On some types of λ-continuous function in Bitopological Spaces

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Abstract:
[singal and singal ,[1968] introduced several properties of almost continuous mapping and [T.Noiri , 1990] introduced a weak form of faint continuity . [Hassna H. and sajda K. , 2005] introduced λ-continuity in bitopological spaces .the purpose of this paper is to introduce and study some types of λ-continuity in bitopological spaces with some relation between them.

Introduction:
The open set In the special bitopological space $(X\Psi,\Psi^{\alpha})$ denoted by λ-open set and the collection of all λ-open sets forms a topological space on X denoted by Ψ_{λ} greater than Ψ . [N.Levine , 1961] introduced the concept of weak continuity as a generalized of continuity , later [Hussain ,1966] introduced almost continuity as another generalization and [Anderew and whitlesy , 1966]introduced the concept of closure continuity which is stronger than weak continuity .[singal and singal , 1968] introduced anew almost continuity which is different from that of hussain.the purpose of this paper is to further the study of the concept of strong λ-continuity and almost strongly λ-continuity ,faintly λ-continuity ,and weakly λ-continuity in bitopological spaces.

A function f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$is weakly λ-continuous function at a point $x\in X$ if given any ξ-open set V in Y containing f(x) ,there exist λ-open set U containing x such that $f(U)\subseteq cl_{\xi}(V)$.if the condition is satisfied at each $x\in X$ then f is said to be weakly continuous function. the function f is strongly λ-continuous function at $x\in X$ if given any ξ-open set V in Y containing f(x) ,there exist λ-open set U containing x such that $f(cl_{\Psi}(U)\subseteq(V)$.if the condition is satisfied at each $x\in X$ then f is said to be strongly λ-continuous function, and it is called almost λ-continuous if for each point $x\in X$ and each ξ-open set V in Y containing f(x) there exist λ-open set U in X containing x such that $f(U)\subseteq int_{\xi}(cl_{\xi}(V))$ .the function f is said to be almost strongly λ-continuous if and only if for ach $x\in X$ and ξ-nbd V of f(x) there exist λ-open set U containing x such that $f(cl_{\Psi}(U))\subseteq int_{\xi}(cl_{\xi}(V))$. A bitopological space (X,Ψ,Ψ^{α}) is called urysohn if for every $x\neq y$ in X there exist Ψ-open sets U,V in X such that $x\in U$,$y\in V$ and $cl_{\Psi}(U)\cap cl_{\Psi}(V)=\emptyset$ and if (X,Ψ) is Urysohn space then (X, Ψ,Ψ^{α}) is also urysohn space..

From the definitions stated above ,one can easily obtain the following diagram

Strongly λ-continuous⇒ λ-continuous⇐Faintly-continuous
⇓ ⇓
Almost strongly λ-continuous⇒almost λ-continuous⇒weakly λ-continuous

1- λ-continuous and strongly λ-continuous function

Theorem (1-1): let f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ be strongly λ-continuous and g: $(Y,\xi,\xi^{\alpha})\rightarrow(Z,\omega,\omega^{\alpha})$ be λ-continuous function then gof is strongly λ-continuous.
Proof: exist by definition.

Theorem(1-2): if f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is λ-continuous such that X is extremely disconnected then f is strongly λ-continuous.
Proof: let $x \in X$ and V is ξ-nbd of f(x) in Y , since f is λ-continuous there exist λ-open set U containing x such that $f(U)\subseteq V$ and since (X,Ψ) extremely disconnected then $f(cl_{\Psi}(U)\subseteq V$ and then f is strongly λ-continuous.

Theorem(1-3): A function f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is λ-continuous if and only if $cl_{\Psi}(f^{-1}(V))\subseteq f^{-1}(cl_{\xi}(V))$ for each ξ-open set V in Y.
Poof: let V is ξ-open set in Y , $f^{-1}(cl_{\xi}(V))$ is λ-closed set in X , now $V\subseteq cl_{\xi}(V)$ then $f^{-1}(V)\subseteq f^{-1}(cl_{\xi}(V))$and then $cl_{\Psi}(f^{-1}(V)) \subseteq cl_{\Psi}(f^{-1}(cl_{\xi}(V))= f^{-1}(cl_{\xi}(V))$.
Conversely , let V is ξ-closed set in Y then $cl_{\Psi}(f^{-1}(V)) \subseteq f^{-1}(cl_{\xi}(V))= f^{-1}(V)$. now since $f^{-1}(V) \subseteq cl_{\Psi}(f^{-1}(V))$ then $cl_{\Psi}(f^{-1}(V))= f^{-1}(V)$ there for $f^{-1}(V)$ is λ-closed set. f is λ-continuous function.

Theorem (1-4): let : $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ be strongly λ-continuous such that X is regular space and g: $(X,\Psi,\Psi^{\alpha})\rightarrow(X\times Y, \Psi\times \xi, \Psi^{\alpha}\times \xi^{\alpha})$ be the graph mapping given by g(x)=(x,f(x)) for each $x\in X$ then g is strongly λ-continuous.
Proof: let $x\in X$ and w be $\Psi\times\xi$-open set in $X\times Y$ containing g(x) .then there exist Ψ-open set G in X and ξ-open set H in Y such that $g(x)=(x,f(x))\in G\times H \subseteq w$, since X is regular space then there exist Ψ-open set B in X such that $cl_{\Psi}(B)\subseteq G$. since f is strongly λ-continuous there exist λ-open set U containing x such that $U\subseteq G$ and $f(cl_{\Psi}(U))\subseteq V$. let $C=U\cap B$ then C is λ-open set and $g(cl_{\Psi}(U))\subseteq cl_{\Psi}\times H\subseteq G\times H\subseteq w$ then g is strongly λ-continuous function.

Theorem(1-5) : let $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ be λ-continuous and g: $(X,\Psi,\Psi^{\alpha})\rightarrow(X\times Y, \Psi\times \xi, \Psi^{\alpha}\times \xi^{\alpha})$ be the graph mapping given by g(x)=(x,f(x)) for each $x\in X$ then g is λ-continuous.
Proof: let $x \in X$ and let w be $\Psi\times \xi$,-open set in $X\times Y$ containing g(x) then there exist Ψ-open set G in X and ξ-open set H in Y such that $g(x)=(x,f(x)) \in G\times H\subseteq w$ since f is λ-continuous there exist λ-open set U containing x such that $U\subseteq G$ and $f(U)\subseteq H$. there for $g(U)\subseteq G\times H\subseteq w$ then g is λ-continuous.

Definition(1-6): let f: $(X,\Psi,\Psi^{\alpha})\rightarrow(X,\Psi_{A},\Psi^{\alpha}_{A})$ be a function such that X is Urysohn space where $A \subseteq X$ and f/A is the identity function on A then f is λ-retraction

Theorem(1-7) : let A be a subset of X and f: $(X,\Psi,\Psi^{\alpha})\rightarrow(X,\Psi_{A},\Psi^{\alpha}_{A})$ be λ-retraction of X onto A if X is Urysohn space then A is λ-closed subset of X.
Proof : suppose that there exist $x\in cl(A)/A$ since f is λ-retraction we have $f(x)\neq x$. since X is Urysohn space then there exist λ-open sets U,V such that $x\in U$ and $f(x)\in V$, $cl_{\Psi}(U) \cap cl_{\Psi}(V)=\emptyset$. Let w be any λ-open set containing x then $U \cap w$ is λ-open sets containing x and then $cl_{\Psi}(U \cap w) \cap A\neq\emptyset$. Now since $x\in cl(A)$ then $y \in cl_{\Psi}(U \cap w) \cap A$. since $y \in A$, $f(y)=y \in cl_{\Psi}(U)$and hence $f(y)\notin cl_{\xi}(V)$, from that we get $f(cl_{\Psi}(w))$ is not contained in $cl_{\xi}(V)$ and this is contradiction.

2-λ-continuous function and weakly λ-continuous.

Theorem(2-1): if let f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is weakly λ-continuous such that (X,Ψ) extremely disconnected then $cl_{\Psi}(f^{-1}(V)) \subseteq f^{-1}(cl_{\xi}(V))$ for each ξ-open set V in Y.
Proof: let V is ξ-open se in Y ,then $f^{-1}(cl_{\xi}(V))$ is λ-open set in X and $f^{-1}(V) \subseteq (f^{-1}(cl_{\xi}(V))$, then $cl_{\Psi}(f^{-1}(V)) \subseteq cl_{\Psi}(f^{-1}(cl_{\xi}(V)$, since X is extremely disconnected $cl_{\Psi}(f^{-1}(V)) \subseteq f^{-1}(cl_{\xi}(V)$.

Theorem(2-2): a mapping f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is weakly λ-continuous if and only if $f^{-1}(V) \subseteq int_{\Psi}(f^{-1}(cl_{\xi}(V))$.
Proof: let f is weakly λ-continuous and V is ξ-open set in Y then $f^{-1}(cl_{\xi}(V))$ is λ-open set in X ,since $int_{\Psi}(f^{-1}(cl_{\xi}(V)).= f^{-1}(cl_{\xi}(V))$ then
$f^{-1}(V) \subseteq f^{-1}(cl_{\xi}(V))= int_{\Psi}(f^{-1}(cl_{\xi}(V))$.
Sufficiency, let $x\in X$ and V is ξ-nbd of f(x) in Y then $x\in f^{-1}(V) \subseteq int_{\Psi}(f^{-1}(cl_{\xi}(V))$.let $U=int_{\Psi}(f^{-1}(cl_{\xi}(V))$.then U is λ-open set containing x and $f(U) \subseteq cl_{\xi}(V)$ and there for f is weakly λ-continuous

Theorem(2-3): let f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is weakly λ-continuous such that Y is extremely disconnected space then f is λ-continuous.
Proof: let $x\in X$ and V is ξ-nbd of f(x) , since f is weakly λ-continuous there exist λ-open set U containing x such that $f(U) \subseteq cl_{\xi}(V)$, since (Y,ξ) extremely disconnected then $f(U)\subseteq V$ and then f is λ-continuous.

Theorem(2-4): let f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is weakly λ-continuous such that (Y,ξ) is extremely disconnected space then f is almost λ-continuous.
Proof: see the proof of theorem(2-3)

III- λ-continuous function and almost λ-continuous.

Theorem(3-1): if the mapping f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is λ-continuous such that (X,Ψ) is extremely disconnected then $cl_{\Psi}(f^{-1}(V)) \subseteq f^{-1}(cl_{\xi}(V)$for each ξ-open set V in Y.
Proof: .let V is ξ-open set in Y and since f is λ-continuous $f^{-1}(V)$ is λ-open set ,since we have that $f^{-1}(V)\subseteq f^{-1}(cl_{\xi}(V))$ and (X,Ψ) is extremely disconnected ,there for $cl_{\Psi}(f^{-1}(V)\subseteq f^{-1}(cl_{\xi}(V))$.

Theorem(3-2): let f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is almost λ-continuous and (Y,ξ) extremely disconnected then f is λ-continuous function.
Proof: let $x\in X$ and V is ξ-nbd of f(x) since f is almost λ-continuous there exist λ-open set U containing x such that $f(U)\subseteq int_{\xi} (cl_{\xi}(V)\subseteq cl_{\xi}(V)$ and since (Y,ξ) is extremely disconnected $f(U)\subseteq V$ and then f is λ-continuous.

Theorem(3-4): let f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is almost λ-continuous $f^{-1}(V)$ λ-pen set in X for each regular -open set V(resp. semi- open set V) in Y.
Proof: exist by definition.

Proposition(3-5): a function f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is almost λ-continuous if and only if $f^{-1}(V) \subseteq int_{\Psi}(f^{-1}(int_{\Psi} (cl_{\xi}(V))$ for each ξ-open set V in Y
proof: let f is almost λ-continuous and V is ξ-open set in Y then $f^{-1}(int_{\Psi} (cl_{\xi}(V))$ is λ-pen set in X and $V \subseteq int_{\xi}(cl_{\xi}(V))$ then we have that
$f^{-1}(V) \subseteq f^{-1}(int_{\xi} (cl_{\xi}(V))= int_{\Psi}(f^{-1}(int_{\xi} (cl_{\xi}(V))$.
sufficiency, let $x\in X$ and V is ξ-nbd of f(x) in Y then

$x\in f^{-1}(V)\subseteq int_\Psi(f^{-1}(\, int_\xi\,(cl_\xi(V))$. let $U = int_\Psi(\, f^{-1}(\, int_\xi\,(cl_\xi(V))$ then U is and of x and $U\subseteq f^{-1}(\, int_\xi\,(cl_\xi(V))$ and then $f(U)\subseteq(\, int_\xi\,(cl_\xi(V)$ there for f is almost λ-continuous .

Proposition(3-6): let $f: (X,\Psi,\Psi^\alpha)\to(Y,\xi,\xi^\alpha)$ is almost λ-continuous then $f^{-1}(V)$is λ-open set in X for each ξ-regular open set in Y.
Proof: let V is ξ-regular pen set in Y then V is ξ-open set , since f is almost λ-continuous there exist λ-open set U in X such that $f(U)\subseteq int_\xi\,(cl_\xi(V)$ and then $f(U)\subseteq V$, there for $f^{-1}(V)$ is λ-open set in X.

Theorem(3-7): almost λ-continuous function $f:(X,\Psi,\Psi^\alpha)\to(Y,\xi,\xi^\alpha)$ such that Y is ξ-extremely disconnected is λ-continuous if and only if $int_\Psi(\, f^{-1}(V))= f^{-1}(\, int_\xi V)$
Proof: let $x \in X$ and V is ξ-nbd of f(x), since f is almost λ-continuous there exist λ-open set U containing x and $f(U)\subseteq(\, int_\xi\,(cl_\xi(V)$ and since (Y,ξ) is extremely disconnected $f(U)\subseteq V$,there for f is λ-continuous.
Conversely, let f is λ-continuous and V is ξ-open set , by theorem(3-5)
$f^{-1}(V)\subseteq int_\Psi(\, f^{-1}(\, int_\xi\,(cl_\xi(V)) = int_\Psi(\, f^{-1}\,(\, int_\xi\,(V))= int_\Psi(\, f^{-1}\,(V))$and then
$f^{-1}\,(\, int_\xi\,(V))\subseteq int_\Psi(\, f^{-1}\,(V))$, also $int_\Psi(\, f^{-1}\,(V))\subseteq f^{-1}(V)= f^{-1}\,(\, int_\xi\,(V))$ from that we get $int_\Psi(\, f^{-1}\,(V))= f^{-1}\,(\, int_\xi\,(V))$.

Theorem(3-8) : let $f: (X,\Psi,\Psi^\alpha)\to(Y,\xi,\xi^\alpha)$ is almost λ-continuous such that (X,Ψ) extremely disconnected then f is almost strongly λ-continuous.
Proof :let $x\in X$ and V is ξ-open set containing f(x) , since f is almost λ-continuous there exist λ-open set U containing x such that $f(U)\subseteq int_\xi(cl_\xi(V))$, since Xis Ψ-extremely disconnected $f(cl_\Psi\,(U))\subseteq int_\xi(cl_\xi(V))$ and then f is almost strongly λ-continuous.

Example(3-9): let X={a,b,c,d} and Ψ={X,∅,{a},{a,b}}
Ψ_λ={X,∅,{a},{a,b},{b},{d},{a,d},{b,d},{a,b,d}}
Y={r,p,q} and ξ={Y,∅,{r},{q},{r,q}}.let $f: (X,\Psi,\Psi^\alpha)\to(Y,\xi,\xi^\alpha)$ defined by f(a)=f(b)=p , f(c)=q then f is weakly λ-continuous but it is neither λ-continuous nor almost λ-continuous

Example(3-10): let X=R and Ψ=co-countable topology and Y={a,b}, ξ={Y,∅,{a}}.let $f: (X,\Psi,\Psi^\alpha)\to(Y,\xi,\xi^\alpha)$ define by f(x)=a if $x\in Q$ and f(x)=b if $x\in Q^c$ then f is almost λ-continuous and strongly λ-continuous but not λ-continuous.

Definition(3-11): A function $f: (X,\Psi,\Psi^\alpha)\to(Y,\xi,\xi^\alpha)$ is said to be faintly λ-continuous if for each $x\in X$ and each λ-open set V containing f(x) there exist Ψ-open set U containing x such that $f(U)\subseteq V$.

Lemma(3-12): if the function $f: (X,\Psi,\Psi^\alpha)\to(Y,\xi,\xi^\alpha)$ is faintly λ-continuous and $g: (Y,\xi,\xi^\alpha)\to(Z,\omega,\omega^\alpha)$ is strongly λ-continuous then gof is strongly λ-continuous
proof: let $x\in X$ and Vis ω-open set containing g(y) where y=f(x) , since g is strongly λ-continuous there exist λ-open set U in Y such that $g(cl_\xi(U))\subseteq V$ .since f is faintly λ-continuous there exist

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Lemma(3-13): if the function $f: (X,\Psi,\Psi^\alpha)\to(Y,\xi,\xi^\alpha)$ is λ-continuous and $g: (Y,\xi,\xi^\alpha)\to(Z,\omega,\omega^\alpha)$ is faintly λ-continuous then gof is λ-continuous
proof: let V is ω-open set n Z since g is faintly λ-continuous $g^{-1}(V)$ is ξ-open set in Y , since f is λ-continuous $f^{-1}(g^{-1}(V))$ is λ-open set in X. since $f^{-1}(g^{-1}(V))=(gof)^{-1}$ then gof is λ-continuous.

Theorem(3-14): if f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is faintly λ-continuous such that X is extremely disconnected then f is strongly λ-continuous(almost λ-continuous)

Proof: let $x\in X$ and V is ξ-open set containing f(x) since f is faintly λ-continuous and V is λ-open set in Y there exist Ψ-open set U containing x such that $f(U)\subseteq V$, now since X is Ψ-extremely disconnected and U is λ-open set in X $f(cl_{\Psi}(U))\subseteq V$ and there for f is strongly λ-continuous . similar argument uses to proof f is almost λ-continuous.

V-Super λ-continuous and completely λ-continuous

Definintion(4-1): a mapping f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is super λ-continuous if and only if for each $x\in X$ and ξ-nbd V of f(x) there exist λ-open set U in X such that $f(int_{\Psi}(cl_{\Psi}(U))\subseteq V$.

Definition (4-2): a mapping f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$ is completely λ-continuous if and only if for each $x\in X$ and ξ-nbd V of f(x) there exist Ψ-regular open set U in X such that $f(U))\subseteq V$.

From these definitions we obtain the following diagram

Completely λ-continuous⇒faintly λ-continuous ⇒λ-continuous
⇓ ⇓
Strongly λ-continuous⇒super λ-continuous⇒⇒⇒⇒⇒ almost λ-continuous
⇓
Weakly λ-continuous

Theorem(4-3): if the mapping f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$is super λ-continuous such that (X,Ψ) is semi regular space then f is completely λ-continuous .

Proof: let $x\in X$ and V is ξ-nbd of f(x), since f is super λ-continuous there exist λ-open set U in X such that $f(int_{\Psi}(cl_{\Psi}(U))\subseteq V$ and since X is Ψ-semi regular space then $f(U)\subseteq V$ then f is completely λ-continuous.

Theorem(4-4): if the mapping f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$is completely λ-continuous such that (X,Ψ) is extremely disconnected then f is almost strongly λ-continuous .

Proof: let $x\in X$ and V is ξ-open set containing f(x) , since f is completely λ-continuous thre exist regular open set U containing x such that $f(U)\subseteq V$, since U isregular open then $f(U)\subseteq V\subseteq int_{\xi}(cl_{\xi}(V))$, also since X isΨ-exteremly disconnected then $f(cl_{\Psi}(U))\subseteq int_{\xi}(cl_{\xi}(V))$

Theorem(4-5): if the mapping f: $(X,\Psi,\Psi^{\alpha})\rightarrow(Y,\xi,\xi^{\alpha})$is weakly λ-continuous such that (X,Ψ) is semi regular space and (Y,ξ) is extremely disconnected then f is super λ-continuous .

Poof: let $x\in X$ and V is ξ-open set containing f(x), since f is weakly λ-continuous there exist λ-open set U containing x such that $f(U)\subseteq cl_{\xi}(V)$, now since X is Ψ-semi regular space and Y is ξ-extremely disconnected then $f(int_{\Psi}(cl_{\Psi}(U))\subseteq V$ and then f is super λ-continuous.

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بعض أنواع λ-continuous في الفضاءات ثنائية التبولوجي

ملخص البحث: الغرض من هذا البحث هو دراسة بعض أنواع الدوال المستمرة في الفضاء ثنائي التبولوجي بلا ضافة إلى دراسة بعض النظريات والعلاقات بين هذه الدوال .