# *Numerical Comparison Function for Weibull Distribution Probability Values with Possibility Values for Fuzzy Logic*

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## Abstract

The relationship between possibility and probability is cleared through the rang of functions values for each one of them, which is equivalent to be in the closed interval [0, 1], and it has some variation in values. The goal of this research is to find the type of relation and to decrease the variations in values through finding a combination between several probability distributions and membership function in fuzzy logic to be either continuous or discrete .In this paper the weibull distribution has been exploited with a continuous membership function, Gaussian membership and applied failure rate function with that distribution.

**دالة مقارنة عددية للقيم الاحتمالية لتوزيع ويبل مع قيم الإمكانية للمنطق الضبابي**

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**جامعة بابل,العراق 2010**

**الخـــلاصة:**

وضحت العلاقة بين الاحتمالية والإمكانية من خلال مدى قيم الدوال لكل منهما, والتي تكون متكافئة لتكون في الفترة المغلقة $[0,1]$, وتمتلك بعض التباين في القيم. ان هدف هذا البحث ان نجد نوع العلاقة وتقليل التباين في القيم من خلال إيجاد ربط بين بعض التوزيعات الاحتمالية ودالة عضوية في المنطق الضبابي تكون إما مستمرة أو متقطعة .ففي هذا البحث تم تطبيق توزيع ويبل مع دالة عضوية مستمرة هي دالة كاوس العضوية, وطبقت دالة معامل الفشل لذلك التوزيع .

## *1.Introduction:*

Someone asks, why we don’t use the statistic instead of fuzzy logic? More detailed difference between the concepts led us use one instead of the other. That is, the difference between the degree of membership in the set (of some member), and a probability of being in that set.

The point of difference is, the probability involves a crisp set theory (probability of it belongs to class or not), and don't allow for an element to be a partial member in a class (or a set, as in fuzzy logic[1]).

Probability is an indicator of frequency or likelihood that an element is in a class, while fuzzy set theory deals with the similarity of an element to a class [3] that is between elements in a class.

Anyone who doesn't know and haven't study fuzzy logic and fuzzy sets think ,that fuzziness is just a clever disguise for probability, which is never true for more information see [1].

Although fuzzy logic is known latterly ,it has been communicated with many other sciences for its benefits in practical applications(applicable branches).Since 1991, fuzzy logic is used in technology as an industrial tool in reference[5]to be fuzzy control, but the theoretical side stay requisite .

Probability theory and fuzzy set theory have been communicated since they were depend on same range to be in closed interval $[0,1]$ ,also membership function(MF) that characterize the fuzzy set depend on some parameters(time_ verify parameters)and its values chosen from parameter space(real number).While probability distribution also depend on parameters describe the distribution and determine its values and shape ,which chosen from parameter space, many ways used to locate these parameter values.

Also the values of MF constraints are as $0 \le \mu \le 1$, [6], [9], while probability has a main condition as $\sum p(u) = 1$ .

To shed light on such a relationship, a probability distribution used to compare the values that computed by Weibull distribution function with that values computed by Gaussian MF (both were continuous functions)on tables for values of dependent variable(s) applied for both functions and values for parameters that be in each function .

## *2.Weibull Probability Density Function [8]*

The density functions as;

$$f(x;a,b) = abx^{b-1}e^{-x^b} \qquad \text{…. (1)}$$

where a,b>0 and x>0 is called the weibull density, a distribution that has been successfully used in reliability theory .From the Weibull distribution, the general equation for failure rate is given by:

[1] A fuzzy logic is basically a multi_valued logic that allows intermediate values to be defined between conventional evaluations like; true/false ,black/white ,yes/no ,high/ low (see any reference on fuzzy logic).

$$h(t) = \frac{f(t)}{1 - F(t)} \qquad \text{.... (2)}$$

$$= abt^{b-1}$$

The function $f(t)$ is the time-to-failure probability density function. It gives the probability that a part will fail at any given time $t$ . $F(t)$ is a commutative density function. The mean, or expected value of $f(t)$ is the average time-to-failure. The relation between weibull distribution and failure rate is cleared as follows;

Weibull distribution has increasing failure rate when the $b$ (shape parameter) is greater than one, and has decreasing failure rate when the $b$ is lesser than one .

The Weibull distribution is often used in the field of life data analysis due to its flexibility and it can mimic the behavior of other statistical distributions such as the normal and the exponential. This distribution curve function is with positive skewness. So the weibull distribution may be used in place of the normal distribution because a weibull variant can be generated through inversion .Normal variants are typically generated using the more complicated Box-Muller method, which requires two uniform random variants.

The Weibull distribution is used [3] -

- In survival analysis
- To represent manufacturing and delivery times in industrial engineering
- In extreme value theory
- In weather forecasting
- In reliability engineering and failure analysis
- In radar systems to model the dispersion of the received signals level produced by some types of clutters
- To model fading channels in wireless communications, as the Weibull fading model seems to exhibit good fit to experimental fading channel measurements
- In General insurance to model the size of Reinsurance claims, and the cumulative development of Asbestosis losses
- In forecasting technological change (also known as the Sharif-Islam model)
- To describe wind speed distributions, as the natural distribution often matches the Weibull shape.

***3. Possibility Distribution and Gaussian Membership Function :***

Possibility theory focus primarily on imprecision, which is intrinsic in natural languages and is assumed to be "possibilistic" rather than probabilistic [6]. Therefore the term "variable" is very often used in a more linguistic sense than in a strictly mathematical one, that why the symbolism of possibility differ in some respects from those of fuzzy set theory. Possibility distribution is one of the central concepts of possibility theory (as opposed to a probability distribution), which characterized by possibility distribution function through proposition that equal numerically to the MF.

This function is continuous function has exponential form that expressed for fuzzy numbers (Gaussian Fuzzy Number GFN) as form [5];

$$\mu(x) = e^{-x((x-m)/\sigma)^2} \qquad \text{.... (3)}$$

With $\mu(x) \in [0,1]$ .which where described by parameters $m$ center for distribute values is the magnitude of GFN and $,\sigma$ is width for distribute values as fuzziness parameter.

Some properties for this function helpful in choice this function, which (properties) are powerful in comparison with probability and distribution properties .The GMF has a property to has single maxima over infinite variable values (infinite support), and it is a real valued convex function, $\mu(x): X \to R$, which give values to be every where positive in spite the inputs are negative, with symmetric as it shown in figure (1).

***Note*** :*Since Weibull Distribution uses in Reliability and Quality Control ,in this paper we refer to use the random variable as time $t$ to comparable with the variable $x$ for membership function $t = x$.*

***Note: Probability*** *for mutually exclusive events can not add up to more than 1(one value), but their fuzzy values could.*

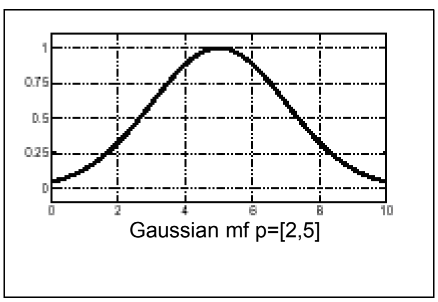



*Figure (1)*: *Gaussian Membership function*

***4. Comparison Idea for Fuzzy logic and Weibull distribution***

Both fuzzy logic and probability are valid approaches to the classification problem [3],for example ,if we were to classify "*old* " ,fuzzy membership make much more sense that probability since in probability each (person)either "*old* " has probability or not has probability ,that is *probability*=0 .

Also in another way a person who is dying of thirst in the desert is given two bottles of fluid, one bottles label says that it has a 0.9 membership degree in the class of fluid known as nonpoisonous drinking water (or sea water, swamp water, cola,…, etc) .The other bottles label states that it has a 90% probability of being pure drinking water and a10% probability of being poison, Which bottle (if you where there) choose?

A fuzzy bottle contains (swamp water, as example) cola; this also makes sense since cola would have a 0.9 membership in the class of nonpoisonous fluids.

This example was given by *Bezdek* [3] as a good example to demonstrate the conceptual difference for statistical and fuzzy classification .The degree of certainty (somewhere) sounds like a probability (perhaps subjective probability), but it is not quit the same .Hot and cold can have 0.6 and 0.5 as their membership degrees in these fuzzy sets (a fuzzy values), but not as probabilities (which could not) [4].

It is become clear that both operate over the same numeric range ,and have similar values as; 0 representing False(or non membership ,in fuzzy) ,and 1 representing True(or full membership in a fuzzy) .

Let us take for instance a possible interferometer coherence $g$ values to be the set $X$ of all real numbers between 0 and1,from this set $X$ as a subset $A$ can be defined as(all values $0 \le g \le 0.2$),that is[2] ;

$$A = \{ g : 0 \le g \le 0.2\} \qquad \dots \quad (4)$$

Since $g$ starts at 0, the lower range of this set ought to be clear; the upper range on the other hand, is rather hard to define. The MF operating in this case on the fuzzy set of interferometer coherence $g$ returns a value between 0.0 and 1.0 ,for example ,an interferometer coherence $g$ of 0.3 has a membership of 0.5 to the set low coherence , see figure(2) .

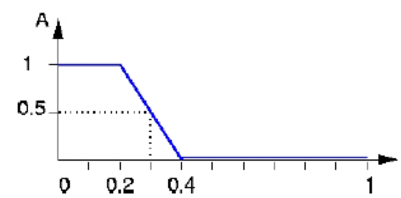



*Figure (2): Characteristic of a fuzzy set*

The probabilistic approach yield the natural language statement *"there is an 50% chance that $g$ is low"*, the probability view suppose that $g$ is or not low it is just that we only have an 50% chance of knowing which set it is in .By contrast, fuzzy terminology supposes that $g$ is "*more or less low",* or in some other term corresponding to value of 0.50 .

The comparison cleared through helpful properties for Weibull distribution and Gaussian Membership Function, that both functions are real valued functions on $t$ to range [0,1]on domain with real infinite variable values, and both are continuous functions with every where positive and with single maxima .All previous properties led to a comparison through numerical values for functions that not exceed than 1 and not less than 0 ; $0 \le F(t) \le 1$ ,which also satisfied for $\mu(t)$ .

Also the comparison depended on parameters values for taking as; $a = \sigma$ , $b = m$ ,or other form at be with extension to a parameters space is the same for both cases ,which will be a real space for this work .An understanding of the rate may provide insight as to what is causing the failures :

- A decreasing failure rate would suggest "infant mortality". That is, defective items fail early and the failure rate decreases over time as they fall out of the population.
- A constant failure rate suggests that items are failing from random events.
- An increasing failure rate suggests "wear out" - parts are more likely to fail as time goes on.

The result for this all will be explained and graphics through practical examples with numerical values.

## 5. *Numerical Results and Discussion*

Parameters values with the numerical results given in the tables (1-7), which represent time, values of parameters that where chosen in real space for t, b and a.

We show that the influence of the time, the shape and scale parameters on the probability distribution function, Gaussian function and failure rate. For different values of times and the parameters under given conditions for each function .Each table fallowed by graphic represent the functions or parameters values or time (at that table), which help us to describe and discuss the difference in values in more accurate.

### *5.1 Influence of Time*

To study influence of time's values on values of weibull function ,Gaussian membership and failure rate the parameters values stay constants and satisfy condition that b >1 ,at a=0.1 and b =2 and the time has values be t<1and t>1 .

*Table (1):The influence of the time on the weibull function, Gaussian membership and failure rate, when a=0.1 and b >1 .*

| a=0.1 , b=2 | | | |
|---|---|---|---|
| t | f(t) | $\mu(t)$ | h(t) |
| 0.2 | 0.0398 | 0.9975 | 0.04 |
| 0.3 | 0.0594 | 0.9900 | 0.06 |
| 0.4 | 0.0681 | 0.9777 | 0.08 |
| 0.5 | 0.0778 | 0.9607 | 0.1 |
| 0.6 | 0.11575 | 0.9394 | 0.12 |
| 1.1 | 0.19492 | 0.7788 | 0.2 |
| 1.2 | 0.2078 | 0.7389 | 0.24 |
| 1.4 | 0.2301 | 0.6554 | 0.28 |
| 1.6 | 0.2477 | 0.5697 | 0.32 |
| 1.8 | 0.2603 | 0.4528 | 0.36 |
| 2 | 0.2681 | 0.4055 | 0.4 |

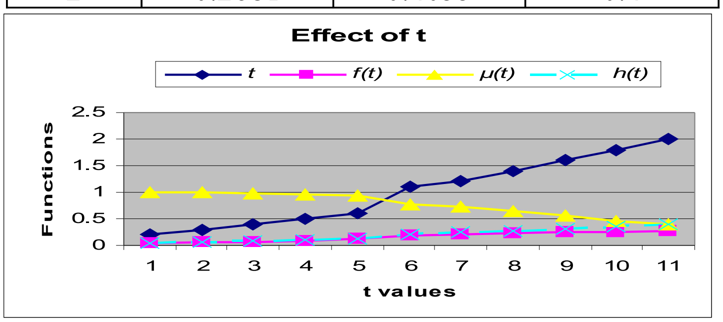
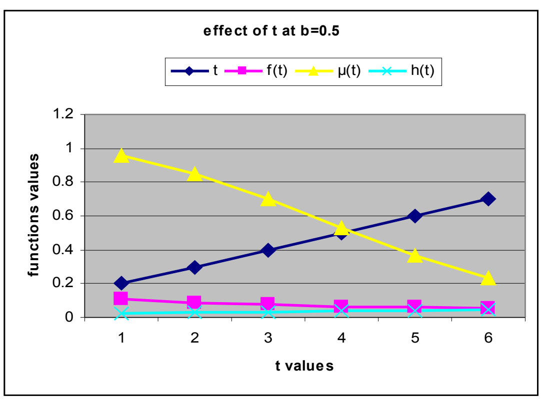



*Figure (3): Graphic of effect of time on the weibull function, Gaussian membership and failure rate, when a=0.1 and b >1 .*

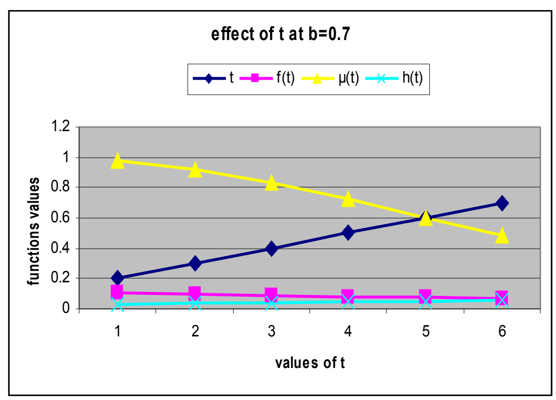
The values of f(t) and h(t) are increased with values of t which increased from 0.2 to 2, while $\mu(t)$ decreased .Failure rate get largest value at largest time.

*Table (2 ): The influence of the time on the weibull function, Gaussian membership and failure rate, when a=0.1 and b <1.*

| b=0.5 | | | |
|---|---|---|---|
| t | f(t) | $\mu(t)$ | h(t) |
| 0.2 | 0.1068 | 0.9607 | 0.0262 |
| 0.3 | 0.0864 | 0.8521 | 0.0308 |
| 0.4 | 0.0742 | 0.6976 | 0.0346 |
| 0.5 | 0.0658 | 0.5272 | 0.0378 |
| 0.6 | 0.0597 | 0.3678 | 0.0407 |
| 0.7 | 0.0549 | 0.2369 | 0.0433 |
| b=0.7 | | | |
| t | f(t) | $\mu(t)$ | h(t) |
| 0.2 | 0.1098 | 0.9797 | 0.0266 |
| 0.3 | 0.0962 | 0.9216 | 0.0339 |
| 0.4 | 0.0874 | 0.8322 | 0.0403 |
| 0.5 | 0.0810 | 0.7214 | 0.0461 |
| 0.6 | 0.0760 | 0.6003 | 0.0515 |
| 0.7 | 0.0720 | 0.4796 | 0.0565 |




*(a)*




*(b)*

*Figure(4):Graphic of effect of time on the weibull function ,Gaussian membership and failure rate ,when a=0.1 and b >1as:(a)b=0.5 . (b) b=0.7 .*

It is clear that $\mu(t)$ has lest value at largest time and it be decreasing when time is increasing ,while f(t) decreasing also but not high difference .The mean of decreasing is about 0.7 .Failure rate is so small and it be less than weibull function ,and has greatest value at greatest time .

### *5.2 Influence of Shape Parameter*

To study influence of shape parameter values on values of weibull function, Gaussian membership and failure rate, time value be constant t<1 and t>1, and parameter a at a<1 and a>1, while b take different values at b<1 with small differences .Also suppose b>1, for different values of t and scale parameter constants at a=0.1 .

*Table (3): The influence of the shape parameter on the weibull function, Gaussian membership and failure rate, when t< 1 and a<1.*

| t=0.3 , a=0.4 | | | |
|---|---|---|---|
| b | f(t) | $\mu(t)$ | h(t) |
| 0.51 | 0.29636 | 0.9622 | 0.3679 |
| 0.52 | 0.29933 | 0.9636 | 0.3707 |
| 0.53 | 0.30221 | 0.9650 | 0.3733 |
| 0.54 | 0.30423 | 0.9660 | 0.3758 |
| 0.55 | 0.30770 | 0.9674 | 0.3781 |

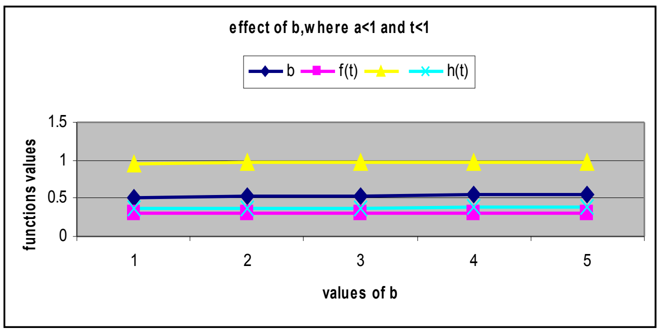



*Figure (5): Graphic of effect of shape parameter on the weibull function, Gaussian membership and failure rate, when t< 1 at t=0.3 and a<1 at a=0.4 .*

It is clear that $\mu(t)$ and f(t) and h(t) increasing with increasing b and they reach largest values at largest b, and the values with small difference .Failure rate values seem so closed with largest value at largest value of b .It be larger than weibull function .

*Table(4):The influence of the shape parameter on the weibull function ,Gaussian membership and failure rate ,when t>1 and a>1 .*

| t=1.5 , a=1.1 | | | |
|---|---|---|---|
| b | f(t) | $\mu(t)$ | h(t) |
| 0.1 | 0.02428 | $112\times 10^{-9}$ | 0.07636 |
| 0.2 | 0.04824 | 0.018315 | 0.15905 |
| 0.3 | 0.07173 | 0.169013 | 0.24845 |
| 0.4 | 0.09461 | 0.36787 | 0.34498 |
| 0.5 | 0.11674 | 0.52729 | 0.44907 |

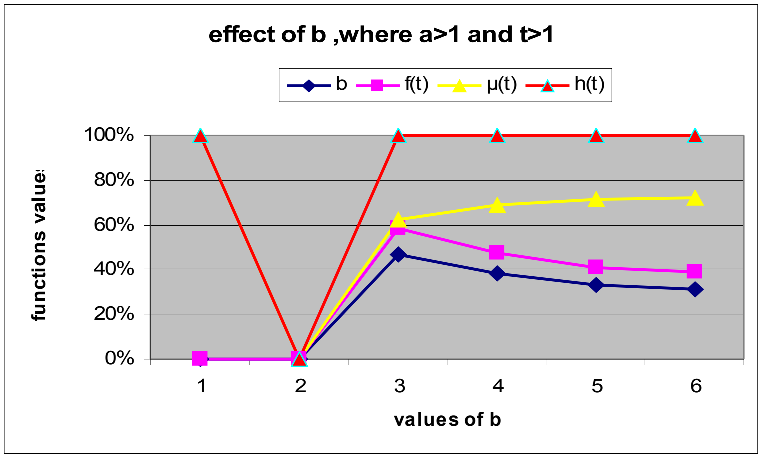
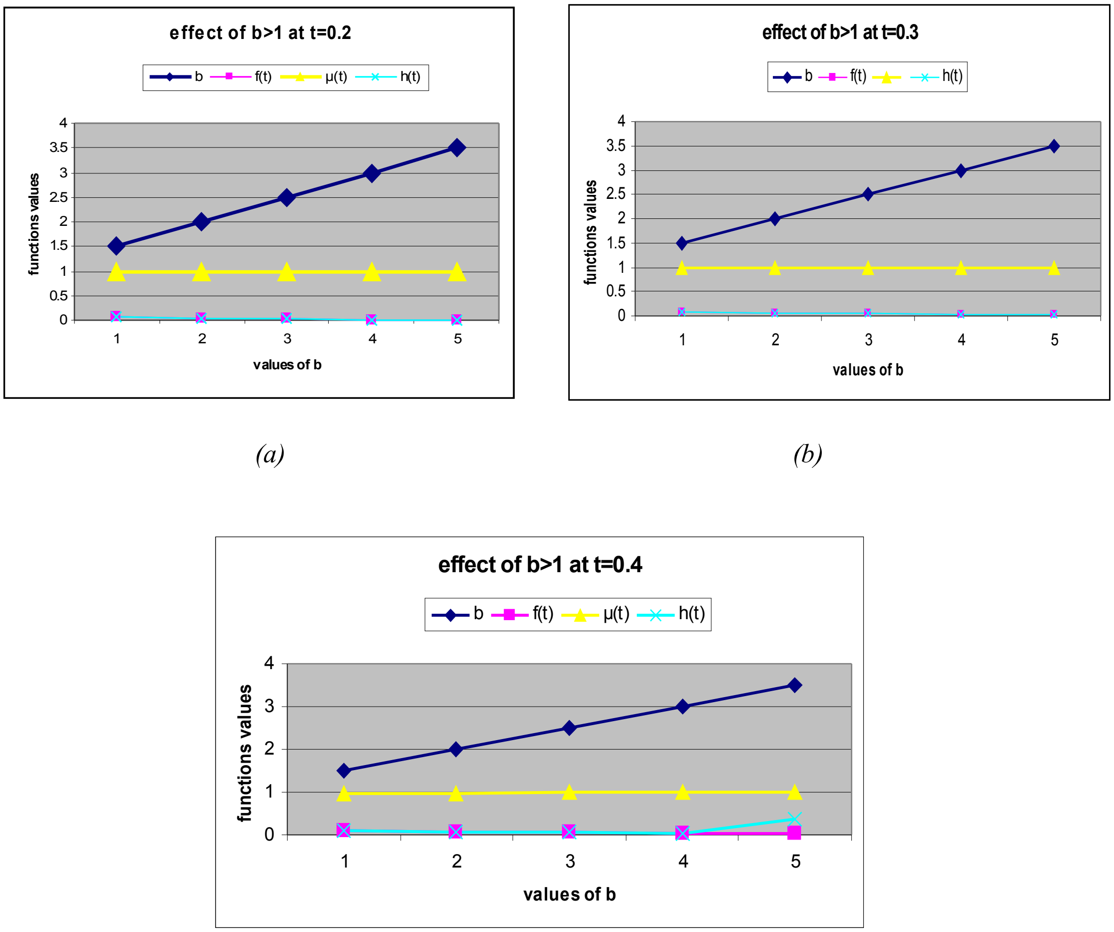



*Figure (6): Graphic of effect of shape parameter on the weibull function, Gaussian membership and failure rate ,when t< 1 at t=1.5 and a>1 at a=1.1 .*

Note that values of $\mu(t)$ ,f(t) and h(t) are increasing in high difference with increasing b and they reach largest values at largest b. Largest values at largest b which b=0.5.

*Table (5):The influence of the shape parameter on the weibull function ,Gaussian membership and failure rate when b>1, for different Values of t and scale parameter a=0.1.*

| t=0.2 , a=0.1 | | | |
|---|---|---|---|
| b | f(t) | $\mu(t)$ | h(t) |
| 1.5 | 0.0641 | 0.9955 | 0.0670 |
| 2 | 0.0398 | 0.9975 | 0.0400 |
| 2.5 | 0.0223 | 0.9984 | 0.0223 |
| 3 | 0.0119 | 0.9988 | 0.0120 |
| 3.5 | 0.00625 | 0.99918 | 0.0060 |
| t=0.3 , a=0.1 | | | |
| b | f(t) | $\mu(t)$ | h(t) |
| 1.5 | 0.0808 | 0.9823 | 0.0821 |
| 2 | 0.0594 | 0.9900 | 0.0600 |
| 2.5 | 0.0408 | 0.9936 | 0.0410 |
| 3 | 0.0269 | 0.9955 | 0.0270 |
| 3.5 | 0.01722 | 0.99674 | 0.01725 |
| t=0.4 , a=0.1 | | | |
| b | f(t) | $\mu(t)$ | h(t) |
| 1.5 | 0.0924 | 0.9607 | 0.0948 |
| 2 | 0.0681 | 0.9777 | 0.0800 |
| 2.5 | 0.0626 | 0.9857 | 0.0632 |
| 3 | 0.0476 | 0.9900 | 0.0480 |
| 3.5 | 0.03527 | 0.99267 | 0.3541 |




*(a)* *(b)*

*(c)*

*Figure (7): Graphics of effect of shape parameter on the weibull function, Gaussian membership and failure rate when b>1, for different values of t and scale parameter a=0.1; (a) t=0.2 .(b) t=0.3 .(c) t=0.4 .*

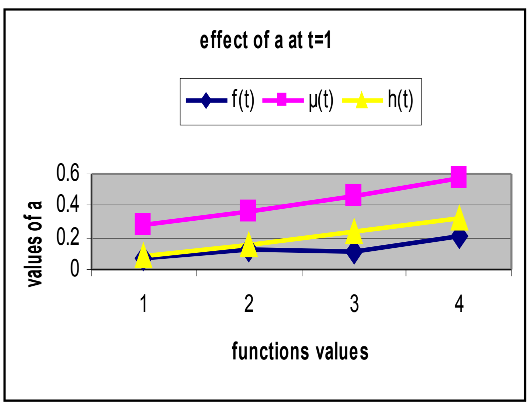
Looking to first two cases; with increasing b values, $\mu(t)$ increasing ,while f(t) and h(t) decreasing ,whit small differences .Failure rate reach largest value at largest for b at all cases ,but in third case it be different decreasing and then increasing .

### *5.3 Influence of Scale Parameter*

Studying influence of scale parameter values on values of weibull function ,Gaussian membership and failure rate require supposing different values for time that be at t<1 and t≥ 1 ,and parameter b at b<1 and b>1,while a take different values at a<1 with small differences .

*Table (6): The influence of the scale parameter on the weibull function, Gaussian membership and failure rate, for different values of t, such that b<1.*

| t=1 , b=0.8 | | | | |
|---|---|---|---|---|
| A<br>Fun, | 0.1 | 0.2 | 0.3 | 0.4 |
| f(t) | 0.0723 | 0.1309 | 0.1185 | 0.2145 |
| $\mu(t)$ | 0.2820 | 0.3678 | 0.4650 | 0.5697 |
| h(t) | 0.0800 | 0.1600 | 0.2400 | 0.3200 |
| Fun, | t=1.1 | | | |
| f(t) | 0.0704 | 0.1265 | 0.1703 | 0.2038 |
| $\mu(t)$ | 0.2096 | 0.2820 | 0.3678 | 0.4650 |
| h(t) | 0.0784 | 0.1569 | 0.2354 | 0.3139 |
| Fun, | t=1.2 | | | |
| f(t) | 0.0687 | 0.1416 | 0.1635 | 0.1949 |
| $\mu(t)$ | 0.1509 | 0.2096 | 0.2820 | 0.3678 |
| h(t) | 0.0771 | 0.1542 | 0.2314 | 0.3085 |

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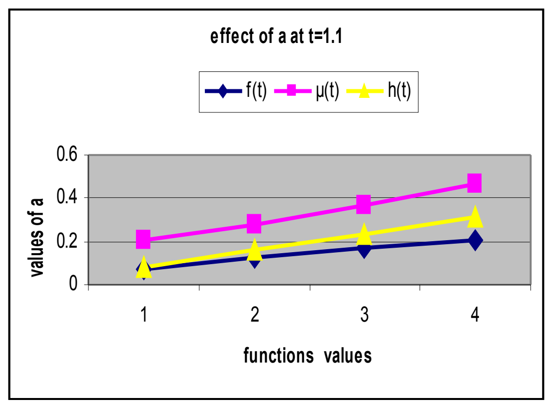


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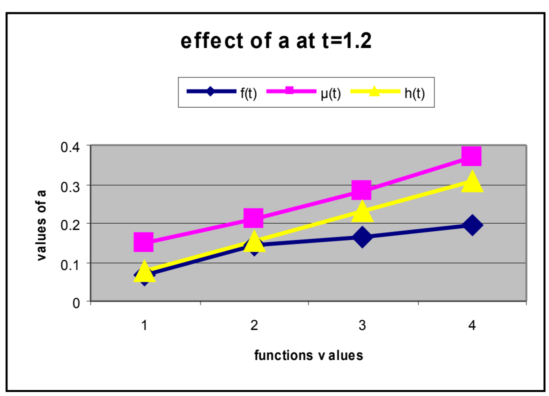


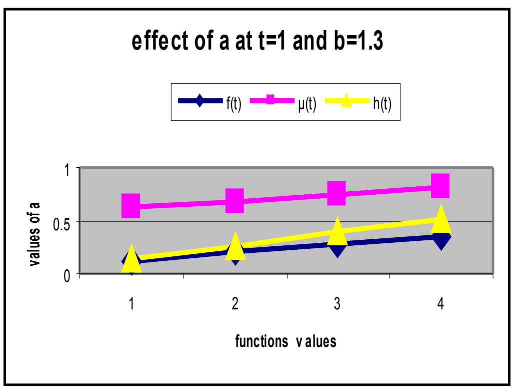

*(c)*

*Figure(8):Graphic of the scale parameter on the weibull function, Gaussian membership and failure rate, for different values of t, such that b<1,as;(a)t=1(b) t=1.1 (c) t=1.2 .*

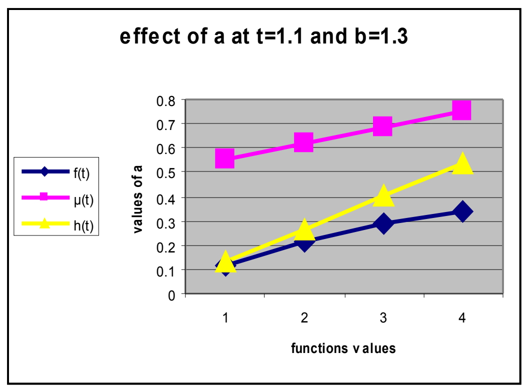
Looking to all cases; with increasing a values, $\mu(t)$ ,f(t) and h(t) are increasing . Failure rate reach largest value at largest for a and at smallest value for t at all cases .

*Table (7):The influence of scale parameter on the weibull function ,Gaussian membership and failure rate for different values of t and b>1 .*

| t=1, b=1.3 | | | | |
|---|---|---|---|---|
| A<br>Fun, | 0.1 | 0.2 | 0.3 | 0.4 |
| f(t) | 0.1176 | 0.2128 | 0.2889 | 0.3485 |
| $\mu(t)$ | 0.6192 | 0.6847 | 0.7483 | 0.80814 |
| h(t) | 0.1300 | 0.2600 | 0.3900 | 0.5200 |
| Fun, | t=1.1 | | | |
| f(t) | 0.1194 | 0.2133 | 0.28576 | 0.3402 |
| $\mu(t)$ | 0.5533 | 0.61922 | 0.6847 | 0.7483 |
| h(t) | 0.1337 | 0.2674 | 0.4013 | 0.5350 |
| Fun, | t=1.2 | | | |
| f(t) | 0.1209 | 0.2131 | 0.2816 | 0.3308 |
| $\mu(t)$ | 0.2981 | 0.5533 | 0.61922 | 0.6847 |
| h(t) | 0.1374 | 0.2746 | 0.4119 | 0.4224 |




*(a)*

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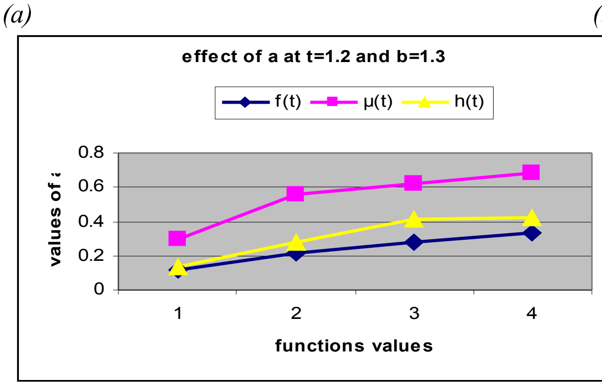


*(c)*

*Figure (9):Graphic of effect of the scale parameter on the weibull function, Gaussian membership and failure rate for different values of t and b>1 ,as;(a)t=1(b) t=1.1 (c) t=1.2 .*

Similarly to previous table and to all cases; increasing values of a ,the values of $\mu(t)$ ,f(t) and h(t) are increasing ,but failure rate reach largest value not at largest value for a ,but at smallest value for t for all cases .

***6.Conclusions***

Some numerical work has been carried out of investigate comparison functions. That is , weibull distribution f(t) ,membership function $\mu(t)$ and failure rate h(t) . Through the numerical examples that shown previously ,we observed that the weibull distribution and failure rate are increase as the time increases, but the Gaussian function decrease as the time increases ,such that b>1. The weibull distribution and Gaussian function are decrease as the time increases and the failure rate increase, but decrease as the time increases ,where b<1. So we show that, the weibull distribution, Gaussian function and failure rate are increases as the shape parameter increases for different values of time and scale parameter ,such that b>1.While the weibull distribution, Gaussian function and failure rate are increases as the scale parameter increases for different values of t, where b>1and b<1, we note that in some numerical results ;If failure rate decreases over time, then b<1.If failure rate constant over time, then b=1. If failure rate increases over time, then b>1. and for all it the Gaussian function is greater than weibull distribution.

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