Solved Problems and Questions on fluid properties

1. The quantities viscosity μ, velocity *V*, and surface tension *Y* may be combined into a dimensionless group. Find the combination which is proportional to μ. This group has a customary name, which begins with *C*. Can you guess its name?

Solution: The dimensions of these variables are {μ} = {M/LT}, {V} = {L/T}, and {Y} = {M/T²}. We must divide μ by Y to cancel mass {M}, then work the velocity into the group:

$$\left\{\frac{\mu}{Y}\right\} = \left\{\frac{\mathrm{M/LT}}{\mathrm{M/T^2}}\right\} = \left\{\frac{L}{T}\right\}.$$

hence multiply by $\{V\} = \left\{\frac{T}{L}\right\}$

finally obtain Ans:

$\left\{\frac{\mu V}{Y}\right\}$ =***dimensionless.***

This dimensionless parameter is commonly called the *Capillary Number.*

2.

The specific weight of water at ordinary pressure and temperature is 9.81 kN/m³. The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Solution

$$\rho_{\text{water}} = \frac{9.81\ \text{kN/m}^3}{9.81\ \text{m/s}^2} = 1.00\ \text{Mg/m}^3 = 1.00\ \text{g/mL} \qquad \textbf{\textit{ANS}}$$

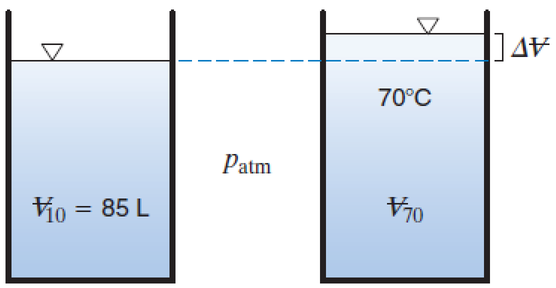
$$\gamma_{\text{mercury}} = s_{\text{mercury}}\gamma_{\text{water}} = 13.56(9.81) = 133.0\ \text{kN/m}^3 \qquad \textbf{\textit{ANS}}$$

$$\rho_{\text{mercury}} = s_{\text{mercury}}\rho_{\text{water}} = 13.56(1.00) = 13.56\ \text{Mg/m}^3 \qquad \textbf{\textit{ANS}}$$

3.

A vessel contains 85 L of water at 10°C and atmospheric pressure. If the water is heated to 70°C, what will be the percentage change in its volume? What weight of water must be removed to maintain the volume at its original value? Use Appendix A.

Solution



Volume, $\quad \forall_{10} = 85\ \text{L} = 0.085\ \text{m}^3$

Table A.1: $\quad \gamma_{10} = 9.804\ \text{kN/m}^3, \quad \gamma_{70} = 9.589\ \text{kN/m}^3$

Weight of water, $W = \gamma\forall = \gamma_{10}\forall_{10} = \gamma_{70}\forall_{70}$

i.e., $\quad 9.804(0.085)\text{ kN} = 9.589\,\mathcal{V}_{70}; \quad \mathcal{V}_{70} = 0.08691\text{ m}^3$

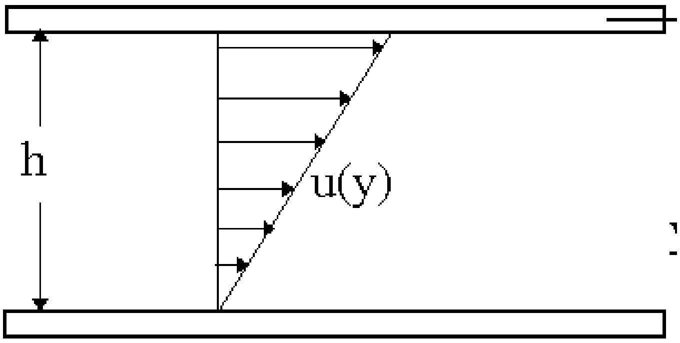
$$\Delta\mathcal{V} = \mathcal{V}_{70} - \mathcal{V}_{10} = 0.08691 - 0.08500 = 0.001906\text{ m}^3 \text{ at } \gamma_{70}$$

$$\Delta\mathcal{V}/\mathcal{V}_{10} = 0.001906/0.085 = 2.24\% \text{ increase} \qquad \textit{ANS}$$

Must remove (at γ_{70}): $\quad W\left(\frac{\Delta\mathcal{V}}{\mathcal{V}_{70}}\right) = \gamma_{70}\Delta\mathcal{V}$

$$= (9589\text{ N/m}^3)(0.001906\text{ m}^3) = 18.27\text{ N} \qquad \textit{ANS}$$

4. In the Fig., if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at V □□5.5 m/s? Note that glycerin viscosity □□1.5 N · s/m^2.



Solution: The shear stress is found from Eq:

$$\tau = \mu\frac{du}{dy} = \mu\frac{V}{h}$$

$$\tau = \frac{\mu v}{h} = \frac{1.5\ Pa.s * 5.5m/s}{0.006m} = 1380\ Pa$$

5. Suppose that the fluid being sheared in Fig. 1.1is SAE 30 oil at 20°C. Compute the shear stress in the oil if $V = 3$ m/s and $h = 2$ cm.

The shear stress is found from Eq:

$$\tau = \mu\frac{du}{dy} = \mu\frac{V}{h}$$

From Table for SAE 30 oil, μ = 0.29 kg/(m . s). Then, for the given values of *V* and *h*,

Eq. (1) predicts:

$$\tau = \frac{0.29kg/m.s*3m/s}{2m} = 43\text{kg/(m.s}^2\text{)} = 43\text{N/m}^2 = 43\text{ Pa}$$

6. When a vehicle such as an automobile slams on its brakes (locking the wheels) on a very wet road it can "hydroplane." In these circumstances a film of water is created between the tires and the road. Theoretically, a vehicle could slide a very long way under these conditions though in practice the film is destroyed before such distances are achieved (indeed, tire treads are designed to prevent the persistence of such films). To analyze this situation, consider a vehicle of mass, M, sliding over a horizontal plane covered with a film of liquid of viscosity, μ. Let the area of the film under all four tires be A and the film thickness (assumed uniform) be h.

a. If the velocity of the vehicle at some instant is V, find the force slowing the vehicle down in terms of A, V, h, and μ.

b. Find the distance, L, that the vehicle would slide before coming to rest assuming that A and h remain constant (this is not, of course, very realistic).

c. What is this distance, L, for a 1000 kg vehicle if A = 0.1 m2 , h = 0.1 mm, V = 10 m/s, and the water viscosity is μ = 0.001 kg/(m·s)?

sol:

a. $\tau = \frac{F}{A} = \frac{\mu V}{h}$

$$\therefore F = \frac{\mu V}{h} A$$

b. $L = \frac{M h V_o}{\mu A}$

c. L= 10,000 m

7. The specific weight of water at ordinary pressure and temperature is 62.4 lb/ft3 . The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Solution:

$$\rho water = \frac{\gamma water}{g} = 1.938\ slugs/ft^3$$

$$\gamma mercury = S.G.mercury * \gamma water = 846\ lb/ft^3$$

$$\rho mercury = S.G.mercury * \rho water = 26.3\ slugs/ft^3$$

8. The specific weight of water at ordinary pressure and temperature is 9.81 kN/m3 . The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury

Sol:

Ans: a. 1000kg/m^3

b. 133.0 kN/m^3

c. 1356 kg/m^3

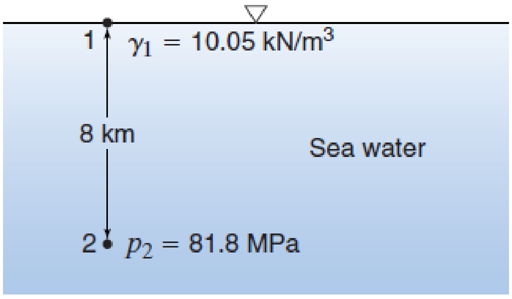
$$\frac{\Delta v}{v} \approx -\frac{\Delta p}{E_v} \qquad (2.3a)$$

$$\frac{v_2 - v_1}{v_1} \approx -\frac{p_2 - p_1}{E_v} \qquad (2.3b)$$

9.

At a depth of 8 km in the ocean the pressure is 81.8 MPa. Assume that the specific weight of seawater at the surface is 10.05 kN/m^3 and that the average volume modulus is 2.34×10^9 N/m^2 for that pressure range. (*a*) What will be the change in specific volume between that at the surface and at that depth? (*b*) What will be the specific volume at that depth? (*c*) What will be the specific weight at that depth?

Solution



(*a*) Eq. (2.2): $v_1 = 1/\rho_1 = g/\gamma_1 = 9.81/10050 = 0.000976$ m^3/kg

Eq. (2.3*a*): $\Delta v = -0.000976(81.8 \times 10^6 - 0)/(2.34 \times 10^9)$

$= -34.1 \times 10^{-6}$ m^3/kg ***ANS***

(*b*) Eq. (2.3*b*): $v_2 = v_1 + \Delta v = 0.000942$ m^3/kg ***ANS***

(*c*) $\gamma_2 = g/v_2 = 9.81/0.000942 = 10410$ N/m^3 ***ANS***

EXERCISES

1 . If the specific weight of a liquid is 52 lb/ft^3 , what is its density? Ans: 1.616 slug/ft^3

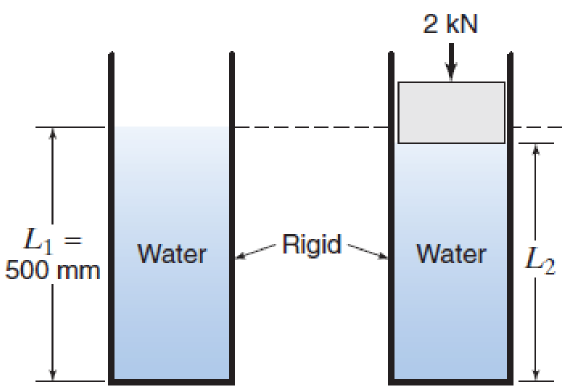
2 . If the specific weight of a liquid is 8.1 kN/m^3 , what is its density? Ans: 826 kg/m^3

3. If the specific volume of a gas is 375 ft^3 /slug, what is its specific weight in lb/ft^3 ? Ans: 0.0858 Ib/ft^3

4. If the specific volume of a gas is 0.70 m^3 /kg, what is its specific weight in N/m^3 ? Ans: 14 N/m^3
5. A certain gas weighs 16.0 N/m^3 at a certain temperature and pressure. What are the values of its density, specific volume, and specific gravity relative to air weighing 12.0 N/m^3 ?Ans: 1.63 kg/m^3, 0.613 m^3/kg, 1.33.
6. The specific weight of glycerin is 78.6 lb/ft^3 . Compute its density and specific gravity. What is its specific weight in kN/m^3 ? 2.44 $slug/ft^3$, 1.26, 40.51KN/m^3
7. If a certain gasoline weighs 43 lb/ft^3 , what are the values of its density, specific volume, and specific gravity relative to water at 60°F? 19.79 $slug/ft^3$, 0.05 $ft^3/slug$, 0.69
8. A rigid cylinder, inside diameter 15 mm, contains a column of water 500 mm long. What will the column length be if a force of 2 kN is applied to its end by a frictionless plunger? Assume no leakage. Ans: 499.974 mm
9. A flat plate 200 mm 750 mm slides on oil (μ= 0.85 Ns/m^2) over a large plane surface . What force F is required to drag the plate at a velocity v of 1.2 m/s, if the thickness t of the separating oil film is 0.6 mm? Ans:11333.33 N.
10. A space 16 mm wide between two large plane surfaces is filled with SAE 30 Western lubricating oil at 35°C . What force F is required to drag a very thin plate of 0.4 m^2 area between the surfaces at a speed v 0.25 m/s (a) if the plate is equally spaced between the two surfaces, and (b) if t 5 mm? Ans: a- 7.25 N, b- 8.436 N
11. Distilled water at 20°C stands in a glass tube of 6.0-mm diameter at a height of 18.0 mm. What is the true static height? Ans: 12.969 mm.
12. compute the capillary depression of mercury at 68°F (θ= 140°) to be expected in a 0.05-in-diameter tube. Ans: $3.65*10^{-4}$ in.
13. Compute the capillary rise in mm of pure water at 10°C expected in an 0.8-mm diameter tube. Ans: 0.037 mm

14.

A rigid cylinder, inside diameter 15 mm, contains a column of water 500 mm long. What will the column length be if a force of 2 kN is applied to its end by a frictionless plunger? Assume no leakage.



15 .

A vessel contains 5.0 ft^3 of water at 40°F and atmospheric pressure. If the water is heated to 80°F, what will be the percentage change in its volume? What weight of water must be removed to maintain the volume at its original value? Use Appendix A.

16. A cylindrical tank (diameter = 8.00 m and depth = 5.00 m) contains water at 15°C and is brimful. If the water is heated to 60°C, how much water will spill over the edge of the tank? Assume the tank does not expand with the change in temperature. Use Appendix A.

Table A.1 Viscosity and Density of Water at 1 atm

T, °C	ρ, kg/m³	μ, N · s/m²	ν, m²/s	T, °F	ρ, slug/ft³	μ, lb · s/ft²	ν, ft²/s
0	1000	1.788 E-3	1.788 E-6	32	1.940	3.73 E-5	1.925 E-5
10	1000	1.307 E-3	1.307 E-6	50	1.940	2.73 E-5	1.407 E-5
20	998	1.003 E-3	1.005 E-6	68	1.937	2.09 E-5	1.082 E-5
30	996	0.799 E-3	0.802 E-6	86	1.932	1.67 E-5	0.864 E-5
40	992	0.657 E-3	0.662 E-6	104	1.925	1.37 E-5	0.713 E-5
50	988	0.548 E-3	0.555 E-6	122	1.917	1.14 E-5	0.597 E-5
60	983	0.467 E-3	0.475 E-6	140	1.908	0.975 E-5	0.511 E-5
70	978	0.405 E-3	0.414 E-6	158	1.897	0.846 E-5	0.446 E-5
80	972	0.355 E-3	0.365 E-6	176	1.886	0.741 E-5	0.393 E-5
90	965	0.316 E-3	0.327 E-6	194	1.873	0.660 E-5	0.352 E-5
100	958	0.283 E-3	0.295 E-6	212	1.859	0.591 E-5	0.318 E-5

Table A.2 Viscosity and Density of Air at 1 atm

T, °C	ρ, kg/m³	μ, N · s/m²	ν, m²/s	T, °F	ρ, slug/ft³	μ, lb · s/ft²	ν, ft²/s
−40	1.52	1.51 E-5	0.99 E-5	−40	2.94 E-3	3.16 E-7	1.07 E-4
0	1.29	1.71 E-5	1.33 E-5	32	2.51 E-3	3.58 E-7	1.43 E-4
20	1.20	1.80 E-5	1.50 E-5	68	2.34 E-3	3.76 E-7	1.61 E-4
50	1.09	1.95 E-5	1.79 E-5	122	2.12 E-3	4.08 E-7	1.93 E-4
100	0.946	2.17 E-5	2.30 E-5	212	1.84 E-3	4.54 E-7	2.47 E-4
150	0.835	2.38 E-5	2.85 E-5	302	1.62 E-3	4.97 E-7	3.07 E-4
200	0.746	2.57 E-5	3.45 E-5	392	1.45 E-3	5.37 E-7	3.71 E-4
250	0.675	2.75 E-5	4.08 E-5	482	1.31 E-3	5.75 E-7	4.39 E-4
300	0.616	2.93 E-5	4.75 E-5	572	1.20 E-3	6.11 E-7	5.12 E-4
400	0.525	3.25 E-5	6.20 E-5	752	1.02 E-3	6.79 E-7	6.67 E-4
500	0.457	3.55 E-5	7.77 E-5	932	0.89 E-3	7.41 E-7	8.37 E-4

1.17 Clouds can weigh thousands of pounds due to their liquid water content. Often this content is measured in grams per cubic meter (g/m^3). Assume that a cumulus cloud occupies a volume of one cubic kilometer, and its liquid water content is 0.2 g/m^3. (a) What is the volume of this cloud in cubic miles? (b) How much does the water in the cloud weigh in pounds?

(a) $\forall\text{olume} = 1\,(\text{km})^3 = 10^9\ \text{m}^3$

Since $1\ \text{m} = 3.281\ \text{ft}$

$$\forall\text{olume} = \frac{(10^9\ \text{m}^3)\left(3.281\ \frac{\text{ft}}{\text{m}}\right)^3}{\left(5.280 \times 10^3\ \frac{\text{ft}}{\text{mi}}\right)^3}$$

$$= \underline{\underline{0.240\ \text{mi}^3}}$$

(b) $\mathcal{W} = \gamma \times \forall\text{olume}$

$$\gamma = \rho g = \left(0.2\ \frac{\text{g}}{\text{m}^3}\right)\left(10^{-3}\ \frac{\text{kg}}{\text{g}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = 1.962 \times 10^{-3}\ \frac{\text{N}}{\text{m}^3}$$

$$\mathcal{W} = \left(1.962 \times 10^{-3}\ \frac{\text{N}}{\text{m}^3}\right)\left(10^9\ \text{m}^3\right) = 1.962 \times 10^6\ \text{N}$$

$$= \left(1.962 \times 10^6\ \text{N}\right)\left(2.248 \times 10^{-1}\ \frac{\text{lb}}{\text{N}}\right) = \underline{\underline{4.41 \times 10^5\ \text{lb}}}$$

1.20 Water flows from a large drainage pipe at a rate of 1200 gal/min. What is this volume rate of flow in (a) m^3/s, (b) liters/min, and (c) ft^3/s?

(a)

$$\text{flowrate} = \left(1200\ \frac{gal}{min}\right)\left(6.309 \times 10^{-5}\ \frac{\frac{m^3}{s}}{\frac{gal}{min}}\right)$$

$$= \underline{\underline{7.57 \times 10^{-2}\ \frac{m^3}{s}}}$$

(b) Since 1 liter $= 10^{-3}\ m^3$,

$$\text{flowrate} = \left(7.57 \times 10^{-2}\ \frac{m^3}{s}\right)\left(\frac{10^3\ liters}{m^3}\right)\left(\frac{60\ s}{min}\right)$$

$$= \underline{\underline{4540\ \frac{liters}{min}}}$$

(c)

$$\text{flowrate} = \left(7.57 \times 10^{-2}\ \frac{m^3}{s}\right)\left(3.531 \times 10\ \frac{\frac{ft^3}{s}}{\frac{m^3}{s}}\right)$$

$$= \underline{\underline{2.67\ \frac{ft^3}{s}}}$$

1.26 An open, rigid-walled, cylindrical tank contains 4 ft^3 of water at 40 °F. Over a 24-hour period of time the water temperature varies from 40 °F to 90 °F. Make use of the data in Appendix B to determine how much the volume of water will change. For a tank diameter of 2 ft, would the corresponding change in water depth be very noticeable? Explain.

mass of water $= \forall \times \rho$

where $\forall$ is the volume and ρ the density. Since the mass must remain constant as the temperature changes

$$\forall_{40°} \times \rho_{40°} = \forall_{90°} \times \rho_{90°} \qquad (1)$$

From Table B.1 $\rho_{H_2O\,@\,40°F} = 1.940 \ \frac{slugs}{ft^3}$

$$\rho_{H_2O\,@\,90°F} = 1.931 \ \frac{slugs}{ft^3}$$

Therefore, from Eq. (1)

$$\forall_{90°} = \frac{(4\,ft^3)\left(1.940\,\frac{slugs}{ft^3}\right)}{1.931\,\frac{slugs}{ft^3}} = 4.0186\ ft^3$$

Thus, the increase in volume is

$$4.0186 - 4.000 = \underline{\underline{0.0186\ ft^3}}$$

The change in water depth, $\Delta \ell$, is equal to

$$\Delta \ell = \frac{\Delta \forall}{area} = \frac{0.0186\ ft^3}{\frac{\pi}{4}(2ft)^2} = 5.92 \times 10^{-3}\ ft = 0.0710\ in.$$

This small change in depth would not be very noticeable. $\underline{\underline{No}}$.

Note: A slightly different value for $\Delta \ell$ will be obtained if specific weight of water is used rather than density. This is due to the fact that there is some uncertainty in the fourth significant figure of these two values, and the solution is sensitive to this uncertainty.