Windowing & Clipping In 2D
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Windowing Concepts

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A scene is made up of a collection of objects specified in world coordinates.
When we display a scene only those objects within a particular window are displayed.
Because drawing things to a display takes time we *clip* everything outside the window.
For the image below consider which lines and points should be kept and which ones should be clipped.

[Diagram showing clipping process with points and bounding box]
Point Clipping

Easy - a point \((x,y)\) is not clipped if:

\[
wx_{\text{min}} \leq x \leq wx_{\text{max}} \text{ AND } wy_{\text{min}} \leq y \leq wy_{\text{max}}
\]

otherwise it is clipped.
Harder - examine the end-points of each line to see if they are in the window or not

<table>
<thead>
<tr>
<th>Situation</th>
<th>Solution</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both end-points inside the window</td>
<td>Don’t clip</td>
<td><img src="image" alt="Example of both end-points inside the window" /></td>
</tr>
<tr>
<td>One end-point inside the window, one outside</td>
<td>Must clip</td>
<td><img src="image" alt="Example of one end-point inside, one outside" /></td>
</tr>
<tr>
<td>Both end-points outside the window</td>
<td>Don’t know!</td>
<td><img src="image" alt="Example of both end-points outside" /></td>
</tr>
</tbody>
</table>
Brute force line clipping can be performed as follows:

- Don’t clip lines with both end-points within the window
- For lines with one end-point inside the window and one end-point outside, calculate the intersection point (using the equation of the line) and clip from this point out
Brute Force Line Clipping (cont.)

- For lines with both endpoints outside the window test the line for intersection with all of the window boundaries, and clip appropriately.

However, calculating line intersections is computationally expensive. Because a scene can contain so many lines, the brute force approach to clipping is much too slow.
An efficient line clipping algorithm
The key advantage of the algorithm is that it vastly reduces the number of line intersections that must be calculated
Cohen-Sutherland Clipping Algorithm

• One of the earliest algorithms with many variations in use.

• Processing time reduced by performing more test before proceeding to the intersection calculation.

• Initially, every line endpoint is assigned a four digit binary value called a region code, and each bit is used to indicate whether the point is inside or outside one of the clipping-window boundaries.
• We can reference the window edges in any order, and here is one possibility.

<table>
<thead>
<tr>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>Bottom</td>
<td>Right</td>
<td>Left</td>
</tr>
</tbody>
</table>

• For this ordering, (bit 1) references the left boundary, and (bit 4) references the top one.
• A value of 1 (true) in any bit position indicate that the endpoint is outside of that border.
• A value of 0 (false) indicates that the endpoint is inside or on that border.
• The four window borders create nine regions

• The Figure below lists the value for the binary code in each of these regions.

<table>
<thead>
<tr>
<th>Region Code</th>
<th>1001</th>
<th>1000</th>
<th>1010</th>
</tr>
</thead>
<tbody>
<tr>
<td>0001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Window</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0101</td>
<td>0100</td>
<td>0110</td>
<td></td>
</tr>
</tbody>
</table>

Thus, an endpoint that is below and to the left of the clipping window is assigned the region (0101).

The region code for any endpoint inside the clipping window is (0000).
Every end-point is labelled with the appropriate region code
Lines completely contained within the window boundaries have region code [0000] for both end-points so are not clipped.
Any lines with 1 in the same bit position for both end-points is completely outside and must be clipped.

For example a line with 1010 code for one endpoint and 0010 for the other (line P11, P12) is completely to the right of the clipping window.
• We can perform inside/outside test for lines using logical operators.
• When the or operation between two endpoint codes is false (0000), the line is inside the clipping window, and we save it.
• When the and operation between two endpoint codes is true (not 0000), the line is completely outside the clipping window, and we can eliminate it.
Cohen-Sutherland: Other Lines

Lines that cannot be identified as completely inside or outside the window may or may not cross the window interior

These lines are processed as follows:

– Compare an end-point outside the window to a boundary (choose any order in which to consider boundaries e.g. left, right, bottom, top) and determine how much can be discarded

– If the remainder of the line is entirely inside or outside the window, retain it or clip it respectively
Otherwise, compare the remainder of the line against the other window boundaries
Continue until the line is either discarded or a segment inside the window is found

We can use the region codes to determine which window boundaries should be considered for intersection
To check if a line crosses a particular boundary we compare the appropriate bits in the region codes of its end-points
If one of these is a 1 and the other is a 0 then the line crosses the boundary
Consider the line $P_9$ to $P_{10}$ below

- Start at $P_{10}$
- From the region codes of the two end-points we know the line doesn’t cross the left or right boundary
- Calculate the intersection of the line with the bottom boundary to generate point $P_{10}'$
- The line $P_9$ to $P_{10}'$ is completely inside the window so is retained
Consider the line $P_3$ to $P_4$ below

- Start at $P_4$
- From the region codes of the two end-points we know the line crosses the left boundary so calculate the intersection point to generate $P_4'$
- The line $P_3$ to $P_4'$ is completely outside the window so is clipped
Consider the line \( P_7 \) to \( P_8 \) below

- Start at \( P_7 \)
- From the two region codes of the two end-points we know the line crosses the left boundary so calculate the intersection point to generate \( P_7' \)
Consider the line $P_7'$ to $P_8$

- Start at $P_8$
- Calculate the intersection with the right boundary to generate $P_8'$
- $P_7'$ to $P_8'$ is inside the window so is retained
Cohen-Sutherland Worked Example

Window

$w_y_{\text{max}}$

$w_y_{\text{min}}$

$w_x_{\text{min}}$

$w_x_{\text{max}}$
Intersection points with the window boundaries are calculated using the line-equation parameters

- Consider a line with the end-points \((x_1, y_1)\) and \((x_2, y_2)\)
- The y-coordinate of an intersection with a vertical window boundary can be calculated using:

\[
y = y_1 + m(x_{\text{boundary}} - x_1)
\]

where \(x_{\text{boundary}}\) can be set to either \(wx_{\text{min}}\) or \(wx_{\text{max}}\)
Calculating Line Intersections (cont...)  

– The x-coordinate of an intersection with a horizontal window boundary can be calculated using:

\[ x = x_1 + \left( y_{\text{boundary}} - y_1 \right) / m \]

where \( y_{\text{boundary}} \) can be set to either \( w_y_{\text{min}} \) or \( w_y_{\text{max}} \)

– \( m \) is the slope of the line in question and can be calculated as \( m = (y_2 - y_1) / (x_2 - x_1) \)
Similarly to lines, areas must be clipped to a window boundary. Consideration must be taken as to which portions of the area must be clipped.
To clip a polygon:

- Modify the line-clipping procedures.
- Polygon boundary processed with a line clipper may be displayed as a series of unconnected line segments as in the following figure:

![Diagram showing area clipping before and after](image)
What really required is a bounded area after clipping as in the following figure:
Thus, what actually required is an algorithm that will generate one or more closed areas that are then scan converted for the appropriate area fill.

The output of a polygon clipper should be a sequence of vertices that defines the clipped polygon boundaries.

Beginning with the initial set of polygon vertices, we could first clip the polygon against the left rectangle boundary to produce a new sequence of vertices.

The new set of vertices could then be successively passed to a right boundary clipper, a bottom boundary clipper and a top boundary clipper as in the following figure:
- At each step, a new sequence of output vertices is generated and passed to the next window boundary clipper.
Various Cases

There are four possible cases when processing vertices in sequence around the perimeter of a polygon. As each pair of adjacent polygon vertices is passed to a window boundary clipper, we make the following tests:

1. If the first vertex is outside the window boundary and the second vertex is inside, both the intersection point and the polygon edge with the window boundary and the second vertex are added to the output vertex list.

2. If both input vertices are inside the window boundary, only the second vertex is added to the output vertex list.

3. If the first vertex is inside the window boundary and the second vertex is outside, only the edge intersection with the window boundary is added to the output vertex list.
(4) If both input vertices are outside the window boundary, nothing is added to the output list. These four cases are illustrated in the following figure for successive pairs of polygon vertices. Once all vertices have been processed for one clip window boundary, the output list of vertices is clipped against the next window boundary.
Example

Start at the left boundary

1,2 (out,out) → clip

2,3 (in,out) → save 1’, 3

3,4 (in,in) → save 4

4,5 (in,in) → save 5

5,6 (in,out) → save 5’

Saved points → 1’, 3, 4, 5, 5’

6,1 (out,out) → clip

Using these points we repeat the process for the next boundary.
Convex polygons are correctly clipped by the Sutherland-Hodgman algorithm, but concave polygons may be displayed with extra areas (area inside the red circle), as demonstrated in the following figure.
• This occurs when the clipped polygon should have two or more separate sections. But since there is only one output vertex list, the last vertex in the list is always joined to the first vertex.
• There are several things we could do to correctly display concave polygons.
• For one, we could split the concave polygon into two or more convex polygons and process each convex polygon separately.
• Another possibility is to modify the Sutherland-Hodgman approach to check the final vertex list for multiple vertex points along any Clip window boundary and correctly join pairs of vertices.
• Finally, we could use a more general polygon clipper, such as either the Weiler-Atherton algorithm or the Weiler algorithm.
In Weiler-Atherton Polygon Clipping, the vertex-processing procedures for window boundaries are modified so that concave polygons are displayed correctly. This clipping procedure was developed as a method for identifying visible surfaces, and so it can be applied with arbitrary polygon-clipping regions.
Weiler-Atherton Polygon Clipping

- The basic idea in this algorithm is that instead of always proceeding around the polygon edges as vertices are processed, we sometimes want to follow the window boundaries.

- Which path we follow depends on the polygon-processing direction (clockwise or counterclockwise) and whether the pair of polygon vertices currently being processed represents an outside-to-inside pair or an inside-to-outside pair.
For clockwise processing of polygon vertices, we use the following rules:

1. For an outside-to-inside pair of vertices, follow the polygon boundary
2. For an inside-to-outside pair of vertices, follow the window boundary in a clockwise direction.
Example

• In the following figure, the processing direction in the Weiler-Atherton algorithm and the resulting clipped polygon is shown for a rectangular clipping window.
Text Clipping

All-or-none String-clipping

All-or-none Character-clipping

Before Clipping

After Clipping
Clipping on components of individual Characters
Areas with curved boundaries can be clipped with methods similar to those discussed in the previous sections. Curve-clipping procedures will involve nonlinear equations, however, and this requires more processing than for objects with linear boundaries.