mixing of fluids

\[
m = \frac{1.0 - \log_{10} 2.121}{40} = -0.0582
\]

\[
N_{Fr}^{m} = 0.14^{-0.0582} = 1.1212
\]

Therefore, power

\[
P = N_{P} \rho N^{3} \Delta_{A} \frac{N_{Fr}^{m}}{m} = 2.0 \times 950 \times 1.5^{3} \times 0.61^{5} \times 1.1212 \left( \frac{\text{kg}}{\text{m}^{3}} \cdot \frac{\text{rev}^{3}}{\text{sec}^{3}} \cdot \text{m}^{3} \right)
\]

\[
= 607.24 \text{ W}
\]

\[
= 0.61 \text{ kW} \ (0.81 \text{ hp})
\]

Studies on various turbine agitators have shown that geometric ratios that vary from the standard design can cause different effects on the Power number \(N_{P}\) in the turbulent regions [24].

• For the flat, six-blade open turbine, \(N_{P} \propto (W/D_{A})^{1.0}\).
• For the flat, six-blade open turbine, varying \(D_{A}/D_{T}\) from 0.25 to 0.5 has no effect on \(N_{P}\).
• When two six-blade open turbines are installed on the same shaft and the spacing between the two impellers (vertical distance between the bottom edges of the two turbines) is at least equal to \(D_{A}\), the total power is 1.9 times a single flat-blade impeller. For two six-blade pitched-blade (45°) turbines, the power is about 1.9 times that of a single pitched-blade impeller.
• A baffled, vertical square tank or a horizontal cylindrical tank has the same Power number as a vertical cylindrical tank.

**SCALE-UP OF MIXING SYSTEMS**

The calculation of power requirements for agitation is only a part of the mixer design. In any mixing problem, there are several defined objectives such as the time required for blending two immiscible liquids, rates of heat transfer from a heated jacket per unit volume of the agitated liquid, and mass transfer rate from gas bubbles dispersed by agitation in a liquid. For all these objectives, the process results are to achieve the optimum mixing and uniform blending.
The process results are related to variables characterizing mixing, namely geometric dimensions, stirrer speed (rpm), agitator power, and physical properties of the fluid (e.g., density, viscosity, and surface tension) or their dimensionsless combinations (e.g., the Reynolds number, Froude number, and Weber number, \( \rho N^2 D^3 / \sigma \)). Sometimes, empirical relationships are established to relate process results and agitation parameters. Often, however, such relationships are nonexistent. Laboratory scales of equipment using the same materials as on a large scale are then experimented with, and the desired process result is obtained. The laboratory system can then be scaled-up to predict the conditions on the larger system.

For some scale-up problems, generalized correlations as shown in Figures 8-11, 8-12, 8-13, and 8-14 are available for scale-up. However, there is much diversity in the process to be scaled-up, and as such no single method can successfully handle all types of scale-up problems.

Various methods of scale-up have been proposed; all based on geometric similarity between the laboratory equipment and the full-scale plant. It is not always possible to have the large and small vessels geometrically similar, although it is perhaps the simplest to attain. If geometric similarity is achievable, dynamic and kinematic similarity cannot often be predicted at the same time. For these reasons, experience and judgment are relied on with aspects to scale-up.

The main objectives in a fluid agitation process are [25]:

- Equivalent liquid motion (e.g., liquid blending where the liquid motion or corresponding velocities are approximately the same in both cases).
- Equivalent suspension of solids, where the levels of suspension are identical.
- Equivalent rates of mass transfer, where mass transfer is occurring between a liquid and a solid phase, between liquid-liquid phases, or between gas and liquid phases, and the rates are identical.

A scale ratio \( R \) is used for scale-up from the standard configuration as shown in Table 8-2. The procedure is:

1. Determine the scale-up ratio \( R \), assuming that the original vessel is a standard cylinder with \( D_{T1} = H_1 \). The volume \( V_1 \) is

\[
V_1 = \frac{\pi D_{T1}^2}{4} \cdot H_1 = \frac{\pi D_{T1}^3}{4}
\]

(8-35)
The ratio of the volumes is then
\[
\frac{V_2}{V_1} = \frac{\pi D_{T2}^3/4}{\pi D_{T1}^3/4} = \frac{D_{T2}^3}{D_{T1}^3} = \left(\frac{V_{T2}}{V_{T1}}\right)^{1/3} \tag{8-36}
\]

The scale-up ratio \( R \) is
\[
R = \frac{D_{T2}}{D_{T1}} = \left(\frac{V_{T2}}{V_{T1}}\right)^{1/3} \tag{8-37}
\]

Using the value of \( R \), calculate the new dimensions for all geometric sizes. That is,
\[
D_{A2} = RD_{A1}, \quad J_2 = RJ_1, \quad W_2 = RW_1,
\]
\[
E_2 = RE_1, \quad L_2 = RL_1, \quad H_2 = RH_1
\]

or
\[
R = \frac{D_{A2}}{D_{A1}} = \frac{D_{T2}}{D_{T1}} = \frac{W_2}{W_1} = \frac{H_2}{H_1} = \frac{J_2}{J_1} = \frac{E_2}{E_1}
\]

2. The selected scale-up rule is applied to determine the agitator speed \( N_2 \) from the equation:
\[
N_2 = N_1 \left(\frac{1}{R}\right)^n = N_1 \left(\frac{D_{T1}}{D_{T2}}\right)^n \tag{8-38}
\]

- where \( n = 1 \) for equal liquid motion
- \( n = 3/4 \) for equal suspension of solids
- \( n = 2/3 \) for equal rates of mass transfer (corresponding equivalent power per unit volume, which results in equivalent interfacial area per unit volume)

The value of \( n \) is based on theoretical and empirical considerations and depends on the type of agitation problem.

3. Knowing the value of \( N_2 \), the required power can be determined using Equation 8-17 and the generalized Power number correlation.
Other possible ways of scaling up are constant tip speed \( u_t (\pi N D_A) \), and a constant ratio of circulating capacity to head \( Q/h \).

Since \( P \propto N^3 D_A^5 \) and \( V \propto D_A^3 \) then

\[
\frac{P}{V} \propto N^3 D_A^2 \quad (8-39)
\]

For scale-up from system 1 to system 2 involving geometrically similar tanks and same liquid properties, the following equations can be applied:

\[ N_1 D_{A1} = N_2 D_{A2} \]

For a constant tip speed,

\[
\frac{N_2}{N_1} = \frac{D_{A1}}{D_{A2}} \quad (8-40)
\]

For a constant ratio of circulating capacity to head, \( Q/h \),

\[ N_1^3 D_{A1}^2 = N_2^3 D_{A2}^2 \quad (8-41) \]

**Example 8-3**

Scraper blades set to rotate at 35 rpm are used for a pilot plant addition of liquid ingredients into a body-wash product. What should the speed of the blades be in a full-scale plant, if the pilot and the full-scale plants are geometrically similar in design? Assume scale-up is based on constant tip speed, diameter of the pilot plant scraper blades is 0.6 m, and diameter of the full-scale plant scraper blades is 8 ft.

**Solution**

The diameter of the full scale plant scraper blades = \( 8.0 \times 0.3048 = 2.4384 \) m (2.4 m).

Assuming constant tip speed,

\[
\frac{N_2}{N_1} = \frac{D_{A1}}{D_{A2}} \quad (9-42)
\]
where \( N_1 \) = scraper speed of pilot plant
\( N_2 \) = scraper speed of full-scale plant
\( D_{A1} \) = diameter of pilot plant scraper blades
\( D_{A2} \) = diameter of full-scale plant scraper blades

\[
N_2 = \frac{N_1 D_{A1}}{D_{A2}}
\]

\[
= \frac{(35)(0.6)}{(2.4)}
\]

\[
= 8.75 \text{ rpm}
\]

**Example 8-4**

During liquid makeup production, color pigments (i.e., solid having identical particle size) are added to the product via a mixer. In the pilot plant, this mixer runs at 6,700 rpm and has a diameter head of 0.035 m. Full-scale production is geometrically similar and has a mixer head diameter of 0.12 m. Determine the speed of the full-scale production mixer head. What additional information is required for the motor to drive this mixer? Assume that power curves are available for this mixer design, and the scale-up basis is constant power/unit volume.

**Solution**

For constant power per unit volume, Equation 8-39 is applied: \( P/V \propto N^3D_A^2 \) or \( N_1^3D_{A1}^2 = N_2^3D_{A2}^2 \). Therefore,

\[
N_2 = N_1 \left( \frac{D_{A1}}{D_{A2}} \right)^{2/3}
\]

where \( N_1 = 6,700 \) rpm
\( D_{A1} = 0.035 \) m
\( D_{A2} = 0.12 \) m

\[
N_2 = 6,700 \left( \frac{0.035}{0.12} \right)^{2/3}
\]
$N_2 = 2,946.7 \text{ rpm}$

$N_2 = 2,950 \text{ rpm}$

The power required for mixing is $P = N_p \rho N^3 D^5 A$, where the Power number ($N_p$) is a function of the Reynolds number \( [i.e., N_p = f(N_{Re})] \):

$$N_{Re} = \frac{\rho ND^2}{\mu}$$

The plant must be provided with the viscosity of the product and its density after addition of the pigments.

**Example 8-5**

A turbine agitator with six flat blades and a disk has a diameter of 0.203 m. It is used in a tank with a diameter of 0.61 m and height of 0.61 m. The width is $W = 0.0405$ m. Four baffles are used with a width of 0.051 m. The turbine operates at 275 rpm in a liquid having a density of 909 kg/m$^3$ and viscosity of 0.02 Pas.

Calculate the kW power of the turbine and kW/m$^3$ of volume. Scale up this system to a vessel whose volume is four times as large, for the case of equal mass transfer rate.

**Solution**

The Reynolds number for mixing is $N_{Re}$. The number of revolutions per sec, $N = 275/60 = 4.58$ rev/sec.

$$N_{Re} = \frac{\rho ND^2}{\mu}$$

$$= \frac{(909)(4.58)(0.203)^2}{0.02} \left\{ \frac{\text{kg}}{\text{m}^3 \cdot \text{sec}} \cdot \frac{\text{rev}}{\text{m}^2} \cdot \frac{\text{m}}{\text{sec}} \cdot \frac{\text{kg}}{\text{m}^3} \right\}$$

$$= 8,578.1$$

$N_{Re} \approx 8,600$
Using curve 6 in Figure 8-14, the Power number $N_p = 6.0$. The power of the turbine $P = N_p \rho N^3 D^2$:

$$P = (6.0)(909)(4.58^3)(0.2035) \left\{ \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{rev}^3}{\text{sec}^3} \cdot \text{m}^5 \right\}$$

$$= 0.1806 \text{ kW (0.24 hp)}$$

The original tank volume $V_1 = \pi D_{T1}^3/4$. The tank diameter $D_{T1} = 0.61$:

$$V_1 = \frac{(\pi)(0.61)^3}{4}$$

$$V_1 = 0.178 \text{m}^3$$

The power per unit volume is $P/V$

$$\frac{P}{V} = \frac{0.1806}{0.178}$$

$$= 1.014 \text{ kW/m}^3$$

For the scale-up of the system, the scale-up ratio $R$ is

$$R = \frac{V_2}{V_1} = \frac{\pi D_{T2}^3}{\pi D_{T1}^3} = \frac{D_{T2}^3}{D_{T1}^3}$$

$$R = \left( \frac{V_2}{V_1} \right)^{\frac{1}{3}} = \frac{D_{T2}}{D_{T1}}$$

(8-37)

where $V_2 = 4V_1$

$$V_2 = 4(0.178)$$

$$= 0.712 \text{ m}^3$$

$$R = (4)^{\frac{1}{3}} = 1.587$$

mixing of fluids
The dimensions of the larger agitator and tank are:

\[ D_{A2} = RDA_1 = 1.587 \times 0.203 = 0.322 \text{ m} \]

\[ D_{T2} = RDT_1 = 1.587 \times 0.61 = 0.968 \text{ m} \]

For equal mass transfer rate \( n = \frac{2}{3} \).

\[ N_2 = N_1 \left( \frac{1}{R} \right)^\frac{2}{3} \]

\[ = 4.58 \left( \frac{1}{1.587} \right)^\frac{2}{3} \]

\[ = 3.37 \text{ rev/sec} \]

The Reynolds number \( N_{Re} \) is

\[ N_{Re} = \frac{\rho N_2^2 A_2^2}{\mu} \]

\[ = \frac{(909)(3.37)(0.322)^2}{0.02} \left\{ \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{rev}}{\text{sec}} \cdot \frac{\text{m}^2}{\text{m} \cdot \text{sec}} \cdot \frac{\text{m}}{\text{kg}} \right\} \]

\[ = 15,880.9 \]

\( N_{Re} \approx 16,000 \)

Using curve 6 in Figure 8-14, \( N_p = 6.0 \). Power required by the agitator is \( P_2 = N_p \rho N_2^3 D_{A2}^5 \)

\[ P_2 = (6.0)(909)(3.37)^3(0.322)^5 \left\{ \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{rev}^3}{\text{sec}^3} \cdot \frac{\text{m}^5}{\text{m}^5} \right\} \]

\[ P_2 = 722.57 \text{ W} \]

\[ = 0.723 \text{ kW (0.97 hp)} \]
The power per unit volume $P/V$ is:

$$\frac{P_2}{V_2} = \frac{0.723}{0.712} = 1.015 \text{ kW/m}^3$$

**MIXING TIME SCALE-UP**

Predicting the time for obtaining concentration uniformity in a batch mixing operation can be based on model theory. Using the appropriate dimensionless groups of the pertinent variables, a relationship can be developed between mixing times in the model and large-scale systems for geometrically similar equipment.

Consider the mixing in both small and large-scale systems to occur in the turbulent region, designated as S and L respectively. Using the Norwood and Metzner's correlation [26], the mixing time for both systems is

$$\frac{t_S(N_SD_S^2D_{AS}^{1/2})^{2/3}}{H_L^{1/2}D_T^{3/2}} = \frac{t_L(N_LD_{AL}^2D_{AL}^{1/2})^{2/3}}{H_T^{1/2}D_T^{3/2}}$$

Applying the scale-up rule of equal mixing times, and rearranging Equation 8-43, yields

$$\left(\frac{N_L}{N_S}\right)^{2/3} = \left(\frac{D_{TL}}{D_{TS}}\right)^{3/2}\left(\frac{D_{AS}}{D_{AL}}\right)^{4/7}\left(\frac{D_{AS}}{D_{AL}}\right)^{1/2}\left(\frac{H_L}{H_S}\right)^{1/2}$$

Assuming geometric similarity,

$$\frac{H_L}{H_S} = \frac{D_{AL}}{D_{AS}}$$

$$\frac{D_{TL}}{D_{TS}} = \frac{D_{AL}}{D_{AS}}$$
Substituting Equations 8-45 and 8-46 into Equation 8-44 gives

\[
\left( \frac{N_L}{N_S} \right)^{\frac{2}{3}} = \left( \frac{D_{AL}}{D_{AS}} \right)^{\frac{3}{2}} \left( \frac{D_{AS}}{D_{AL}} \right)^{\frac{4}{3}} \left( \frac{D_{AS}}{D_{AL}} \right)^{\frac{1}{2}} \left( \frac{D_{AL}}{D_{AS}} \right)^{\frac{1}{2}}
\]  

(8-47)

\[
\left( \frac{N_L}{N_S} \right)^{\frac{2}{3}} = \left( \frac{D_{AL}}{D_{AS}} \right)^{\frac{1}{6}}
\]

or

\[
\left( \frac{N_L}{N_S} \right) = \left( \frac{D_{AL}}{D_{AS}} \right)^{\frac{1}{4}}
\]  

(8-48)

The exponent \( n \) for the mixing time scale-up rule is 0.25.

The power \( P \) of the agitator for both large and small systems is

\[
\frac{P_L}{\rho N_L^2 D_{AL}^5} = \frac{P_S}{\rho N_S^3 D_{AS}^5}
\]  

(8-49)

where

\[
\frac{P_L}{P_S} = \left( \frac{N_L}{N_S} \right)^{3} \left( \frac{D_{AL}}{D_{AS}} \right)^{5}
\]  

(8-50)

Substituting Equation 8-48 into Equation 8-50 yields

\[
\frac{P_L}{P_S} = \left( \frac{D_{AL}}{D_{AS}} \right)^{0.75} \left( \frac{D_{AL}}{D_{AS}} \right)^{5}
\]  

(8-51)

or

\[
\frac{P_L}{P_S} = \left( \frac{D_{AL}}{D_{AS}} \right)^{5.75}
\]  

(8-52)
The power per unit volume $P/V$ for both large- and small-scale systems is:

$$\frac{P_L}{V_L} = \frac{P_L}{\pi D_{TL}^3} \quad \frac{P_S}{V_S} = \frac{P_S}{\pi D_{TS}^3}$$

$$= \frac{P_L}{P_S} \left( \frac{D_{TS}}{D_{TL}} \right)^3$$  \hspace{1cm} (8-53)

Substituting Equations 8-46 and 8-52 into Equation 8-53 gives

$$\left( \frac{P/V}{V} \right)_L = \left( \frac{D_{AL}}{D_{AS}} \right)^{5.75} \left( \frac{D_{AS}}{D_{AL}} \right)^3$$

$$= \left( \frac{D_{AL}}{D_{AS}} \right)^{2.75}$$  \hspace{1cm} (8-54)

Table 8-7 summarizes correlations for the effects of equipment size on the rotational speed needed for the same mixing time by various investigators.

The relationships in Table 8-7 show that the rotational speed to obtain the same batch mixing time is changed by a small power of the increase in linear equipment dimension as equipment size is changed. Equation 8-49 shows that greater power is required for a large-scale system compared to a smaller system. Often, the power required for a larger system may be prohibitive, thus modification of the scale-up rule is needed (e.g., $t_L = 10t_S$ or $t_L = 100t_S$) to obtain a lower power requirement. It should be noted that relaxation of mixing time requirements may not pose other problems. For example, if the mixing is accompanied by a chemical reaction in a CSTR, assuming that the Norwood-Metzner [26] correlation for mixing time ($t$) is still applicable, it must be ensured that the mixing time in the larger scale ($t_L = 10t_S$ or $t_L = 100t_S$) is less than 5% of the average residence time of the liquids in the reactor, otherwise the conversion
## Table 8-7

Effect of equipment size on rotational speed needed for the same mixing time

<table>
<thead>
<tr>
<th>Relationship between N and D</th>
<th>Equipment</th>
<th>∆ρ</th>
<th>Equation</th>
<th>Investigator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N \propto D^{-1/6} )</td>
<td>Propellers, no baffles</td>
<td>Not zero</td>
<td>( \left( \frac{\theta ND^2 p}{V} \right) \left( \frac{\rho D^2 N^2}{g Z_1 \Delta \rho} \right)^{0.25} = 9 )</td>
<td>van de Vusse [17]</td>
</tr>
<tr>
<td>( N \propto D^{-0.1 \text{ to } -0.2} )</td>
<td>Paddles, turbines</td>
<td>Not zero</td>
<td>( \left( \frac{\theta Q}{V} \right) \propto \left( \frac{\rho D^2 N^2}{g Z_1 \Delta \rho} \right)^{-0.3} )</td>
<td>van de Vusse [17]</td>
</tr>
<tr>
<td>( N = \text{constant} )</td>
<td>Propellers, paddles, turbines</td>
<td>Zero</td>
<td>( \theta = \frac{C[Z_L^{0.2} \tau]}{N_{Re}^2 \left( ND^2 \right)^{1/6} \frac{g^{4/6}}{e^{4/6}}} )</td>
<td>Fox and Gex [18]</td>
</tr>
<tr>
<td>( N \propto D^{1/5} )</td>
<td>Propellers</td>
<td>Zero</td>
<td>( \text{Zero} )</td>
<td>Norwood and Metzner [26]</td>
</tr>
<tr>
<td>( N \propto D^{1/4} )</td>
<td>Turbines</td>
<td>Zero</td>
<td>( \text{Zero} )</td>
<td></td>
</tr>
</tbody>
</table>

The power per unit volume $P/V$ for both large- and small-scale systems is:

$$\frac{P_L/V_L}{P_S/V_S} = \frac{P_L/\pi D_{TL}^3}{P_S/\pi D_{TS}^3} = \frac{P_L}{P_S} \left( \frac{D_{TS}}{D_{TL}} \right)^3$$

Substituting Equations 8-46 and 8-52 into Equation 8-53 gives

$$\frac{(P/V)_L}{(P/V)_S} = \left( \frac{D_{AL}}{D_{AS}} \right)^{5.75} \left( \frac{D_{AS}}{D_{AL}} \right)^3$$

$$= \left( \frac{D_{AL}}{D_{AS}} \right)^{2.75}$$

Table 8-7 summarizes correlations for the effects of equipment size on the rotational speed needed for the same mixing time by various investigators.

The relationships in Table 8-7 show that the rotational speed to obtain the same batch mixing time is changed by a small power of the increase in linear equipment dimension as equipment size is changed. Equation 8-49 shows that greater power is required for a large-scale system compared to a smaller system. Often, the power required for a larger system may be prohibitive, thus modification of the scale-up rule is needed (e.g., $t_L = 10t_S$ or $t_L = 100t_S$) to obtain a lower power requirement. It should be noted that relaxation of mixing time requirements may not pose other problems. For example, if the mixing is accompanied by a chemical reaction in a CSTR, assuming that the Norwood-Metzner [26] correlation for mixing time ($t$) is still applicable, it must be ensured that the mixing time in the larger scale ($t_L = 10t_S$ or $t_L = 100t_S$) is less than 5% of the average residence time of the liquids in the reactor, otherwise the conversion...
scale-up chart only applies to systems of similar geometry. When the geometry is different, special and specific analyses of the system are required.

Samant and Ng [28] compared various scale-up rules for agitated reactors. They suggested that a scale-up rule of power per unit volume and constant average residence time (where the power per unit volume and average residence time cannot be increased) is the most suited in many operations. However, this still may not improve or preserve the performance of the systems. Therefore, adequate consideration must be given to a tradeoff between performance and operating constraints.

REFERENCES