EXTERNAL FORCED CONVECTION

we considered the general and theoretical aspects of forced convection, with emphasis on differential formulation and analytical solutions. In this chapter we consider the practical aspects of forced convection to or from flat or curved surfaces subjected to external flow, characterized by the freely growing boundary layers surrounded by a free flow region that involves no velocity and temperature gradients.

PARALLEL FLOW OVER FLAT PLATES

Consider the parallel flow of a fluid over a flat plate of length $L$ in the flow direction, as shown in Figure.

The transition from laminar to turbulent flow depends on the surface geometry, surface roughness, upstream velocity, surface temperature, and the type of fluid, among other things, and is best characterized by the Reynolds number. The Reynolds number at a distance $x$ from the leading edge of a flat plate is expressed as

$Re_x = \frac{\rho v_x}{\mu}$

For flow over a flat plate, transition from laminar to turbulent is usually taken to occur at the critical Reynolds number of

$Re_{xc} = \frac{\rho V_{xc} \mu}{2} = 5 \times 10^5$

Friction Coefficient: Based on analysis, the boundary layer thickness

and the local friction coefficient at location $x$ for laminar flow over a flat plate

$C_f = \frac{1}{L} \int_0^L C_{f,x} \, dx \text{ where } C_f: \text{average friction coefficient}$

$drag \ force \ and \ heat \ transfer \ rate \ F_D = C_f A \frac{\rho v_x^2}{2} \text{ where } F_D \ drag \ force$

Laminar: $\delta_{x,x} = \frac{5x}{Re_x^{1/2}} \text{ and } C_{f,x} = \frac{0.664}{Re_x^{1/2}}, \ Re_x < 5 \times 10^5$

The corresponding relations for turbulent flow are

Turbulent: $\delta_{x,x} = \frac{0.382x}{Re_x^{1/5}} \text{ and } C_{f,x} = \frac{0.0592}{Re_x^{1/5}}, \ 5 \times 10^5 \leq Re_x \leq 10^7$

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the average friction coefficient over the entire plate is determined by performing the integration over two parts: the laminar region $0 < x < x_{cr}$ and the turbulent region $x_{cr} < x < L$ as

$C_f = \frac{1}{L} \int_0^{x_{cr}} C_{f,x,Laminar} \, dx + \int_{x_{cr}}^L C_{f,x,Turbulent} \, dx$

Heat Transfer Coefficient:

$Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{0.5} Pr^{1/3} \quad (Pr \geq 0.6)$

The corresponding relation for turbulent flow is

$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{4/5} Pr^{1/3} \quad (0.6 \leq Pr \leq 60)$

$h = \frac{1}{L} \int_0^L h_x \, dx \text{ where } h: \text{average heat transfer coefficient}$
The average Nusselt number over the entire plate is determined by substituting the relations above and performing the integrations. We get

\[ \text{Laminar: } Nu = \frac{hL}{k} = 0.664 \left( \frac{ReL^{0.5} Pr^{1/3}}{Pr} \right) \quad (Pr \geq 0.6) \]

The average fraction factor and Nusselt number over entire plate.

\[ \text{Turbulent: } Nu = \frac{hL}{k} = 0.0296 \left( \frac{ReL^{4/5} Pr^{1/3}}{Pr} \right) \quad (0.6 \leq Pr \leq 60) \quad \text{and} \quad 5 \times 10^5 \leq ReL \leq 10^7 \]

The first relation gives the average heat transfer coefficient for the entire plate when the flow is laminar over the entire plate. The second relation gives the average heat transfer coefficient for the entire plate only when the flow is turbulent over the entire plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region.

**Combined Laminar and Turbulent flow over flat plates**

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the average heat transfer coefficient over the entire plate is determined by

\[
h = \frac{1}{L} \int_0^{x_{cr}} h_{x,\text{Laminar}} \, dx + \int_{x_{cr}}^L h_{x,\text{Turbulent}} \, dx
\]

Again taking the critical Reynolds number to be \( Re_{cr} = 5 \times 10^5 \) and performing the integrations in above Eq. after substituting the indicated expressions, the average Nusselt number over the entire plate is determined to be (Fig.)

\[ Nu = \frac{hL}{k} = (0.037 ReL^{4/5} - 871) Pr^{1/3} \quad (0.6 \leq Pr \leq 60) \quad \text{and} \quad 5 \times 10^5 \leq ReL \leq 10^7 \]

The average fraction factor

\[
C_f = \frac{0.074}{ReL^{1/5}} - \frac{1742}{ReL} \left( 5 \times 10^5 \leq ReL \leq 10^7 \right)
\]

**Flat Plate with Unheated Starting Length**

So far we have limited our consideration to situations for which the entire plate is heated from the leading edge. But many practical applications involve surfaces with an unheated starting section of length \( \xi \), shown in Figure and thus there is no heat transfer for \( 0 < x < \xi \). In such cases, the velocity boundary layer starts to develop at the leading edge \( (x = 0) \), but the thermal boundary layer starts to develop where heating starts \( (x = \xi) \). Consider a flat plate whose heated section is maintained at a constant temperature \( T = T_s \) constant for \( x > \xi \). Using integral solution methods (see Kays and Crawford, 1994), the local Nusselt numbers for both laminar and turbulent flows are determined to be

\[
\text{Laminar: } Nu_x = \frac{2(1 - (\xi/x)^{1/4})}{1 - (\xi/x)^{1/4}}
\]

\[
\text{Turbulent: } Nu_x = \frac{5(1 - (\xi/x)^{1/4})}{1 - (\xi/x)^{1/4}}
\]

for \( x > \xi \). Note that for \( \xi = 0 \), these \( Nu_x \) relations reduce to \( Nu_x \) (for \( \xi = 0 \)), which is the Nusselt number relation for a flat plate without an unheated starting length. Therefore, the terms in brackets in the denominator serve as correction factors for plates with unheated starting lengths.

The determination of the average Nusselt number for the heated section of a plate requires the integration of the local Nusselt number relations above, which cannot be done analytically. Therefore, integrations must be done numerically. The results of numerical integrations have been correlated for the average convection coefficients [Thomas, (1977) Ref. 11] as
Uniform Heat Flux: When a flat plate is subjected to *uniform heat flux* instead of uniform temperature, the local Nusselt number is given by

**Laminar:**
\[
Nu_x = 0.453 \text{Re}_x^{0.5} \text{Pr}^{1/3} \quad \text{Eq.c}
\]

**Turbulent:**
\[
Nu_x = 0.0308 \text{Re}_x^{0.8} \text{Pr}^{1/3} \quad \text{Eq.d}
\]

These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case. When the plate involves an unheated starting length, the relations developed for the uniform surface temperature case can still be used provided that Eqs. c and d are used for \( Nu_x \) (for \( \xi = 0 \)) in Eqs. a and b, respectively.

When heat flux \( q' \) is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance \( x \) are determined from

\[
\dot{Q} = \dot{q}_x A_x
\]

and

\[
\dot{q}_x = h_x [T_f(x) - T_x] \quad \rightarrow \quad T_f(x) = T_x + \frac{\dot{q}_x}{h_x}
\]

where \( A_x \) is the heat transfer surface area.

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**EXAMPLE 1: Flow of Hot Oil over a Flat Plate**

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s. Determine the total drag force and the rate of heat transfer per unit width of the entire plate. The properties of engine oil at the film temperature of \( T_f = \frac{(T_s + T_\infty)}{2} = \frac{(20 + 60)}{2} = 40^\circ C \) are \( \rho = 867 \text{ Kg/m}^3 \quad \text{Pr}=2870 \quad k=0.144 \text{ W/m.s} \quad \nu=242 \times 10^{-6} \text{ m}^2/\text{s} \)

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**EXAMPLE 2: Cooling of a Hot Block by Forced Air at High Elevation**

The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5 m X 6 m flat plate whose temperature is 140°C. Determine the rate of heat transfer from the plate if the air flows parallel to the (a) 6-m-long side and (b) the 1.5-m side. \( T_f = \frac{(T_s + T_\infty)}{2} = \frac{(140 + 20)}{2} = 80^\circ C \) are \( \rho = 867 \text{ Kg/m}^3 \quad \text{Pr}=0.7154 \quad k=0.02953 \text{ W/m.s} \quad 2.097 \times 10^{-5} \text{ m}^2/\text{s} \). The atmospheric pressure in Denver is \( P = (83.4 \text{ kPa})/(101.325 \text{ kPa/atm}) = 0.823 \text{ atm} \). Then the kinematic viscosity of air in Denver becomes \( \nu = \nu \text{ at } 1 \text{ atm} / P = (2.097 \times 10^{-5} \text{ m}^2/\text{s})/0.823 = 2.548 \times 10^{-5} \text{ m}^2/\text{s} \).
Flow across cylinders and spheres is frequently encountered in practice. For example, the tubes in a shell-and-tube heat exchanger involve both internal flow through the tubes and external flow over the tubes, and both flows must be considered in the analysis of the heat exchanger. Also, many sports such as soccer, tennis, and golf involve flow over spherical balls.

Note: The characteristic length for a circular cylinder or sphere is taken to be the external diameter $D$. Thus, the Reynolds number is defined as $Re = \frac{V_\infty D}{v}$ where $V_\infty$ is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $Re_{cr} = 2 \times 10^5$. That is, the boundary layer remains laminar for about $Re \leq 2 \times 10^5$ and becomes turbulent for $Re \geq 2 \times 10^5$.

The characteristic length for a circular cylinder or spheres is taken to external diameter $D$ and Reynolds number as:

$$Re = \frac{V_\infty D}{v}$$

$$F_D = C_D A_N \frac{\rho V_\infty^2}{2} \quad \text{where } F_D \text{ drag force and } A_N$$

$$= LD \text{ in cylinder and } A_N$$

$$= \frac{1}{4} \pi D^2 \text{ for spheres}$$

Nusselt number for cross flow over circular cylinder when $Re \ Pr > 0.2$ and all properties at film temperature $T_f$:

$$Nu_{cyl} = \frac{h D}{k} = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

Nusselt number for flow over sphere when $3.5 \leq Re \leq 80000$ and $0.7 > Pr > 380$ all properties $T_\infty$ except $\mu_s$ at $T_s$:

$$Nu_{sph} = \frac{h D}{k} = 2 + \left[0.4 Re^{1/2} + 0.06 Re^{2/3} Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)\right]^{1/4}$$