Lesson 1 Chapter Four
Effect of Source Inductance on the Performance of
AC to DC Converters

1. Single phase fully controlled converter with source inductance

Fig.1(a) shows a single phase fully controlled converter with source inductance. For simplicity it has been assumed that the converter operates in the continuous conduction mode. Further, it has been assumed that the load current ripple is negligible and the load can be replaced by a dc current source the magnitude of which equals the average load current. Fig.1(b) shows the corresponding waveforms. It is assumed that the thyristors $T_3$ and $T_4$ were conducting at $t = 0$. $T_1$ and $T_2$ are fired at $\omega t = \alpha$. If there were no source inductance $T_3$ and $T_4$ would have commutated as soon as $T_1$ and $T_2$ are turned ON. The input current polarity would have changed instantaneously. However, if a source inductance is present the commutation and change of input current polarity can not be instantaneous. Therefore, when $T_1$ and $T_2$ are turned ON $T_3$ and $T_4$ does not commutate immediately. Instead, for some interval all four thyristors continue to conduct as shown in Fig. 1(b). This interval is called “overlap” interval.
During this period the load current freewheels through the thyristors and the output voltage is clamped to zero. On the other hand, the input current starts changing polarity as the current through $T_1$ and $T_2$ increases and $T_3$ $T_4$ current decreases. At the end of the overlap interval the current through $T_3$ and $T_4$ becomes zero and they commutate, $T_1$ and $T_2$ starts conducting the full load current. The same process repeats during commutation from $T_1$ $T_2$ to $T_3$ $T_4$ at $\omega t = \pi + \alpha$. From Fig.1(b) it is clear that, commutation overlap not only reduces average output dc voltage but
also reduces the extinction angle $\gamma$ which may cause commutation failure in the inverting mode of operation if $\alpha$ is very close to 180º. In the following analysis an expression of the overlap angle “$\mu$” will be determined.

**Fig. 2: Equivalent circuit during overlap period**

From the equivalent circuit of the converter during overlap period

\[
\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{V}{L}, \quad \text{for} \quad \alpha \leq \omega t \leq \alpha + \mu \tag{1}
\]

\[i(\omega t = \alpha) = -I_0 \tag{2}\]

\[i = \frac{\sqrt{2}V}{\omega L} \cos \omega t \tag{3}\]

\[i|_{\omega t = \alpha} = \frac{\sqrt{2}V}{\omega L} \cos \alpha = -I_b \tag{4}\]

\[I = \frac{\sqrt{2}V}{\omega L} \cos \alpha \cdot I_b \tag{5}\]

\[i = \frac{\sqrt{2}V}{\omega L} (\cos \alpha \cdot \cos(\omega t)) \cdot I_b \tag{6}\]

\[\text{at} \quad \omega t = \alpha + \mu \quad i = I_0 \]

\[I_b = \frac{\sqrt{2}V}{\omega L} (\cos \alpha \cdot \cos(\alpha + \mu)) \cdot I_b \tag{7}\]

\[\cos \alpha \cdot \cos(\alpha + \mu) = \frac{\sqrt{2}V}{V_i} I_b \tag{8}\]

\[V_b = \frac{1}{\pi} \int_0^\pi V_i \, \mathrm{d}t \tag{9}\]

\[V_b = \frac{1}{\pi} \int_0^\pi \sqrt{2}V \sin \omega t \, \mathrm{d}t \quad \text{or} \]

\[= \frac{\sqrt{2}V}{\pi} \left[ \cos(\alpha + \mu) - \cos(\pi + \alpha) \right] \]

\[= \frac{\sqrt{2}V}{\pi} \left[ \cos \alpha \cdot \cos(\alpha + \mu) \right] \tag{10}\]

\[V_b = 2\sqrt{2} \frac{V_i}{\pi} \cos \alpha - \frac{\sqrt{2}V}{\pi} \left[ \cos \alpha - \cos(\alpha + \mu) \right] \]

\[= \frac{2\sqrt{2}V}{\pi} \cos \alpha - \frac{2}{\pi} \omega L I_b \tag{11}\]
Equation 11 can be represented by the following equivalent circuit

Fig. 3: Equivalent circuit representation of the single phase fully controlled rectifier with source inductance

The simple equivalent circuit of Fig. 3 represents the single phase fully controlled converter with source inductance as a practical dc source as far as its average behavior is concerned. The open circuit voltage of this practical source equals the average dc output voltage of an ideal converter (without source inductance) operating at a firing angle of $\alpha$. The voltage drop across the internal resistance “$R_c$” represents the voltage lost due to overlap shown in Fig.1(b) by the hatched portion of the $v_o$ waveform. Therefore, this is called the “Commuation resistance”. Although this resistance accounts for the voltage drop correctly there is no power loss associated with this resistance since the physical process of overlap does not involve any power loss. Therefore this resistance should be used carefully where power calculation is involved.

2. Three phase fully controlled converter with source inductance

When the source inductance is taken into account, the qualitative effects on the performance of the converter is similar to that in the case of a single phase converter. Fig.4(a) shows such a converter. As in the case of a single phase converter the load is assumed to be highly inductive such that the load can be replaced by a current source.
As in the case of a single phase converter, commutations are not instantaneous due to the presence of source inductances. It takes place over an overlap period of “μ₁” instead. During the overlap period three thyristors instead of two conducts. Current in the outgoing thyristor gradually decreases to zero while the incoming thyristor current increases and equals the total load current at the end of the overlap period. If the duration of the overlap period is greater than 60° four thyristors may also conduct clamping the output voltage to zero for some time. However, this situation is not very common and will not be discussed any further in this lesson. Due to the conduction of two devices during commutation either from the top group or the bottom group the instantaneous output voltage during the overlap period drops (shown by the hatched portion of Fig.4 (b)) resulting in reduced average voltage. The exact amount of this reduction can be calculated as follows.

In the time interval $\alpha < \omega t \leq \alpha + \mu$, $T_6$ and $T_2$ from the bottom group and $T_1$ from the top group conducts. The equivalent circuit of the converter during this period is given by the circuit diagram of Fig.5.
Fig. 5: Equivalent circuit during commutation from T6 to T2

Therefore, in the interval $\alpha < \omega t \leq \alpha + \mu$

\[
v_b = L \frac{di_b}{dt} - L \frac{di_c}{dt} + v_i \tag{12}
\]

or,

\[
v_{bc} = L \frac{d}{dt}(i_b - i_c) \tag{13}
\]

but $i_b + i_c + I_o = 0$.

\[
2L \frac{di_b}{dt} = v_{bc} = \sqrt{2}V_L \sin \omega t \tag{14}
\]

\[
. = i_b = C - \frac{\sqrt{2}V_L}{2\omega L} \cos \omega t \tag{15}
\]

\[
. = i_b = \frac{\sqrt{2}V_L}{2\omega L} (\cos \alpha - \cos \omega t) - I_o \tag{16}
\]

at $\omega t = \alpha$, $i_b = - I_0$.

\[
. = i_b = \frac{\sqrt{2}V_L}{2\omega L} (\cos \alpha - \cos \omega t) - I_o \tag{17}
\]

at $\omega t = \alpha + \mu$, $i_b = 0$.

\[
. = \frac{\sqrt{2}V_L}{2\omega L} (\cos \alpha - \cos (\alpha + \mu)) = I_0 \tag{18}
\]

\[
. = \cos \alpha - \cos (\alpha + \mu) = \frac{\sqrt{2}V_L}{V_L} I_0 \tag{19}
\]

Or,

\[
I_0 \leq \frac{V_L}{\sqrt{2}\omega L} \cos \left(\alpha - \frac{\omega t}{3}\right) \tag{20}
\]

Equation 20 holds for $\mu \leq 60^\circ$. It can be shown that for this condition to be satisfied

\[
\frac{V_L}{\sqrt{2}\omega L} \cos \left(\alpha - \frac{\omega t}{3}\right) \tag{21}
\]

To calculate the dc voltage

For $\alpha \leq \omega t \leq \alpha + \mu$

\[
v_o = v_a - v_b + L \frac{di_b}{dt} = \frac{3}{2}v_i \tag{22}
\]

\[
\alpha + \mu \leq \omega t \leq \alpha + \frac{\pi}{3} \quad v_o = v_{ac}
\]
\[ V_0 = \frac{3}{\pi} \int_{a}^{\alpha_b} \left( \frac{3}{2} v_s \right) \, d\phi + \int_{a}^{\alpha_b} v_{se} \, d\phi \]

\[ = \frac{3}{\pi} \int_{a}^{\alpha_b} \left( v_s + \frac{3}{2} v_s \cdot v_s \right) \, d\phi + \int_{a}^{\alpha_b} v_{se} \, d\phi \]

\[ = \frac{3}{\pi} \left( \int_{a}^{\alpha_b} v_{se} \, d\phi + \int_{a}^{\alpha_b} \left( \frac{v_s}{2} + v_s \right) \, d\phi \right) \]

\[ = \frac{3\sqrt{2}}{\pi} V_L \cos \alpha - \frac{3}{2\pi} \int_{a}^{\alpha_b} v_{se} \, d\phi \]

or

\[ V_0 = \frac{3\sqrt{2}}{\pi} V_L \cos \alpha - \frac{3\sqrt{2} V_L}{2\pi} \int_{a}^{\alpha_b} \sin \phi \, d\phi \]

\[ = \frac{3\sqrt{2}}{\pi} V_L \cos \alpha - \frac{3\sqrt{2} V_L}{2\pi} \left[ \cos \alpha - \cos(\alpha + \mu) \right] \]  

(23)

Substituting Equation 20 into 24

\[ V_0 = \frac{3\sqrt{2}}{\pi} V_L \cos \alpha - \frac{3}{\pi} \cos \alpha L \]

(25)

Equation 25 suggests the same dc equivalent circuit for the three phase converter with source inductance as shown in Fig. 3 with

\[ V_{oc} = \frac{3\sqrt{2}}{\pi} V_L \cos \alpha \]

and commutation resistance

\[ R_C = \frac{3}{\pi} \omega L. \]

It should be noted that \( R_C \) is a “loss less” resistance, since the overlap process does not involve any active power loss.

**Exercise 1**

1. Fill in the blank(s) with appropriate word(s)

   i. The internal impedance of an ac source supplying a converter is largely __________ in nature.

   ii. Due to the presence of source __________ commutation in a converter is not __________.

   iii. The period over which the commutation process continues is called the __________ period.

   iv. Length of the overlap period depends on the value of the source inductance and load __________.

   v. In a single phase converter __________ thyristors conduct during the overlap period.

   vi. In a three phase converter __________ thyristors conduct during the overlap period provided the overlap angle is less than __________.
degrees.

vii. The average output voltage of a ac-dc converter ______________ as a result of commutation overlap.

viii. In the dc equivalent circuit of a converter the input ac source inductor appears as a loss less resistance called the ______________ resistance.

ix. Commutation overlap decreases the ______________ angle of a converter and may cause commutation failure during ______________ mode of operation.

x. Commutation overlap introduces ______________ in the supply voltage waveform.

**Answer:** (i) inductive; (ii) inductance, instantaneous; (iii) overlap; (iv) current; (v) four; (vi) three, sixty; (vii) decreases; (viii) commutation; (ix) inverter, (x) notches.

2. A 220V, 1450 RPM, 100A separately excited dc motor has an armature resistance to 0.1Ω. It is supplied from a 3 phase fully controlled converter connected to a 3 phase 50 Hz ac source. The ac source has an inductive reactance of 0.5Ω at 50 Hz. The line voltage is adjusted such that at α = 0; the motor operates at rated speed and torque. The motor is to be braked regeneratively in the reverse direction at rated speed using the converter. What is the maximum braking torque the motor will be able to produce under this condition without causing commutation failure?

**Answer:** Under rated operating condition, the motor terminal voltage is 220V and it draws 100 Amps current. Therefore from eqn. 25.

\[ 220 = \frac{3\sqrt{2}}{\pi} V_L - \frac{3}{\pi} \times .5 \times 100 \]

or \( V_L = 198 \) volts \( E_b |_{\text{rated speed}} = 220 - 100 \times 0.1 = 210V \)

Under regenerative braking in the reverse direction at rated speed

\[ \frac{3\sqrt{2}}{\pi} \times 198 \cos \alpha - \left( \frac{3}{\pi} \times 0.5 + 0.1 \right) I_o = -210V \]

Also from equation 15.20

\[ \cos \alpha - \cos (\alpha + \mu) = \frac{\sqrt{2} \times 0.5}{198} I_o \]

At the limiting condition of commutation failure

\[ \alpha + \mu \approx 180^\circ \]

\[ \therefore \cos \alpha = \frac{I_o}{198\sqrt{2}} - 1 \]
\[
\therefore \frac{3}{\pi} I_o - \frac{3\sqrt{2}}{\pi} \times 198 - \left( \frac{3}{\pi} \times 0.5 + 0.1 \right) I_o = -210
\]

or \[0.377 I_o = 57.4 \quad \therefore I_o = 152.24 \text{ Amps}\]

∴ Maximum braking torque will be approximately 150% of the rated motor torque.