Vigenère cipher



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The Vigenère cipher is named for Blaise de Vigenère (pictured), although Giovan Battista Bellaso had invented the cipher earlier. Vigenère did invent a stronger autokey cipher.



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A reproduction of the Confederacy's cipher disk on display at the National Cryptologic Museum

The **Vigenère cipher** is a method of encrypting alphabetic text by using a series of different Caesar ciphers based on the letters of a keyword. It is a simple form of polyalphabetic substitution.

The Vigenère (French pronunciation: $[vi3n\epsilon \ B]$) cipher has been reinvented many times. The method was originally described by Giovan Battista Bellaso in his 1553 book *La cifra del. Sig. Giovan Battista Bellaso*; however, the scheme was later misattributed to Blaise de Vigenère in the 19th century, and is now widely known as the "Vigenère cipher".

This cipher is well known because while it is easy to understand and implement, it often appears to beginners to be unbreakable; this earned it the description **le chiffre indéchiffrable** (French for 'the indecipherable cipher'). Consequently, many people have tried to implement encryption schemes that are essentially Vigenère ciphers, only to have them broken.^[1]

Contents

- 1 History
- 2 Description
- 3 Algebraic description
- 4 Cryptanalysis
 - 4.1 Kasiski examination
 - o 4.2 Friedman test
 - 4.3 Frequency analysis
 - o 4.4 Key elimination
- 5 Variants

History

The first well documented description of a polyalphabetic cipher was formulated by Leon Battista Alberti around 1467 and used a metal cipher disc to switch between cipher alphabets. Alberti's system only switched alphabets after several words, and switches were indicated by writing the letter of the corresponding alphabet in the ciphertext. Later, in 1508, Johannes Trithemius, in his work *Poligraphia*, invented the tabula recta, a critical component of the Vigenère cipher. The Trithemius cipher, however, only provided a progressive, rigid and predictable system for switching between cipher alphabets.

What is now known as the Vigenère cipher was originally described by Giovan Battista Bellaso in his 1553 book *La cifra del. Sig. Giovan Battista Bellaso*. He built upon the tabula recta of Trithemius, but added a repeating "countersign" (a key) to switch cipher alphabets every letter. Whereas Alberti and Trithemius used a fixed pattern of substitutions, Bellaso's scheme meant the pattern of substitutions could be easy changed simply by selecting a new key. Keys were typically single words or short phrases, known to both parties in advance, or transmitted "out of band" along with the message. Bellaso's method thus required strong security for only the key. As it is relatively easy to secure a short key phrase, say by a previous private conversation, Bellaso's system was considerably more secure.

Blaise de Vigenère published his description of a similar but stronger autokey cipher before the court of Henry III of France, in 1586. Later, in the 19th century, the invention of Bellaso's cipher was misattributed to Vigenère. David Kahn in his book *The Codebreakers* lamented the misattribution by saying that history had "ignored this important contribution and instead named a regressive and elementary cipher for him [Vigenère] though he had nothing to do with it".^[2]

The Vigenère cipher gained a reputation for being exceptionally strong. Noted author and mathematician Charles Lutwidge Dodgson (Lewis Carroll) called the Vigenère cipher unbreakable in his 1868 piece "The Alphabet Cipher" in a children's magazine. In 1917, *Scientific American* described the Vigenère cipher as "impossible of translation".^[3] This reputation was not deserved. Charles Babbage was known to have broken a variant of the cipher as early as 1854; however, he didn't publish his work.^[4] Kasiski entirely broke the cipher and published the technique in the 19th century.

Even before this, though, some skilled cryptanalysts could occasionally break the cipher in the 16th century.^[2]

The Vigenère cipher is simple enough to be a field cipher if it is used in conjunction with cipher disks.^[5] The Confederate States of America, for example, used a brass cipher disk to implement the Vigenère cipher during the American Civil War. The Confederacy's messages were far from secret and the Union regularly cracked their messages. Throughout the war, the Confederate leadership primarily relied upon three key phrases, "Manchester Bluff", "Complete Victory" and, as the war came to a close, "Come Retribution".^[6]

Gilbert Vernam tried to repair the broken cipher (creating the Vernam-Vigenère cipher in 1918), but, no matter what he did, the cipher was still vulnerable to cryptanalysis. Vernam's work, however, eventually led to the one-time pad, a provably unbreakable cipher.

Description



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The Vigenère square or Vigenère table, also known as the *tabula recta*, can be used for encryption and decryption.

In a Caesar cipher, each letter of the alphabet is shifted along some number of places; for example, in a Caesar cipher of shift 3, A would become D, B would become E, Y would become B and so on. The Vigenère cipher consists of several Caesar ciphers in sequence with different shift values.

To encrypt, a table of alphabets can be used, termed a *tabula recta*, *Vigenère square*, or *Vigenère table*. It consists of the alphabet written out 26 times in different rows, each alphabet shifted cyclically to the left compared to the previous alphabet,

corresponding to the 26 possible Caesar ciphers. At different points in the encryption process, the cipher uses a different alphabet from one of the rows. The alphabet used at each point depends on a repeating keyword.

For example, suppose that the plaintext to be encrypted is:

ATTACKATDAWN

The person sending the message chooses a keyword and repeats it until it matches the length of the plaintext, for example, the keyword "LEMON":

LEMONLEMONLE

Each row starts with a key letter. The remainder of the row holds the letters A to Z (in shifted order). Although there are 26 key rows shown, you will only use as many keys (different alphabets) as there are unique letters in the key string, here just 5 keys, {L, E, M, O, N}. For successive letters of the message, we are going to take successive letters of the key string, and encipher each message letter using its corresponding key row. Choose the next letter of the key, go along that row to find the column heading that matches the message character; the letter at the intersection of [key-row, msg-col] is the enciphered letter.

For example, the first letter of the plaintext, A, is paired with L, the first letter of the key. So use row L and column A of the Vigenère square, namely L. Similarly, for the second letter of the plaintext, the second letter of the key is used; the letter at row E and column T is X. The rest of the plaintext is enciphered in a similar fashion:

Plaintext: ATTACKATDAWN Key: LEMONLEMONLE Ciphertext: LXFOPVEFRNHR

Decryption is performed by going to the row in the table corresponding to the key, finding the position of the ciphertext letter in this row, and then using the column's label as the plaintext. For example, in row L (from *L*EMON), the ciphertext L appears in column A, which is the first plaintext letter. Next we go to row E (from *L*EMON), locate the ciphertext X which is found in column T, thus T is the second plaintext letter.

Algebraic description

Vigenère can also be viewed algebraically. If the letters A–Z are taken to be the numbers 0–25, and addition is performed modulo 26, then Vigenère encryption E using the key K can be written,

$$C_i = E_K(M_i) = (M_i + K_i) \mod 26$$

and decryption *D* using the key *K*,

$$M_i = D_K(C_i) = (C_i - K_i) \mod 26$$

whereas $M = M_0 \dots M_n$ is the message, $C = C_0 \dots C_n$ is the ciphertext and $K = K_0 \dots K_m$ is the used key.

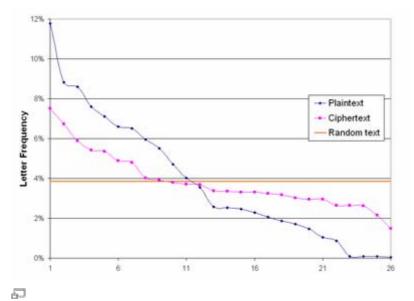
Thus using the previous example, to encrypt $A \cong 0$ with key letter $L \cong 11$ the calculation would result in $11 \cong L$.

$$11 = (0 + 11) \mod 26$$

Therefore to decrypt $R \triangleq 17$ with key letter $E \triangleq 4$ the calculation would result in $13 \triangleq N$.

$$13 = (17 - 4) \mod 26$$

Cryptanalysis



The Vigenère cipher masks the characteristic letter frequencies of English plaintexts, but some patterns remain.

The idea behind the Vigenère cipher, like all polyalphabetic ciphers, is to disguise plaintext letter frequencies, which interferes with a straightforward application of frequency analysis. For instance, if P is the most frequent letter in a ciphertext whose plaintext is in English, one might suspect that P corresponds to E, because E is the most frequently used letter in English. However, using the Vigenère cipher, E can be enciphered as different ciphertext letters at different points in the message, thus defeating simple frequency analysis.

The primary weakness of the Vigenère cipher is the repeating nature of its key. If a cryptanalyst correctly guesses the key's length, then the cipher text can be treated as interwoven Caesar ciphers, which individually are easily broken. The Kasiski and Friedman tests can help determine the key length.

Kasiski examination

In 1863 Friedrich Kasiski was the first to publish a successful general attack on the Vigenère cipher. Earlier attacks relied on knowledge of the plaintext, or use of a recognizable word as a key. Kasiski's method had no such dependencies. Kasiski was the first to publish an account of the attack, but it's clear that there were others who were aware of it. In 1854, Charles Babbage was goaded into breaking the Vigenère cipher when John Hall Brock Thwaites submitted a "new" cipher to the Journal of the Society of the Arts. When Babbage showed that Thwaites' cipher was essentially just another recreation of the Vigenère cipher, Thwaites challenged Babbage to break his cipher encoded twice, with keys of different length. Babbage succeeded in decrypting a sample, which turned out to be the poem "The Vision of Sin", by Alfred Tennyson, encrypted according to the keyword "Emily", the first name of Tennyson's wife. Babbage never explained the method he used. Studies of Babbage's notes reveal that he had used the method later published by Kasiski, and suggest that he had been using the method as early as 1846.^[7]

The Kasiski examination, also called the Kasiski test, takes advantage of the fact that repeated words may, by chance, sometimes be encrypted using the same key letters, leading to repeated groups in the ciphertext. For example, Consider the following encryption using the keyword ABCD:

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Key: ABCDABCDABCDABCDABCDABCDABCD
Plaintext: CRYPTOISSHORTFORCRYPTOGRAPHY
Ciphertext: CSASTPKVSIQUTGQUCSASTPIUAQJB
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There is an easily seen repetition in the ciphertext, and the Kasiski test will be effective. Here the distance between the repetitions of CSASTP is 16. Assuming that the repeated segments represent the same plaintext segments, this implies that the key is 16, 8, 4, 2, or 1 characters long. (All factors of the distance are possible key lengths – a key of length one is just a simple shift cipher, where cryptanalysis is much easier.) Since key lengths 2 and 1 are unrealistically short, one only needs to try lengths 16, 8, or 4. Longer messages make the test more accurate because they usually contain more repeated ciphertext segments. The following ciphertext has two segments that are repeated:

Ciphertext: VHVSSPQUCEMRVBVBBBVHVSURQGIBDUGRNICJQUCERVUAXSSR

The distance between the repetitions of VHVS is 18. Assuming that the repeated segments represent the same plaintext segments, this implies that the key is 18, 9, 6, 3, 2, or 1 characters long. The distance between the repetitions of QUCE is 30 characters. This means that the key length could be 30, 15, 10, 6, 5, 3, 2, or 1 characters long. By taking the intersection of these sets one could safely conclude that the most likely key length is 6, since 3, 2, and 1 are unrealistically short.

Friedman test

The Friedman test (sometimes known as the kappa test) was invented during the 1920s by William F. Friedman. Friedman used the index of coincidence, which

measures the unevenness of the cipher letter frequencies to break the cipher. By knowing the probability κ_p that any two randomly chosen source-language letters are the same (around 0.067 for monocase English) and the probability of a coincidence for a uniform random selection from the alphabet κ_r (1/26 = 0.0385 for English), the key length can be estimated as:

$$\frac{\kappa_p - \kappa_r}{\kappa_o - \kappa_r}$$

from the observed coincidence rate

$$\kappa_o = \frac{\sum_{i=1}^{c} n_i (n_i - 1)}{N(N - 1)}$$

where *c* is the size of the alphabet (26 for English), *N* is the length of the text, and n_1 through n_c are the observed ciphertext letter frequencies, as integers.

This is, however, only an approximation whose accuracy increases with the size of the text. It would in practice be necessary to try various key lengths close to the estimate.^[8] A better approach for repeating-key ciphers is to copy the ciphertext into rows of a matrix having as many columns as an assumed key length, then compute the average index of coincidence with each column considered separately; when this is done for each possible key length, the highest average I.C. then corresponds to the most likely key length.^[9] Such tests may be supplemented by information from the Kasiski examination.

Frequency analysis

Once the length of the key is known, the ciphertext can be rewritten into that many columns, with each column corresponding to a single letter of the key. Each column consists of plaintext that has been encrypted by a single Caesar cipher; the Caesar key (shift) is just the letter of the Vigenère key that was used for that column. Using methods similar to those used to break the Caesar cipher, the letters in the ciphertext can be discovered.

An improvement to the Kasiski examination, known as Kerckhoffs' method, matches each column's letter frequencies to shifted plaintext frequencies to discover the key letter (Caesar shift) for that column. Once every letter in the key is known, the cryptanalyst can simply decrypt the ciphertext and reveal the plaintext.^[10] Kerckhoffs' method is not applicable when the Vigenère table has been scrambled, rather than using normal alphabetic sequences, although Kasiski examination and coincidence tests can still be used to determine key length in that case.

Key elimination

The Vigenère cipher function is essentially modulo arithmetic, and thus commutative. So if the key length is known (or guessed) then subtracting the cipher text from itself, offset by the key length will produce the cipher text encrypted with itself. If any words in the cipher text are known or can be guessed, then the plain text and also the key, will be revealed. This is useful if the key is an obscure sequence of letters because the plain text will generally be ordinary words. Key elimination is useful for making short versions of the plain text.

Variants

The running key variant of the Vigenère cipher was also considered unbreakable at one time. This version uses as the key a block of text as long as the plaintext. Since the key is as long as the message the Friedman and Kasiski tests no longer work (the key is not repeated). In 1920, Friedman was the first to discover this variant's weaknesses. The problem with the running key Vigenère cipher is that the cryptanalyst has statistical information about the key (assuming that the block of text is in a known language) and that information will be reflected in the ciphertext.

If using a key which is truly random, is at least as long as the encrypted message and is used only once, the Vigenère cipher is theoretically unbreakable. However, in this case it is the key, not the cipher, which provides cryptographic strength and such systems are properly referred to collectively as one-time pad systems, irrespective of which ciphers are employed.

Vigenère actually invented a stronger cipher: an autokey cipher. The name "Vigenère cipher" became associated with a simpler polyalphabetic cipher instead. In fact, the two ciphers were often confused, and both were sometimes called "*le chiffre indéchiffrable*". Babbage actually broke the much stronger autokey cipher, while Kasiski is generally credited with the first published solution to the fixed-key polyalphabetic ciphers.

A simple variant is to encrypt using the Vigenère decryption method, and decrypt using Vigenère encryption. This method is sometimes referred to as "Variant Beaufort". This is different from the Beaufort cipher, created by Sir Francis Beaufort, which nonetheless is similar to Vigenère but uses a slightly modified enciphering mechanism and tableau. The Beaufort cipher is a reciprocal cipher.

Despite the Vigenère cipher's apparent strength it never became widely used throughout Europe. The Gronsfeld cipher is a variant created by Count Gronsfeld which is identical to the Vigenère cipher, except that it uses just 10 different cipher alphabets (corresponding to the digits 0 to 9). The Gronsfeld cipher is strengthened because its key is not a word, but it is weakened because it has just 10 cipher alphabets. Gronsfeld's cipher did become widely used throughout Germany and Europe, despite its weaknesses.

Beaufort cipher

The **Beaufort cipher**, created by Sir <u>Francis Beaufort</u>, is a <u>substitution cipher</u> that is similar to the <u>Vigenère cipher</u> but uses a slightly modified enciphering mechanism and <u>tableau</u>. ^[1]. Its most famous application was in a rotor-based cipher machine, the <u>Hagelin M-209^[2]</u>.

Using the cipher

To encrypt (or decrypt for that matter), first choose the plaintext character from the top row of the tableau,call this column P. Secondly, travel down column P to the corresponding keyLetter K. Finally, move directly left from the Key letter to the left edge of the tableau, the CipherText encryption of Plaintext P with Key K will be there. For example if encrypting Plain text character "d" with Key "m" the steps would be: 1) find the column with "d" on the top, 2) travel down that column to find Key "m", 3) travel to the left edge of the tableau to find the CipherText letter ("J" in this case). The Beaufort cipher is a <u>reciprocal cipher</u>, that is, Decryption and Encryption algorithms are the same.

Distinguished from 'variant Beaufort'

The Beaufort cipher should not be confused with the "variant Beaufort" cipher. In variant Beaufort, encryption is performed by performing the decryption step of the standard Vigenère cipher, and likewise decryption is performed by using Vigenère encryption.