CHAPTER 16

Drying

16.1. INTRODUCTION

The drying of materials is often the final operation in a manufacturing process, carried out immediately prior to packaging or dispatch. Drying refers to the final removal of water, or another solute, and the operation often follows evaporation, filtration, or crystallisation. In some cases, drying is an essential part of the manufacturing process, as for instance in paper making or in the seasoning of timber, although, in the majority of processing industries, drying is carried out for one or more of the following reasons:

(a) To reduce the cost of transport.
(b) To make a material more suitable for handling as, for example, with soap powders, dyestuffs and fertilisers.
(c) To provide definite properties, such as, for example, maintaining the free-flowing nature of salt.
(d) To remove moisture which may otherwise lead to corrosion. One example is the drying of gaseous fuels or benzene prior to chlorination.

With a crystalline product, it is essential that the crystals are not damaged during the drying process, and, in the case of pharmaceutical products, care must be taken to avoid contamination. Shrinkage, as with paper, cracking, as with wood, or loss of flavour, as with fruit, must also be prevented. With the exception of the partial drying of a material by squeezing in a press or the removal of water by adsorption, almost all drying processes involve the removal of water by vaporisation, which requires the addition of heat. In assessing the efficiency of a drying process, the effective utilisation of the heat supplied is the major consideration.

16.2. GENERAL PRINCIPLES

The moisture content of a material is usually expressed in terms of its water content as a percentage of the mass of the dry material, though moisture content is sometimes expressed on a wet basis, as in Example 16.3. If a material is exposed to air at a given temperature and humidity, the material will either lose or gain water until an equilibrium condition is established. This equilibrium moisture content varies widely with the moisture content and the temperature of the air, as shown in Figure 16.1. A non-porous insoluble solid, such as sand or china clay, has an equilibrium moisture content approaching zero.
for all humidities and temperatures, although many organic materials, such as wood, textiles, and leather, show wide variations of equilibrium moisture content. Moisture may be present in two forms:

*Bound moisture.* This is water retained so that it exerts a vapour pressure less than that of free water at the same temperature. Such water may be retained in small capillaries, adsorbed on surfaces, or as a solution in cell walls.

*Free moisture.* This is water which is in excess of the equilibrium moisture content.

The water removed by vaporisation is generally carried away by air or hot gases, and the ability of these gases to pick up the water is determined by their temperature and humidity. In designing dryers using air, the properties of the air–water system are essential, and these are detailed in Volume 1, Chapter 13, where the development of the humidity chart is described. For the air–water system, the following definitions are of importance:

*Humidity* \( \mathcal{H} \), mass of water per unit mass of dry air.

\[
\text{moles of water vapour} \quad \frac{\text{moles of dry air}}{P_w} = \frac{P_w}{(P - P_w)}
\]

then:

\[
\mathcal{H} = \frac{18P_w}{29(P - P_w)}
\]

where \( P_w \) is the partial pressure of water vapour and \( P \) is the total pressure.

*Humidity of saturated air* \( \mathcal{H}_0 \). This is the humidity of air when it is saturated with water vapour. The air then is in equilibrium with water at the given temperature and pressure.

*Percentage humidity*

\[
= \frac{\text{Humidity of air}}{\text{Humidity of saturated air}} \times 100 = \frac{\mathcal{H}}{\mathcal{H}_0} \times 100
\]
**Percentage relative humidity, \( \mathcal{R} \)**

\[
\mathcal{R} = \frac{\text{Partial pressure of water vapour in air}}{\text{Vapour pressure of water at the same temperature}} \times 100
\]

The distinction between percentage humidity and percentage relative humidity is of significance though, the difference in the values of the two quantities does not usually exceed 7 to 8 per cent. Reference may be made here to Volume 1, Section 13.2.1.

**Humid volume**, is the volume of unit mass of dry air and its associated vapour. Then, under ideal conditions, at atmospheric pressure:

\[
\text{humid volume} = \frac{22.4}{29} \left( \frac{T}{273} \right) + \frac{22.4}{18} \left( \frac{T}{273} \right) \text{ m}^3/\text{kg}
\]

where \( T \) is in degrees K,

or :

\[
\frac{359}{29} \left( \frac{T}{492} \right) + \frac{359}{18} \left( \frac{T}{492} \right) \text{ ft}^3/\text{lb}
\]

where \( T \) is in degrees Rankine.

**Saturated volume** is the volume of unit mass of dry air, together with the water vapour required to saturate it.

**Humid heat** is the heat required to raise unit mass of dry air and associated vapour through 1 degree K at constant pressure or 1.00 + 1.88\( \mathcal{R} \) kJ/kg K.

**Dew point** is the temperature at which condensation will first occur when air is cooled.

**Wet bulb temperature**. If a stream of air is passed rapidly over a water surface, vaporisation occurs, provided the temperature of the water is above the dew point of the air. The temperature of the water falls and heat flows from the air to the water. If the surface is sufficiently small for the condition of the air to change inappreciably and if the velocity is in excess of about 5 m/s, the water reaches the wet bulb temperature \( \theta_w \) at equilibrium.

The rate of heat transfer from gas to liquid is given by:

\[
Q = hA(\theta - \theta_w)
\]

(16.1)

The mass rate of vaporisation is given by:

\[
G_v = \frac{hDAM_w}{RT}(P_{w0} - P_w) = \frac{hDAM_w}{RT}[(P - P_w)_{\text{mean}}(\mathcal{H}_w - \mathcal{H})] = hDAP_A(\mathcal{H}_w - \mathcal{H})
\]

(16.2)

The rate of heat transfer required to effect vaporisation at this rate is given by:

\[
G_v = hDAP_A(\mathcal{H}_w - \mathcal{H})\lambda
\]

(16.3)
At equilibrium, the rates of heat transfer given by equations 16.1 and 16.3 must be equal, and hence:

\[ H - H_w = -\frac{h}{h_D \rho_A \lambda} (\theta - \theta_w) \]  

(16.4)

In this way, it is seen that the wet bulb temperature \( \theta_w \) depends only on the temperature and humidity of the drying air.

In these equations:
- \( h \) is the heat transfer coefficient,
- \( h_D \) is the mass transfer coefficient,
- \( A \) is the surface area,
- \( \theta \) is the temperature of the air stream,
- \( \theta_w \) is the wet bulb temperature,
- \( P_{w0} \) is the vapour pressure of water at temperature \( \theta_w \),
- \( M_A \) is the molecular weight of air,
- \( M_w \) is the molecular weight of water,
- \( R \) is the universal gas constant,
- \( T \) is the absolute temperature,
- \( H \) is the humidity of the gas stream,
- \( H_w \) is the humidity of saturated air at temperature \( \theta_w \),
- \( \rho_A \) is the density of air at its mean partial pressure, and
- \( \lambda \) is the latent heat of vaporisation of unit mass of water.

Equation 16.4 is identical with equation 13.8 in Volume 1, and reference may be made to that chapter for a more detailed discussion.

### 16.3. RATE OF DRYING

#### 16.3.1. Drying periods

In drying, it is necessary to remove free moisture from the surface and also moisture from the interior of the material. If the change in moisture content for a material is determined as a function of time, a smooth curve is obtained from which the rate of drying at any given moisture content may be evaluated. The form of the drying rate curve varies with the structure and type of material, and two typical curves are shown in Figure 16.2. In curve 1, there are two well-defined zones: AB, where the rate of drying is constant and BC, where there is a steady fall in the rate of drying as the moisture content is reduced. The moisture content at the end of the constant rate period is represented by point B, and this is known as the critical moisture content. Curve 2 shows three stages, DE, EF and FC. The stage DE represents a constant rate period, and EF and FC are falling rate periods. In this case, the Section EF is a straight line, however, and only the portion FC is curved. Section EF is known as the first falling rate period and the final stage, shown as FC, as the second falling rate period. The drying of soap gives rise to a curve of type 1, and sand to a curve of type 2. A number of workers, including Sherwood\(^1\) and Newitt and co-workers\(^2-7\), have contributed various theories on the rate of drying at these various stages.
During the constant rate period, it is assumed that drying takes place from a saturated surface of the material by diffusion of the water vapour through a stationary air film into the air stream. Gilliland\(^{(8)}\) has shown that the rates of drying of a variety of materials in this stage are substantially the same as shown in Table 16.1.

**Table 16.1. Evaporation rates for various materials under constant conditions\(^{(8)}\)**

<table>
<thead>
<tr>
<th>Material</th>
<th>Rate of evaporation (kg/m(^2) h)</th>
<th>Rate of evaporation (kg/m(^2) s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>2.7</td>
<td>0.00075</td>
</tr>
<tr>
<td>Whiting pigment</td>
<td>2.1</td>
<td>0.00058</td>
</tr>
<tr>
<td>Brass filings</td>
<td>2.4</td>
<td>0.00067</td>
</tr>
<tr>
<td>Brass turnings</td>
<td>2.4</td>
<td>0.00067</td>
</tr>
<tr>
<td>Sand (fine)</td>
<td>2.0–2.4</td>
<td>0.00055–0.00067</td>
</tr>
<tr>
<td>Clays</td>
<td>2.3–2.7</td>
<td>0.00064–0.00075</td>
</tr>
</tbody>
</table>

In order to calculate the rate of drying under these conditions, the relationships obtained in Volume 1 for diffusion of a vapour from a liquid surface into a gas may be used. The simplest equation of this type is:

\[
W = k_G A (P_s - P_w) \tag{16.5}
\]

where \(k_G\) is the mass transfer coefficient.

Since the rate of transfer depends on the velocity \(u\) of the air stream, raised to a power of about 0.8, then the mass rate of evaporation is:

\[
W = k_G A (P_s - P_w) u^{0.8} \tag{16.6}
\]

where: \(A\) is the surface area,  
\(P_s\) is the vapour pressure of the water, and  
\(P_w\) is the partial pressure of water vapour in the air stream.
This type of equation, used in Volume 1 for the rate of vaporisation into an air stream, simply states that the rate of transfer is equal to the transfer coefficient multiplied by the driving force. It may be noted, however, that \((P_s - P_w)\) is not only a driving force, but it is also related to the capacity of the air stream to absorb moisture.

These equations suggest that the rate of drying is independent of the geometrical shape of the surface. Work by Powell and Griffiths\(^9\) has shown, however, that the ratio of the length to the width of the surface is of some importance, and that the evaporation rate is given more accurately as:

(a) For values of \(u = 1–3 \text{ m/s}\):

\[
W = 5.53 \times 10^{-9} L^{0.77} B (P_s - P_w)(1 + 61u^{0.85}) \text{ kg/s} \tag{16.7}
\]

(b) For values of \(u < 1 \text{ m/s}\):

\[
W = 3.72 \times 10^{-9} L^{0.73} B^{0.8} (P_s - P_w)(1 + 61u^{0.85}) \text{ kg/s} \tag{16.8}
\]

where: \(P_s\), the saturation pressure at the temperature of the surface (N/m\(^2\)), \(P_w\), the vapour pressure in the air stream (N/m\(^2\)), and \(L\) and \(B\) are the length and width of the surface, respectively (m).

For most design purposes, it may be assumed that the rate of drying is proportional to the transfer coefficient multiplied by \((P_s - P_w)\). Chakravorty\(^10\) has shown that, if the temperature of the surface is greater than that of the air stream, then \(P_w\) may easily reach a value corresponding to saturation of the air. Under these conditions, the capacity of the air to take up moisture is zero, while the force causing evaporation is \((P_s - P_w)\). As a result, a mist will form and water may be redeposited on the surface. In all drying equipment, care must therefore be taken to ensure that the air or gas used does not become saturated with moisture at any stage.

The rate of drying in the constant rate period is given by:

\[
W = \frac{\text{d}w}{\text{d}t} = \frac{hA\Delta T}{\lambda} = k_G A (P_s - P_w) \tag{16.9}
\]

where:
- \(W\) is the rate of loss of water,
- \(h\) is the heat transfer coefficient from air to the wet surface,
- \(\Delta T\) is the temperature difference between the air and the surface,
- \(\lambda\) is the latent heat of vaporisation per unit mass,
- \(k_G\) is the mass transfer coefficient for diffusion from the wet surface through the gas film,
- \(A\) is the area of interface for heat and mass transfer, and
- \((P_s - P_w)\) is the difference between the vapour pressure of water at the surface and the partial pressure in the air.
It is more convenient to express the mass transfer coefficient in terms of a humidity difference, so that \( k_G A (P_t - P_w) \approx k A \left( \frac{\theta}{\theta_s} - \frac{\theta_s}{\theta} \right) \). The rate of drying is thus determined by the values of \( h \), \( \Delta T \) and \( A \), and is not influenced by the conditions inside the solid. \( h \) depends on the air velocity and the direction of flow of the air, and it has been found that \( h = C G^{0.8} \) where \( G' \) is the mass rate of flow of air in kg/s m\(^2\). For air flowing parallel to plane surfaces, Shepherd et al.\(^{(11)}\) have given the value of \( C \) as 14.5 where the heat transfer coefficient is expressed in W/m\(^2\) K.

If the gas temperature is high, then a considerable proportion of the heat will pass to the solid by radiation, and the heat transfer coefficient will increase. This may result in the temperature of the solid rising above the wet bulb temperature.

**First falling-rate period**

The points B and E in Figure 16.2 represent conditions where the surface is no longer capable of supplying sufficient free moisture to saturate the air in contact with it. Under these conditions, the rate of drying depends very much on the mechanism by which the moisture from inside the material is transferred to the surface. In general, the curves in Figure 16.2 will apply, although for a type 1 solid, a simplified expression for the rate of drying in this period may be obtained.

**Second falling-rate period**

At the conclusion of the first falling rate period it may be assumed that the surface is dry and that the plane of separation has moved into the solid. In this case, evaporation takes place from within the solid and the vapour reaches the surface by molecular diffusion through the material. The forces controlling the vapour diffusion determine the final rate of drying, and these are largely independent of the conditions outside the material.

**16.3.2. Time for drying**

If a material is dried by passing hot air over a surface which is initially wet, the rate of drying curve in its simplest form is represented by BCE, shown in Figure 16.3.
where: 
- \( w \) is the total moisture,
- \( w_e \) is the equilibrium moisture content (point E),
- \( w - w_e \) is the free moisture content, and
- \( w_c \) is the critical moisture content (point C).

**Constant-rate period**

During the period of drying from the initial moisture content \( w_1 \) to the critical moisture content \( w_c \), the rate of drying is constant, and the time of drying \( t_c \) is given by:

\[
 t_c = \frac{w_1 - w_c}{R_c A} \quad \text{(16.10)}
\]

where: 
- \( R_c \) is the rate of drying per unit area in the constant rate period, and
- \( A \) is the area of exposed surface.

**Falling-rate period**

During this period the rate of drying is, approximately, directly proportional to the free moisture content \( (w - w_e) \), or:

\[
 -\left( \frac{1}{A} \right) \frac{dw}{dt} = m(w - w_e) = mf \quad \text{(say)} \quad \text{(16.11)}
\]

Thus:

\[
 -\frac{1}{mA} \int_{w_e}^{w} \frac{dw}{w - w_e} = \int_0^{t_f} dt
\]

or:

\[
 \frac{1}{mA} \ln \left[ \frac{w_e - w_c}{w - w_c} \right] = t_f
\]

and:

\[
 t_f = \frac{1}{mA} \ln \left( \frac{f_c}{f} \right) \quad \text{(16.12)}
\]

**Total time of drying**

The total time \( t \) of drying from \( w_1 \) to \( w \) is given by \( t = (t_c + t_f) \).

The rate of drying \( R_c \) over the constant rate period is equal to the initial rate of drying in the falling rate period, so that \( R_c = mf_c \).

Thus:

\[
 t_c = \frac{(w_1 - w_c)}{mA f_c} \quad \text{(16.13)}
\]

and the total drying time:

\[
 t = \frac{(w_1 - w_c)}{mA f_c} + \frac{1}{mA} \ln \left( \frac{f_c}{f} \right) = \frac{1}{mA} \left[ \frac{(f_1 - f_c)}{f_c} + \ln \left( \frac{f_c}{f} \right) \right] \quad \text{(16.14)}
\]
Example 16.1

A wet solid is dried from 25 to 10 per cent moisture under constant drying conditions in 15 ks (4.17 h). If the critical and the equilibrium moisture contents are 15 and 5 per cent respectively, how long will it take to dry the solid from 30 to 8 per cent moisture under the same conditions?

Solution

For the first drying operation:

\[ w_1 = 0.25 \text{ kg/kg}, \, w = 0.10 \text{ kg/kg}, \, w_c = 0.15 \text{ kg/kg} \text{ and } w_e = 0.05 \text{ kg/kg} \]

Thus:

\[ f_1 = (w_1 - w_e) = (0.25 - 0.05) = 0.20 \text{ kg/kg} \]
\[ f_c = (w_c - w_e) = (0.15 - 0.05) = 0.10 \text{ kg/kg} \]
\[ f = (w - w_e) = (0.10 - 0.05) = 0.05 \text{ kg/kg} \]

From equation 16.14, the total drying time is:

\[ t = \left( \frac{1}{mA} \right) \left( \frac{f_1 - f_c}{f_c} + \ln \left( \frac{f_c}{f} \right) \right) \]

or:

\[ 15 = \left( \frac{1}{mA} \right) \left( \frac{0.20 - 0.10}{0.10} + \ln \left( \frac{0.10}{0.05} \right) \right) \]

and:

\[ mA = 0.0667(1.0 + 0.693) = 0.113 \text{ kg/s} \]

For the second drying operation:

\[ w_1 = 0.30 \text{ kg/kg}, \, w = 0.08 \text{ kg/kg}, \, w_c = 0.15 \text{ kg/kg} \text{ and } w_e = 0.05 \text{ kg/kg} \]

Thus:

\[ f_1 = (w_1 - w_e) = (0.30 - 0.05) = 0.25 \text{ kg/kg} \]
\[ f_c = (w_c - w_e) = (0.15 - 0.05) = 0.10 \text{ kg/kg} \]
\[ f = (w - w_e) = (0.08 - 0.05) = 0.03 \text{ kg/kg} \]

The total drying time is then:

\[ t = \left( \frac{1}{0.113} \right) \left( \frac{0.25 - 0.10}{0.10} + \ln \left( \frac{0.10}{0.03} \right) \right) \]

\[ = 8.856(1.5 + 1.204) \]

\[ = 23.9 \text{ ks (6.65 h)} \]

Example 16.2

Strips of material 10 mm thick are dried under constant drying conditions from 28 to 13 per cent moisture in 25 ks (7 h). If the equilibrium moisture content is 7 per cent, what is the time taken to dry 60 mm planks from 22 to 10 per cent moisture under the same conditions assuming no loss from the edges? All moistures are given on a wet basis.

The relation between \( E \), the ratio of the average free moisture content at time \( t \) to the initial free moisture content, and the parameter \( J \) is given by:

\[
\begin{array}{cccccccc}
E & 1 & 0.64 & 0.49 & 0.38 & 0.295 & 0.22 & 0.14 \\
J & 0 & 0.1 & 0.2 & 0.3 & 0.5 & 0.6 & 0.7
\end{array}
\]
It may be noted that $J = kt/L^2$, where $k$ is a constant, $t$ the time in ks and $2l$ the thickness of the sheet of material in millimetres.

**Solution**

*For the 10 mm strips*

Initial free moisture content $= (0.28 - 0.07) = 0.21$ kg/kg.

Final free moisture content $= (0.13 - 0.07) = 0.06$ kg/kg.

Thus: when $t = 25$ ks, $E = (0.06/0.21) = 0.286$

and from Figure 16.4, a plot of the given data,

$$J = 0.52$$

Thus:

$$0.52 = (k \times 25)/(10/2)^2$$

and:

$$k = 0.52$$

![Figure 16.4. Drying data for Example 6.2](image)

*For the 60 mm planks*

Initial free moisture content $= (0.22 - 0.07) = 0.15$ kg/kg.

Final free moisture content $= (0.10 - 0.07) = 0.03$ kg/kg.

$$E = (0.03/0.15) = 0.20$$

From Figure 16.4: $J = 0.63$

and hence: $t = Jl^2/k$

$$= 0.63(60/2)^2/0.52 = 1090 \text{ ks (12.6 days)}$$
Example 16.3

A granular material containing 40 per cent moisture is fed to a countercurrent rotary dryer at a temperature of 295 K and is withdrawn at 305 K, containing 5 per cent moisture. The air supplied, which contains 0.006 kg water vapour/kg dry air, enters at 385 K and leaves at 310 K. The dryer handles 0.125 kg/s wet stock.

Assuming that radiation losses amount to 20 kJ/kg dry air used, determine the mass flowrate of dry air supplied to the dryer and the humidity of the exit air.

The latent heat of water vapour at 295 K = 2449 kJ/kg, specific heat capacity of dried material = 0.88 kJ/kg K, the specific heat capacity of dry air = 1.00 kJ/kg K, and the specific heat capacity of water vapour = 2.01 kJ/kg K.

Solution

This example involves a heat balance over the system. 273 K will be chosen as the datum temperature, and it will be assumed that the flowrate of dry air = \( G \) kg/s.

Heat in:

(a) Air

\[ G \text{ kg/s dry air enter with } 0.006G \text{ kg/s water vapour and hence the heat content of this stream} \]

\[ = [(1.00G) + (0.006G \times 2.01)](385 - 273) = 113.35G \text{ kW} \]

(b) Wet solid

0.125 kg/s enter containing 0.40 kg water/kg wet solid, assuming the moisture is expressed on a wet basis.

Thus:

mass flowrate of water = \((0.125 \times 0.40) = 0.050 \text{ kg/s}\)

and:

mass flowrate of dry solid = \((0.125 - 0.050) = 0.075 \text{ kg/s}\)

Hence:

the heat content of this stream = \([(0.050 \times 4.18) + (0.075 \times 0.88)](295 - 273) = 6.05 \text{ kW}\)

Heat out:

(a) Air

Heat in exit air = \([(1.00G) + (0.006G \times 2.01)](310 - 273) = 37.45G \text{ kW}\).

Mass flowrate of dry solids = 0.075 kg/s containing 0.05 kg water/kg wet solids.

Hence:

water in the dried solids leaving = \((0.05 \times 0.075)/(1 + 0.05) = 0.0036 \text{ kg/s}\)

and:

the water evaporated into gas steam = \((0.050 - 0.0036) = 0.0464 \text{ kg/s}\).

Assuming evaporation takes place at 295 K, then:

heat in the water vapour = \(0.0464[2.01(310 - 295) + 2449 + 4.18(295 - 273)]\)

\[= 119.3 \text{ kW}\)
and:

the total heat in this stream = \((119.30 + 37.45G)\) kW.

(b) Dried solids
The dried solids contain 0.0036 kg/s water and hence heat content of this stream is:

\[ = [(0.075 \times 0.88) + (0.0036 \times 4.18)/(305 - 273)] = 2.59\ kW \]

(c) Losses
These amount to 20 kJ/kg dry air or 20m kW.

Heat balance

\[(113.35 \ G + 6.05) = (119.30 + 37.45 \ G + 2.59 + 20 \ G)\]

and:

\[ G = 2.07 \text{ kg/s} \]

Water in the outlet air stream = \((0.006 \times 2.07) + 0.0464 = 0.0588 \text{ kg/s}\)

and:

the humidity \(\mathcal{H} = (0.0588/2.07) = 0.0284 \text{ kg/kg dry air}\)

16.4. THE MECHANISM OF MOISTURE MOVEMENT DURING DRYING

16.4.1. Diffusion theory of drying

In the general form of the curve for the rate of drying of a solid shown in Figure 16.2, there are two and sometimes three distinct sections. During the constant-rate period, moisture vapourises into the air stream and the controlling factor is the transfer coefficient for diffusion across the gas film. It is important to understand how the moisture moves to the drying surface during the falling-rate period, and two models have been used to describe the physical nature of this process, the diffusion theory and the capillary theory. In the diffusion theory, the rate of movement of water to the air interface is governed by rate equations similar to those for heat transfer, whilst in the capillary theory the forces controlling the movement of water are capillary in origin, arising from the minute pore spaces between the individual particles.

Falling rate period, diffusion control

In the falling-rate period, the surface is no longer completely wetted and the rate of drying steadily falls. In the previous analysis, it has been assumed that the rate of drying per unit effective wetted area is a linear function of the water content, so that the rate of drying is given by:

\[
\left(\frac{1}{A}\right) \frac{dw}{dr} = -m(w - w_e) \quad \text{(equation 16.11)}
\]

In many cases, however, the rate of drying is governed by the rate of internal movement of the moisture to the surface. It was initially assumed that this movement was a process