding. On the other hand, fluids exhibit friction effects so that the suspension's shock absorbers (containing a fluid that is forced through a small opening as the car bounces) dissipate energy due to the fluid friction, which stops the bouncing after a few oscillations. If your shocks are "shot," the fluid they contained has leaked out so that there is almost no friction as the car bounces, and it bounces several times rather than quickly coming to rest. The idea that substances can be categorized as being either a solid or a liquid holds for most substances, but a number of substances exhibit both springiness and friction; they are viscoelastic. Many biological tissues are viscoelastic. For example, the synovial fluid in human knee joints lubricates those joints but also absorbs some of the shock occurring during walking or running. Note that the system of springs and shock absorbers comprising the car suspension is also viscoelastic, although the individual components are not

Dimensions and Units

Since in our study of fluid mechanics we will be dealing with a variety of fluid characteristics, it is necessary to develop a system for describing these characteristics both qualitatively and quantitatively. The qualitative aspect serves to identify the nature, or type, of the characteristics 1such as length, time, stress, and velocity2, whereas the quantitative aspect provides a numerical measure of the characteristics. The quantitative description requires both a number and a standard by which various quantities can be compared. A standard for length might be a meter or foot, for time an hour or second, and for mass a slug or kilogram. Such standards are called units, and several systems of

units are in common use as described in the following section. The qualitative description is conveniently given in terms of certain primary quantities, such as length, L, time, T, mass, M, and temperature, These primary quantities can then be used to provide a qualitative description of any other secondary quantity: for example, area = L^2, velocity = LT^{-1}, density = ML^{-3}, and so on, where the symbol is used to indicate the dimensions of the secondary quantity in terms of the primary quantities. Thus, to describe qualitatively a velocity, V, we would write

$$V = LT^{-1}$$

and say that "the dimensions of a velocity equal length divided by time." The primary quantities are also referred to as basic dimensions. For a wide variety of problems involving fluid mechanics, only the three basic dimensions, L, T, and M are required. Alternatively, L, T, and F could be used, where F is the basic dimensions of

force. Since Newton's law states that force is equal to mass times acceleration, it follows $F = MLT^{-2}$ or $M = FL^{-1}\ T^{2.}$ Thus, secondary quantities expressed in terms of M can be expressed $\sigma = FL^{-2}$, in terms of F through the relationship above. For example, stress, is a force per unit area, so that but an equivalent dimensional equation is Table 1.1 provides a list of dimensions

for a number of common physical quantities. All theoretically derived equations are dimensionally homogeneous—that is, the dimensions of the left side of the equation must be the same as those on the right side, and all additive separate terms must have the same dimensions. We accept as a fundamental premise that all equations describing physical phenomena must be dimensionally homogeneous. If this were not true, we would be attempting to equate or add unlike physical quantities, which would not make sense. For example, the equation for the velocity, V, of a uniformly accelerated body is

$$V = V_0 + at \qquad (1.1)$$

TABLE 1.1
Dimensions Associated with Common Physical Quantities

	FLT System	MLT System
Acceleration	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of a force	FL	ML^2T^{-2}
Moment of inertia (area)	L^4	L^4
Moment of inertia (mass)	FLT^2	ML^2
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	Θ	Θ
Time	T	T
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Volume	L^3	L^3
Work	FL	ML^2T^{-2}

where is V_O the initial velocity, *a* the acceleration, and *t* the time interval. In terms of dimensions

$$LT^{-1} \doteq LT^{-1} + LT^{-1} \qquad \textbf{(1.2)}$$

and thus Eq. 1.1 is dimensionally homogeneous.
Some equations that are known to be valid contain constants having dimensions. The equation for the distance, d, traveled by a freely falling body can be written as

$$d = \frac{gt^2}{2} \qquad \textbf{(1.3)}$$

in which g is the acceleration of gravity. Equation 1.3 is dimensionally homogeneous and valid in any system of units. For g = 32.2 ft/s^{2}the equation reduces to Eq. 1.2 and thus Eq. 1.2 is valid only for the system of units using feet and seconds. Equations that are restricted to a particular system of units can be denoted as *restricted homogeneous equations*, as opposed to equations valid in any system of units, which are *general homogeneous equations*. The preceding discussion indicates one rather elementary, but important, use of the concept of dimensions: the determination of one aspect of the generality of a given equation simply based on a consideration of the dimensions of the various terms in the equation. The concept of dimensions also forms the basis for the powerful tool of *dimensional analysis*,. ***Note to the users of this text.*** All of the examples in the text use a consistent problem-solving methodology which is similar to that in other engineering courses such as statics. Each example highlights the key elements of analysis: ***Given, Find, Solution,*** and ***Comment.*** The ***Given*** and ***Find*** are steps that ensure the user understands what is being asked in the problem and explicitly list the items provided to help solve the problem. The ***Solution*** step is where the equations needed to solve the problem are formulated and the problem is actually solved. In this step, there are typically several other tasks that help to setup the solution and are required to solve the problem. The first is a drawing of the problem; where appropriate, it is always helpful to draw a sketch of the problem. Here the relevant geometry and coordinate system to be used as well as features such as control volumes, forces and pressures, velocities, and mass flow rates are included. This helps in gaining a visual understanding of the
problem. Making appropriate assumptions to solve the problem is the second task. In a realistic engineering problem-solving environment, the necessary assumptions are developed as an integral part of the solution process. Assumptions can provide appropriate simplifications or offer useful constraints, both of which can help in solving the problem. Throughout the examples in this text, the necessary assumptions are embedded within the Solution step, as they are in solving a real world problem. This provides a realistic problem-solving experience. The final element in the methodology is the Comment. For the examples in the text, thissection is used to provide further insight into the problem or the solution. It can also be a point in the analysis at which certain questions are posed. For example: Is the answer reasonable, and does it make physical sense? Are the final units correct? If a certain parameter were changed, how would the answer change? Adopting the above type of methodology will aid in the development of problem-solving skills for fluid mechanics, as well as other engineering disciplines.

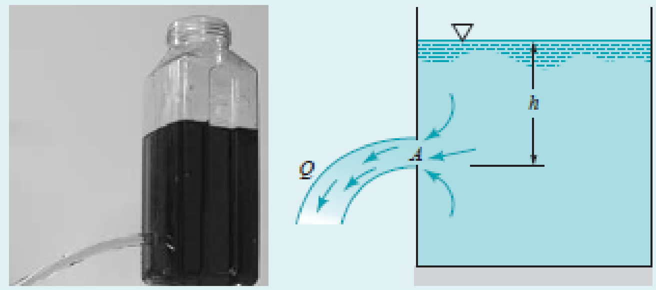
EXAMPLE 1.1 Restricted and General Homogeneous Equations

GIVEN A liquid flows through an orifice located in the side of a tank as shown in Fig. E1.1. A commonly used equation for determining the volume rate of flow, Q, through the orifice is

$$Q = 0.61\,A\sqrt{2gh}$$

where A is the area of the orifice, g is the acceleration of gravity, and h is the height of the liquid above the orifice.

FIND Investigate the dimensional homogeneity of this formula.



(a) (b)

■ FIGURE E1.1

SOLUTION

The dimensions of the various terms in the equation are Q = volume/time $\doteq L^3T^{-1}$, A = area $\doteq L^2$, g = acceleration of gravity $\doteq LT^{-2}$, and h = height $\doteq L$.

These terms, when substituted into the equation, yield the dimensional form:

$$(L^3T^{-1}) \doteq (0.61)(L^2)(\sqrt{2})(LT^{-2})^{1/2}(L)^{1/2}$$

or

$$(L^3T^{-1}) \doteq [(0.61)\sqrt{2}](L^3T^{-1})$$

It is clear from this result that the equation is dimensionally homogeneous (both sides of the formula have the same dimensions of L^3T^{-1}), and the numbers (0.61 and $\sqrt{2}$) are dimensionless.

If we were going to use this relationship repeatedly we might be tempted to simplify it by replacing g with its standard value of 32.2 ft/s^2 and rewriting the formula as

$$Q = 4.90\,A\sqrt{h} \qquad \textbf{(1)}$$

A quick check of the dimensions reveals that

$$L^3T^{-1} \doteq (4.90)(L^{5/2})$$

and, therefore, the equation expressed as Eq. 1 can only be dimensionally correct if the number 4.90 has the dimensions of $L^{1/2}T^{-1}$. Whenever a number appearing in an equation or formula has dimensions, it means that the specific value of the number will depend on the system of units used. Thus, for the case being considered with feet and seconds used as units, the number 4.90 has units of ft$^{1/2}$/s. Equation 1 will only give the correct value for Q (in ft^3/s) when A is expressed in square feet and h in feet. Thus, Eq. 1 is a *restricted* homogeneous equation, whereas the original equation is a *general* homogeneous equation that would be valid for any consistent system of units.

COMMENT A quick check of the dimensions of the various terms in an equation is a useful practice and will often be helpful in eliminating errors—that is, as noted previously, all physically meaningful equations must be dimensionally homogeneous. We have briefly alluded to units in this example, and this important topic will be considered in more detail in the next section.

Systems of Units

In addition to the qualitative description of the various quantities of interest, it is generally necessary to have a quantitative measure of any given quantity. For example, if we measure the width of this page in the book and say that it is 10 units wide, the statement has no meaning until the unit of length is defined. If we indicate that the unit of length is a meter, and define the meter as some standard length, a unit system for length has been established 1and a numerical value can be given to the page width2. In addition to length, a unit must be established for each of the remaining basic quantities (force, mass, time, and temperature). There are several systems of units in use and we shall consider three systems that are commonly used in engineering.

International System (SI). In 1960 the Eleventh General Conference on Weights and Measures, the international organization responsible for maintaining precise uniform standards of measurements, formally adopted the *International System of Units* as the international standard.

This system, commonly termed SI, has been widely adopted worldwide and is widely used (although certainly not exclusively) in the United States. It is expected that the long-term trend will be for all countries to accept SI as the accepted standard and it is imperative that engineering students become familiar with this system. In SI the unit of length is the meter (m), the time unit is the second (s), the mass unit is the kilogram (kg), and the temperature unit is the kelvin (K). Note that there is no degree symbol used when expressing a temperature in kelvin units. The kelvin temperature scale is an absolute scale and is related to the Celsius (centigrade) scale through the relationship

$$K = {}^{\circ}C + 273.15$$

Although the Celsius scale is not in itself part of SI, it is common practice to specify temperatures
in degrees Celsius when using SI units.

The force unit, called the newton (N), is defined from Newton's second law as

$$1\ N = (1\ kg) /(\ m/s^2)$$

Thus, a 1-N force acting on a 1-kg mass will give the mass an acceleration of 1 m/s^2 Standard gravity is 9.807 m/s^2 in SI is (commonly approximated as) so that a 1-kg mass weighs 9.81 N under

standard gravity. Note that weight and mass are different, both qualitatively and quantitatively! The unit of work in SI is the joule (J), which is the work done when the point of application of a 1-N force is displaced through a 1-m distance in the direction of a force. Thus,

$$1 \text{ J} = 1 \text{ N} . \text{m}$$

The unit of power is the watt 1W2 defined as a joule per second. Thus,

$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ N} . \text{m/s}$$

Prefixes for forming multiples and fractions of SI units are given in Table 1.2. For example, the notation kN would be read as "kilonewtons" and stands for 10^3 N Similarly, mm would be read as "millimeters" and stands for 10^{-3} m. The centimeter is not an accepted unit of length in the SI system, so for most problems in fluid mechanics in which SI units are used, lengths will be expressed in millimeters or meters.

TABLE 1.2
Prefixes for SI Units

Factor by Which Unit Is Multiplied	Prefix	Symbol	Factor by Which Unit Is Multiplied	Prefix	Symbol
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10	deka	da	10^{-18}	atto	a
10^{-1}	deci	d			

British Gravitational (BG) System. In the BG system the unit of length is the foot 1ft2, the time unit is the second 1s2, the force unit is the pound (lb), and the temperature unit is the degree Fahrenheit (°F)or the absolute temperature unit is the degree Rankine (°R) where

$$°R = °F + 459.67$$

The mass unit, called the *slug*, is defined from Newton's second law (force = mass × acceleration) as

$$1\ lb= (1\ slug)(1 ft/s^2)$$

This relationship indicates that a 1-lb force acting on a mass of 1 slug will give the mass an acceleration of 1 ft/s^2

The weight, °W (which is the force due to gravity, *g*) of a mass, *m*, is given by the equation

$$°W= mg$$

and in BG units

$$°W(lb)= m\,(slugs)\ g\,(ft/s^2)$$

Since the earth's standard gravity is taken as *g*= 32.174 ft/s^2(commonly approximated as 32.174 ft/s^2, it follows that a mass of 1 slug weighs 32.2 lb under standard gravity.

English Engineering (EE) System. In the EE system, units for force and mass are defined independently; thus special care must be exercised when using this system in conjunction with Newton's second law. The basic unit of mass is the pound mass (lbm), and the unit of force is the pound (lb).1 The unit of length is the foot (ft), the unit of time is the second (s), and the absolute temperature scale is the degree Rankine (°R). To make the equation expressing Newton's second law dimensionally homogeneous we write it as

$$F= ma / gc \qquad (1.4)$$

Where *gc* is a constant of proportionality which allows us to define units for both force and mass. For the BG system, only the force unit was prescribed and the mass unit defined in a consistent manner such that *gc* = 1 Similarly, for SI the mass unit was prescribed and the force unit defined in a consistent manner such that *gc* = 1 For the EE system, a 1-lb force is defined as that force which gives a 1 lbm a standard acceleration of gravity which is taken as 32.174 ft/ s^2 Thus, for Eq. 1.4 to be both numerically and dimensionally correct

$$1 \text{ lb} = \frac{(1 \text{ lbm})(32.174 \text{ ft/s}^2)}{g_c}$$

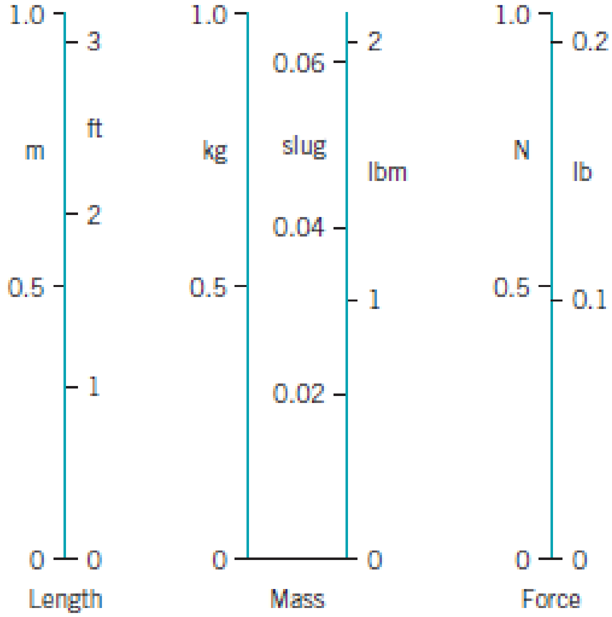


FIGURE 1.2 Comparison of SI, BG, and EE units.

so that

$$g_c = \frac{(1 \text{ lbm})(32.174 \text{ ft/s}^2)}{(1 \text{ lb})}$$

With the EE system, weight and mass are related through the equation

$$\mathcal{W} = \frac{mg}{g_c}$$

where g is the local acceleration of gravity. Under conditions of standard gravity the weight in pounds and the mass in pound mass are numerically equal. Also, since a 1-lb force gives a mass of 1 lbm an acceleration of and a mass of 1 slug an acceleration of it follows that

1 slug =32.174 lbm

In this text we will primarily use the BG system and SI for units. The EE system is used very

sparingly, and only in those instances where convention dictates its use. Approximately one-half the problems and examples are given in BG units and one-half in SI units. We cannot overemphasize the importance of paying close attention to units when solving problems. It is very easy to introduce huge errors into problem solutions through the use of incorrect units. Get in the habit of using a *consistent* system of units throughout a given solution. It really makes no difference which system you use as long as you are consistent; for example, don't mix slugs and newtons. If problem data are specified in SI units, then use SI units throughout the solution. If the data are specified in BG units, then use BG units throughout the solution. The relative sizes of the SI, BG, and EE units of length, mass, and force are shown in Fig. 1.2. Tables 1.3 and 1.4 provide conversion factors for some quantities that are commonly encountered in fluid mechanics. For convenient reference these tables are reproduced on the inside of the back cover. Note that in these tables (and others) the numbers are expressed by using computer exponential notation. For example, the number 5.154 E + 2 is equivalent to $5.154 \text{ E} \times 10^2$ in scientific notation, and the number 2.832 E - 2 is equivalent to 2.832×10^2.

EXAMPLE 1.2 BG and SI Units

GIVEN A tank of liquid having a total mass of 36 kg rests on a support in the equipment bay of the Space Shuttle.

FIND Determine the force (in newtons) that the tank exerts on the support shortly after lift off when the shuttle is accelerating upward as shown in Fig. E1.2*a* at 15 ft/s^2.

■ FIGURE E1.2*a* (Photograph courtesy of NASA.)

SOLUTION

A free-body diagram of the tank is shown in Fig. E1.2*b*, where $\mathcal{W}$ is the weight of the tank and liquid, and F_f is the reaction of the floor on the tank. Application of Newton's second law of motion to this body gives

$$\sum \mathbf{F} = m\,\mathbf{a}$$

or

$$F_f - \mathcal{W} = ma \qquad (1)$$

where we have taken upward as the positive direction. Since $\mathcal{W} = mg$, Eq. 1 can be written as

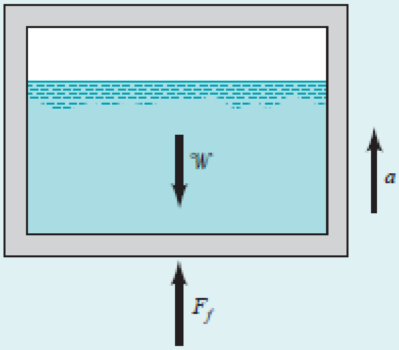
$$F_f = m(g + a) \qquad (2)$$

Before substituting any number into Eq. 2, we must decide on a system of units, and then be sure all of the data are expressed in these units. Since we want F_f in newtons, we will use SI units so that

$$F_f = 36\text{ kg}[9.81\text{ m/s}^2 + (15\text{ ft/s}^2)(0.3048\text{ m/ft})]$$
$$= 518\text{ kg}\cdot\text{m/s}^2$$

Since $1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$, it follows that

$$F_f = 518\text{ N} \qquad \text{(downward on floor)} \qquad \textbf{(Ans)}$$



■ FIGURE E1.2*b*

The direction is downward since the force shown on the free-body diagram is the force of the support *on the tank* so that the force the tank exerts *on the support* is equal in magnitude but opposite in direction.

COMMENT As you work through a large variety of problems in this text, you will find that units play an essential role in arriving at a numerical answer. Be careful! It is easy to mix units and cause large errors. If in the above example the acceleration had been left as 15 ft/s^2 with m and g expressed in SI units, we would have calculated the force as 893 N and the answer would have been 72% too large!

Analysis of Fluid Behavior

The study of fluid mechanics involves the same fundamental laws you have encountered in physics and other mechanics courses. These laws include Newton's laws of motion, conservation of mass, and the first and second laws of thermodynamics. Thus, there are strong similarities between the general approach to fluid mechanics and to rigid-body and deformable-body solid mechanics. This is indeed helpful since many of the concepts and techniques of analysis used in fluid mechanics will be ones you have encountered before in other courses. The broad subject of fluid mechanics can be generally subdivided into *fluid statics,* in which the fluid is at rest, and *fluid dynamics,* in which the fluid is moving. In the following chapters we will consider both of these areas in detail. Before we can proceed, however, it will be necessary to define and discuss certain fluid *properties* that are intimately related to fluid behavior. It is obvious that different fluids can have grossly different characteristics. For example, gases are light and compressible, whereas liquids are heavy (by comparison) and relatively incompressible. A syrup flows slowly from a container, but water flows rapidly when poured from the same container. To quantify these differences, certain fluid properties are used. In the following several sections the properties that play an important role in the analysis of fluid behavior are considered.

Measures of Fluid Mass and Weight

density

The ***density*** of a fluid, designated by the Greek symbol ρ(rho), is defined as its mass per unit volume. Density is typically used to characterize the mass of a fluid system. In the BG system, ρ has units of slugs / $ft^3$and in SI the units are kg /m^3.The value of density can vary widely between different fluids, but for liquids, variations in pressure and temperature generally have only a small effect on the value of ρ The small change in the density of water with large variations in temperature is illustrated in Fig. 1.3. Tables 1.5 and 1.6 list values of density for several common liquids. The density of water at 60 ^oF is 1.94 slugs / ft^3or 999 kg / m^3.The large difference between those two values illustrates the importance of paying attention to units! Unlike liquids, the density of a gas is strongly influenced $\rho$by both pressure and temperature, and this difference will be discussed in the next section.

The *specific volume,*υ , is the *volume* per unit mass and is therefore the reciprocal of the density—that is,

$$\upsilon = 1/\rho \qquad (1.5)$$

This property is not commonly used in fluid mechanics but is used in thermodynamics.

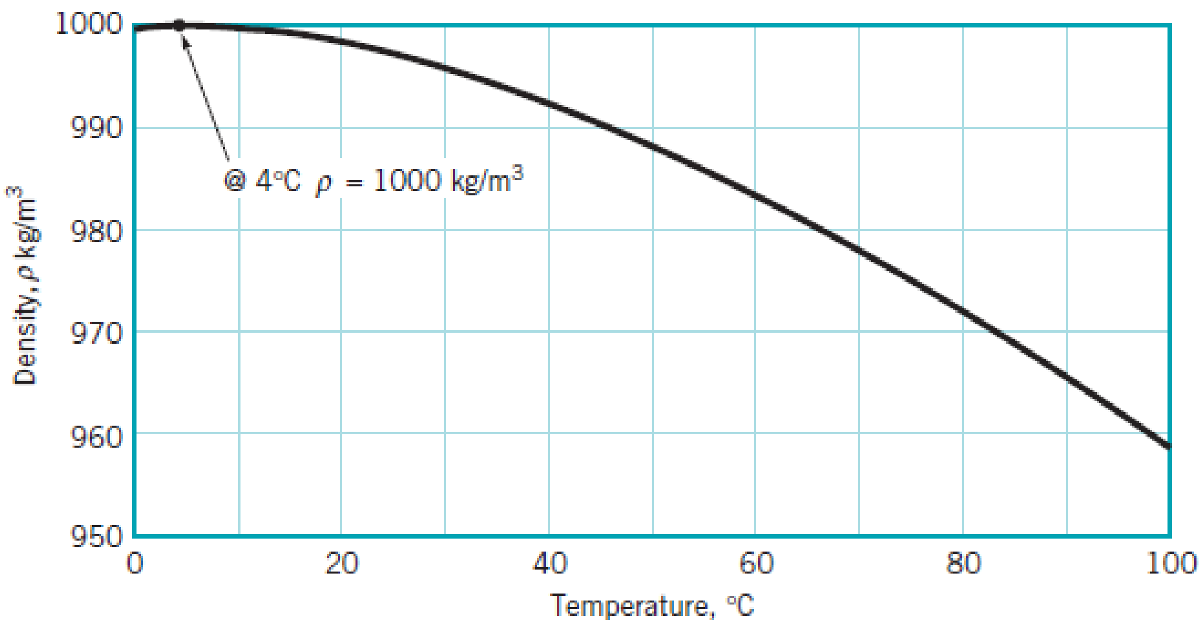


FIGURE 1.3 **Density of water as a function of temperature.**

Specific Weight

The ***specific weight*** of a fluid, designated by the Greek symbol γ(gamma), is defined as its *weight* per unit volume. Thus, specific weight is related to density through the equation

$$\gamma = \rho . g \qquad (1.6)$$

where g is the local acceleration of gravity. Just as density is used to characterize the mass of a fluid system, the specific weight is used to characterize the weight of the system. In the BG system, γ has units of lb/ft^3 and in SI the units are N/m^3Under conditions of standard gravity (g= 32.174 ft/s^2 = 9.807 m/s^2) water at 60 ^oF has a specific weight of 62.4

lb/ ft^3 and 9.80 KN/m^3Tables 1.5 and 1.6 list values of specific weight for several common liquids (based on standard gravity).

Specific Gravity

The ***specific gravity*** of a fluid, designated as *SG*, is defined as the ratio of the density of the fluid to the density of water at some specified temperature. Usually the specified temperature is taken as 4 ^oC (39.2 ^oF)and at this temperature the density of water is 1.94 slugs / ft^3 or 1000 kg/m^3.In equation form, specific gravity is expressed as

$$SG = \frac{\rho}{\rho_{H_2O@4\,^\circ C}} \qquad \textbf{(1.7)}$$

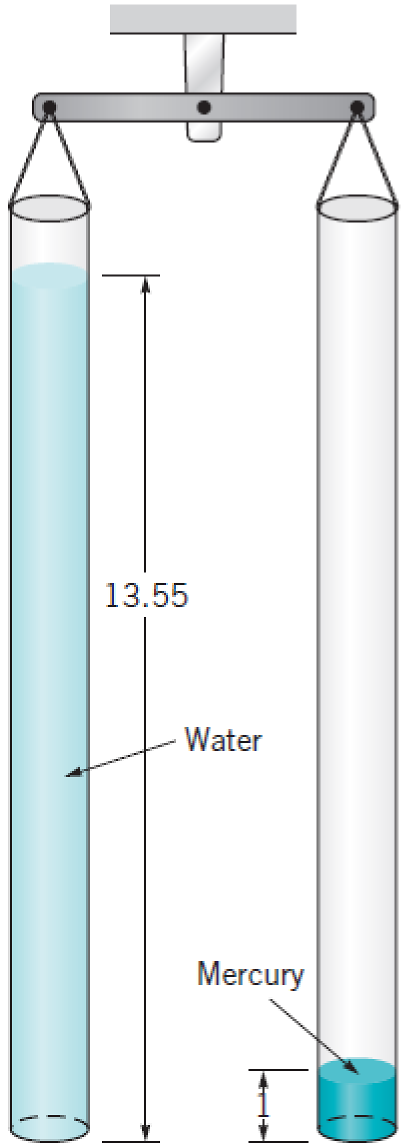
and since it is the *ratio* of densities, the value of *SG* does not depend on the system of units used. For example, the specific gravity of mercury at 20 ^oC is 13.55. This is illustrated by the figure in
the margin. Thus, the density of mercury can be readily calculated in either BG or SI units through
the use of Eq. 1.7 as

$$\rho_{Hg} = (13.55)(1.94\ \text{slugs/ft}^3) = 26.3\ \text{slugs/ft}^3$$

or

$$\rho_{Hg} = (13.55)(1000\ \text{kg/m}^3) = 13.6 \times 10^3\ \text{kg/m}^3$$

It is clear that density, specific weight, and specific gravity are all interrelated, and from a knowledge of any one of the three the others can be calculated.


13.55
Water
Mercury
1

Ideal Gas Law

Gases are highly compressible in comparison to liquids, with changes in gas density directly related to changes in pressure and temperature through the equation

$$\rho = p/ RT \qquad \textbf{(1.8)}$$

where p is the absolute pressure, ρ the density, T the absolute temperature,2 and R is a gas constant. Equation 1.8 is commonly termed the ***ideal*** or *perfect* ***gas law,*** or the *equation of state* for an ideal gas. It is known to closely approximate the behavior of real gases under normal conditions when the gases are not approaching liquefaction.

Pressure in a fluid at rest is defined as the normal force per unit area exerted on a plane surface (real or imaginary) immersed in a fluid and is created by the bombardment of the surface with the fluid molecules. From the definition, pressure has the dimension of $FL^{-2}$and in BG units is expressed as lb/ft^2 (psf) or lb/in^2 (psi) and in SI units as N/m^2 In SI, 1 N/m^2 defined as a pascal, abbreviated as Pa, and pressures are commonly specified in pascals. The pressure in the ideal gas law must be expressed as an absolute pressure, denoted (abs), which means that it is measured relative to absolute zero pressure (a pressure that would only occur in a perfect vacuum). Standard sea-level atmospheric pressure (by international agreement) is 14.696 psi 1abs2 or 101.33 kPa (abs). For most calculations these pressures can be rounded to 14.7 psi and 101 kPa, respectively. In engineering it is common practice to measure pressure relative to the local atmospheric pressure, and when measured in this fashion it is called gage pressure. Thus, the absolute pressure can be obtained from the gage pressure by adding the value of the atmospheric pressure. For example, as shown by the figure in the margin on the next page, a pressure of 30 psi (gage) in a tire is equal to 44.7 psi (abs) at standard atmospheric pressure.

EXAMPLE 1.3 Ideal Gas Law

GIVEN The compressed air tank shown in Fig. E1.3*a* has a volume of 0.84 ft^3. The temperature is 70 °F and the atmospheric pressure is 14.7 psi (abs).

FIND When the tank is filled with air at a gage pressure of 50 psi, determine the density of the air and the weight of air in the tank.

SOLUTION

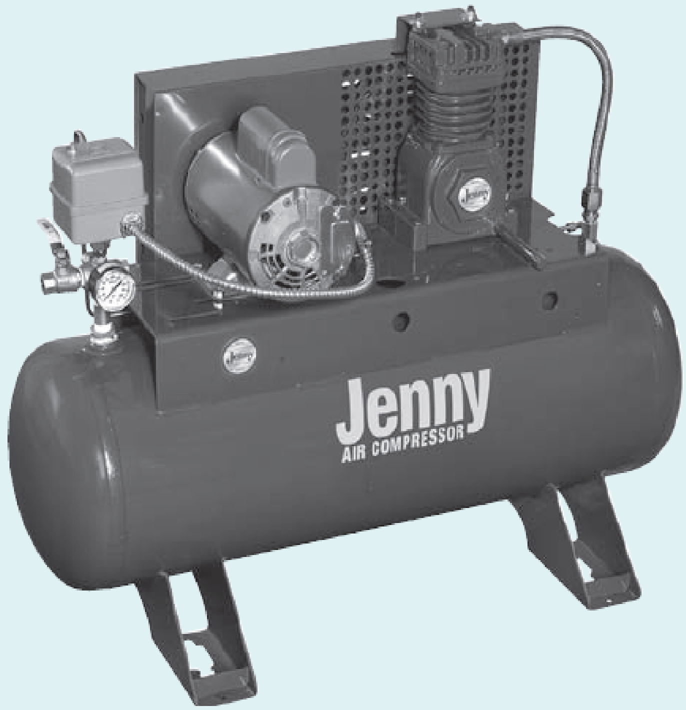
The air density can be obtained from the ideal gas law (Eq. 1.8)

$$\rho = \frac{p}{RT}$$

so that

$$\rho = \frac{(50 \text{ lb/in.}^2 + 14.7 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)}{(1716 \text{ ft} \cdot \text{lb/slug} \cdot {}^\circ\text{R})[(70 + 460)^\circ\text{R}]}$$
$$= 0.0102 \text{ slugs/ft}^3 \qquad \textbf{(Ans)}$$

Note that both the pressure and temperature were changed to absolute values.



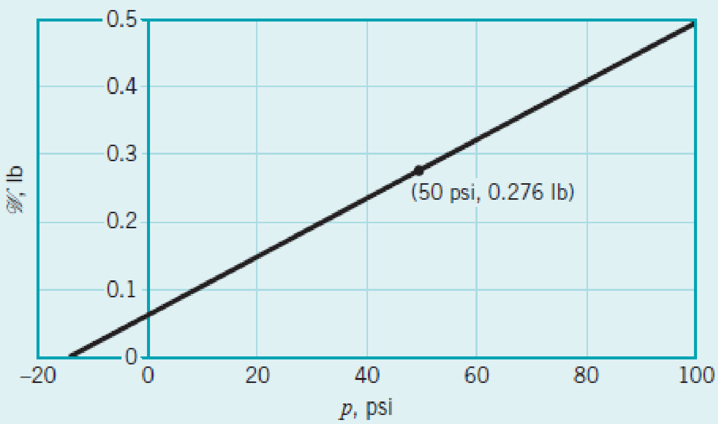
■ FIGURE E1.3*a* (Photograph courtesy of Jenny Products, Inc.)

The weight, $\mathcal{W}$, of the air is equal to

$$\mathcal{W} = \rho g \times (\text{volume})$$
$$= (0.0102 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(0.84 \text{ ft}^3)$$
$$= 0.276 \text{ slug} \cdot \text{ft/s}^2$$

so that since 1 lb = 1 slug · ft/s^2

$$\mathcal{W} = 0.276 \text{ lb} \qquad \textbf{(Ans)}$$



■ FIGURE E1.3*b*

COMMENT By repeating the calculations for various values of the pressure, *p*, the results shown in Fig. E1.3*b* are obtained. Note that doubling the gage pressure does not double the amount of air in the tank, but doubling the absolute pressure does. Thus, a scuba diving tank at a gage pressure of 100 psi does not contain twice the amount of air as when the gage reads 50 psi.