2-1 SOLUTION METHODS:

Solutions to the problems of statics may be obtained in one or more of the following ways:

1. **Graphical Solution:**
   By drowning all the force act on the body with a specific scale. We can find the equivalent force (resultant) by close the force system.

   **Ex. 3**
   Combine the two forces \( P \) and \( T \), which act on the fixed structure at \( B \), into a single equivalent force \( R \).

   **Sol**
   \[ R = 524 \text{ lb} \]
   \[ \theta = 48.6^\circ \]

2. **Algebraic Solution:**
   We obtain mathematical solution by hand, using one or more of three theorems to get the solution. These theorem are :
   
   **Pythagorean Theorem:**
   \[ R^2 = a^2 + b^2 \]  
   **Eq 3**
   Note: one of the three angles most be 90°.

   \[ \theta + \beta + \alpha = 180^\circ \]  
   **Eq 4**
Law of sines:
In any triangle, the length of side to the sine of the opposite angles is constant, then

\[
\frac{R}{\sin \theta} = \frac{a}{\sin \alpha} = \frac{b}{\sin \beta}
\]

Eq 4

Law of cosines:

\[
R^2 = a^2 + b^2 - 2ab \cos \theta
\]

or

\[
b^2 = R^2 + a^2 - 2Ra \cos \beta
\]

or

\[
a^2 = R^2 + b^2 - 2Rb \cos \alpha
\]

Eq 5

Ex 4
For the forces \( F_1 \) and \( F_2 \) act on the bracket as shown in the figure. Determine the magnitude of the resultant of the two forces and its direction.

\[
F^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos 130^\circ
\]

\[
F^2 = (80)^2 + (100)^2 - 2 \times (40) \times (100) \times \cos 130^\circ
\]

\[
F^2 = 26699.56
\]

\[
F = 163.4 N
\]

using the law of sines for the lower triangle, we have

\[
\frac{163.4}{\sin 130} = \frac{100}{\sin(\alpha+20)}
\]

\[
\sin(\alpha+20) = 0.4688
\]

\[
(\alpha+20) = \sin^{-1} 0.4688
\]

\[
(\alpha+20) = 27.956
\]

\[
\alpha = 27.956 - 20 = 7.956^\circ
\]

See examples 4, 5, 6, and 7 page (15)
Ex 5

The forces $F_1$, $F_2$, and $F_3$, all of which act on point A of the bracket are specified in three different ways. Determine the resultant and its direction.

1. Graphical solution

2. Algebraic solution

Using the law of cosines to find $C$

$$C^2 = 800^2 + 600^2 - 2(800)(600) \cos(81.56)$$

$$C^2 = 859097 \quad \rightarrow \quad C = 926.9$$

Using the law of sines to find $\alpha$

$$\frac{C}{\sin 81.56} = \frac{800}{\sin \alpha}$$

$$\sin \alpha = 0.8535 \quad \rightarrow \quad \alpha = 58.69^\circ$$

$$\beta + \alpha = 71.86 \quad \rightarrow \quad \beta = 71.85 - 58.69$$

$$\beta = 13.72^\circ$$
Using the cosines law to find $R$

$$R^2 = 926.87^2 + 500^2 - 2(926.87)(500) \cos (13.27)$$

$$R^2 = 206.966 \quad R = 455 \, N$$

Using the sine’s law to find $\delta$

$$\frac{R}{\sin 13.27} = \frac{500}{\sin \delta}$$

$\therefore \sin \delta = 0.252 \quad \delta = 14.6^\circ$

To find the angle of the resultant $\theta$

$\varepsilon = 58.69 - 35$

$\varepsilon = 23.69$

$\theta = 360 - (\varepsilon - \delta) \quad \theta = 360 - (23.69 - 14.6)$

$\therefore \theta = 351^\circ$
1. The two structural members, one of which is in tension and the other in compression, exert the indicated force in joint O. Determine the magnitude of the resultant \( R \) of the two forces and the angle \( \theta \) which \( R \) makes with the positive \( x \)-axis. 

   \[ \text{Ans. } R = 8.75kN \quad \theta = 171.6° \]

2. Determine the resultant \( R \) of the two forces shown by applying the parallelogram rule.

   \[ \text{Ans. } R = 529.5N \quad \theta = 100.89° \]
3. While steadily pushing the machine up an incline, a person exerts a 180 N force $P$ as shown. Determine the components of $P$ which are parallel and perpendicular to the incline. 

Answer. $P_t = 163.1$ N $P_n = -76.1$ N

4. If the resultant $F_R$ of the two forces acting on the log is to be directed along the positive $x$-axis and have a magnitude of 10 kN. Determine the angle $\theta$ of the cable, attached to B such that the force $F_B$ in this cable is minimum what is the magnitude of the force in each cable for this situation. 

Answer. $\theta = 60^\circ$, $F_A = 8.66$ kN, $F_B = 5$ kN
5. Determine the x and y components of each force acting on the gusset plate on the bridge truss. Show that the resultant force is zero. [R. C. Hibbeler (2-47)]

Ans.

\[
\begin{align*}
F_{1x} &= 200 \text{ lb} \\
F_{1y} &= 0 \\
F_{2x} &= 320 \text{ lb} \\
F_{2y} &= -240 \text{ lb} \\
F_{3x} &= 180 \text{ lb} \\
F_{3y} &= 240 \text{ lb} \\
F_{4x} &= -300 \text{ lb} \\
F_{4y} &= 0
\end{align*}
\]