Structural and Thermal Analysis of Heat Exchanger with Tubes of Elliptical Shape

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Abstract:
An approach to select the tube wall thickness distribution of streamlined tubes intended for use in heat exchangers is developed in this study. The main goal is to retain a streamlined outer profile (resist deformation) and to prevent strain failure due to the applied internal pressure. The effect of the tube wall thickness distribution on shaped tube efficiency is also considered. The strain is calculated as a function of several dimensionless geometric ratios and the ratio of the internal pressure to elastic material modulus. Using the finite element method, a set of dimensionless design curves is created for elliptical tube geometries. From these curves, a range of possible materials and tube geometries can be selected that meet a specific strain limit. To illustrate the approach, structure-satisfied elliptical designs are selected and their thermal performance is evaluated for an automotive charge air cooler made of polymeric material.

التحليل التركيبي و الحراري لمبادل حراري ذو أنابيب بيضوية الشكل

نورس حيدر         قصي رشيد

الخلاصة

تتضمن هذه الدراسة عملية اختيار توزيع سمك جدار الأنابيب لطية لنظام الأنابيب الانسيابية المستخدمة في المبادلات الحرارية. إن الهدف الرئيسي هو الحفاظ على الشكل الانسيابي الخارجي للأنابيب (أي مقاومة التدفق) ومع الص(stdout) الاحترافي نتيجة الضغط المستمر داخل تلك الأنابيب. كما تم دراسة تأثير توزيع ضغط جدار الأنابيب على كفاءته (الكفاءة الحرارية). تم حساب الانفعال كدالة لنسب هندسية لا ت الحديدية بالإضافة إلى نسبة الضغط الداخلي إلى معامل الموتينة لمادة الأنبوب. تم استخدام طريقة العناصر المحددة لإيجاد مجموعة من منحنى التصميم اللابعادية الخاصة بالنابيب البيضوية المقطع. هذه المنحنى، أعطى مدى لاختيار المادة و الشكل الهندسي المحدد بفاعل معين. و لتوضيح هذه الطريقة تم اختيار مبرد هواء الشحن في السيارة ذات أنابيب بيضوية الشكل مصنعة من مادة لدانية لتحقيق التصميم البيروي و تحسين الأداء الحراري لها.

Keywords: elliptical tube, shaped tube efficiency, polymer, heat exchanger, deformation, strain, stress
1- Introduction

Polymer heat exchangers have been used for decades in corrosive environments [1–4] and are now being considered in other applications where either weight is a concern or innovative geometries are desirable. The most common polymer heat exchanger is a tube bundle made of hundreds of circular tubes connected to headers. Elliptical and lenticular profiles are being considered due to their ability to reduce form drag over a wide range of flow rates [6–12]. There are three primary challenges to design the streamlined tubes:

a. The tube wall thickness distribution (i.e. the geometry of the inner flow passage) must be selected so that the maximum stress in the tubes is less than the mechanical strength of the material.

b. The deformation of the tubes must be within a range to avoid strain failure and to maintain a streamlined profile.

c. The tube wall thickness distribution should be selected to reduce the wall thermal conductive resistance.

For polymers, which have low thermal conductivity and strength compared with metals, there is a significant trade-off between the thermal performance and the mechanical design requirements. Based on earlier work [13], it is anticipated that a non-uniform wall distribution will emerge as the optimum solution.

Two mechanical failure modes must be considered: burst failure and strain failure [14]. Because polymers creep, a strain limit may be exceeded before the stresses exceed the burst strength. Young’s modulus for polymeric material is significantly lower than those for most metals. Consequently, for the same loading, a polymer structure must be thicker than a metal structure to avoid strain failure.

Stress distribution and deformation in circular and elliptical tubes with uniform wall thickness are well understood. Exact solutions of stress distribution and deformation in internally pressurized circular tubes are available in standard reference texts (see, for example, reference [15]). The hoop and radial stresses are axisymmetric; there is only radial displacement. Stresses and deformation have also been analyzed for non-circular tubes, primarily elliptical tubes [16–18]. In elliptical tubes, stresses concentrate at the major and minor axes [16] and deformation is non-uniform [17, 18].

The use of a non-uniform wall to reduce stress in elliptical tubes was proposed by Holland [13]. He identified optimum wall thickness distributions to minimize the maximum bending stress for minor to major-axis ratios from 0.1 to 1.0. The geometry that he proposed has the maximum thickness at the major axis and the minimum thickness at the minor axis. His analysis is based on a beam approximation which is limited to thin-walled tubes with small deformations.

The objective of the present study is to develop an approach to optimize the inner flow passage of streamlined polymer tubes intended for use in heat exchangers. The goal is to retain a streamlined outer profile (resist deformation) and to prevent failure due to the pressure imposed by the heat transfer fluid flowing in the tube. Dimensionless parameters are utilized throughout the analysis such that the results are applicable to elliptical tubes of all sizes. This approach can be extended to other outer shapes.

2- Approach

The effect of the inner geometry and tube wall thickness on stress and deformation of elliptical tubes was determined numerically using finite element analysis for internally pressurized tubes. Several dimensionless geometric parameters are required to define the relative size and shapes of the outer and inner ellipses (Fig. 1):

(a) \( \lambda_o \), the length ratio of the minor axis to major axis of the outer ellipse.
(b) $\lambda_i$, the length ratio of the minor axis to major axis of the inner ellipse.

(c) $\lambda_i$, the length ratio of the wall thickness at $\theta = 90^\circ$ to the semi minor axis of the outer ellipse.

The optimum inner shape is one in which the stresses and strains are within material limits and the heat transfer is greatest. To select an optimum inner shape for a fixed outer shape $\lambda_o$, a two-step approach is undertaken. The stress and strain limits are imposed and a set of acceptable inner ellipses ($\lambda_i$ and corresponding $\lambda_i$) are identified. From this set, the inner shape that maximizes the thermal performance for a specific application may be selected.

For polymer tubes, the stress failure criterion is the hydrostatic burst strength [19] measured under temperature and pressure conditions similar to those of the application. The strain failure criterion was selected to limit the deformation of the tube. A strain limit of 0.05 is specified in the hydrostatic burst test, such that the burst strength is defined by either the pressure at which the tube ruptures or the pressure at which the tube strain reaches 0.05. Because polymers creep, the 0.05 strain limit is often exceeded before the tube ruptures. Thus, a maximum von Mises strain $\varepsilon_{\text{max}}$ of 0.05 was selected as the criterion for selecting inner elliptical shapes which satisfy the mechanical failure requirements. It will be shown that by limiting the von Mises strain to less than $\varepsilon_{\text{max}}$, the tube will remain streamlined even after deformation.

The strain within the tube was determined by solving the force equilibrium equations for a tube subjected to internal pressure $P_i$. Because the tube geometry is complex, the solution to the equilibrium equations was obtained by the finite element method. Regardless of the solution approach, the tube geometry, material properties, loads, and boundary conditions must be specified. The strain is reported in the form of $\varepsilon_{\text{max}} E/P_i$ as a function of dimensionless parameters $\lambda_i$, $\lambda_o$, and $\lambda_i$. The range of values for each parameter was selected to ensure that the results are applicable to a wide range of materials, geometries, and loading conditions.

2.1- Tube geometry and boundary conditions

Two-dimensional elliptical tubes were drawn using ANSYS. Taking advantage of the tube symmetry, one-quarter of the tube shape was drawn in the x–y plane (Fig. 2) and meshed with the four-node element for plane stress PLANE42. This plane stress element has two degrees of freedom, the displacement in the x and y directions, at each node. A typical quarter-model of the elliptical shape required approximately 2000 elements. Mesh density was varied to determine the appropriate number of elements for convergence.
Tubes of varying geometries were modeled by changing the dimensionless length ratios $\lambda_o$, $\lambda_i$, and $\lambda_t$ (Table 1). Specifically, three outer geometries are considered: $\lambda_o = 0.3$, 0.5, and 0.8 with four inner elliptical shapes are considered: $\lambda_i = 0.3$, 0.5, 0.8, and 1.0. The relative size of the inner to outer ellipse is determined by $\lambda_t$, which was varied from 0.04 to 1.0. As these parameters are decreased, the ellipse becomes more slender. A uniformly distributed constant pressure $P_i$ was applied normal to the inner elliptical surface. At the outer elliptical surface, the pressure was set to zero.

### 2.2-Material properties

The material is modeled to be linear elastic. Only Young’s modulus $E$ and Poisson’s ratio $\nu$ are required to define the material stress–strain relationship. This material model is valid for most metals and for polymers loaded within the linear viscoelastic range. In this study, a polymer tube material is considered. Nylon (PA) material is considered for polymer heat exchangers. Long-term viscoelastic properties for these materials have been recently published [14]. The module of this polymer is 182 MPa for PA in air which represents the high value found among plastics. Aluminum and copper, with module of $7.3 \times 10^4$ MPa and $1.3 \times 10^5$ MPa respectively, are metals often used in heat exchangers. To account for the performance of both metals and polymers over a range of operating conditions, the dimensionless parameter $E/P_i$ was varied from 20 to 1000. Poisson’s ratio for the material was set to 0.45 (that of polymers) regardless of the value of $E/P_i$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer elliptical shape $\lambda_o$</td>
<td>0.3, 0.5, 0.8</td>
</tr>
<tr>
<td>Inner elliptical shape $\lambda_i$</td>
<td>0.3, 0.5, 0.8, 1</td>
</tr>
<tr>
<td>Relative size $\lambda_t$</td>
<td>0.04-1</td>
</tr>
<tr>
<td>Modulus/load $E/P_i$</td>
<td>20-1000</td>
</tr>
<tr>
<td>Poisson's ratio $\nu$</td>
<td>0.45</td>
</tr>
</tbody>
</table>
2.3- Finite element output

Since a strain failure criterion is imposed, the maximum von Mises strain $\varepsilon_{\text{max}}$ is calculated as part of the post-processing option in ANSYS. Because a linear material constitutive law was adopted, the strain results can be represented by the dimensionless parameter $\varepsilon_{\text{max}} E/P_i$. This parameter is a variation of the dimensionless parameter $\sigma_{\text{max}}/P_i$, which is often used for linear elastic materials in which the stresses $\sigma$ vary linearly with the load. Displacement is also of interest, particularly in the case of polymer tubes. Large displacement of the tube indicates that the tube may take a new shape and subsequently heat transfer and pressure drop will be affected.

2.4- Shaped tube efficiency

The shaped tube efficiency is defined as

$$\eta \equiv \frac{q}{q_{\text{max}}} = \frac{T_o - T_\infty}{T_i - T_\infty}$$

where $q$ is the actual heat transfer rate of the shaped tube and $q_{\text{max}}$ is the heat transfer rate that would be achieved if the spatially averaged temperature of the surface of the shaped tube, $T_o$, were equal to the average temperature of the base of the fin, $T_i$ at $r = r_i$.[5]

3- Results and discussion

The effect of tube geometry, materials, and loading on tube strain is illustrated in Fig. 3. This set of graphs shows the thickness ratio $\lambda_i$ as a function of $\varepsilon_{\text{max}} E/P_i$. The three plots correspond to three different values of $\lambda_o$ (0.3, 0.5, and 0.8). Model output for each inner elliptical shape ($\lambda_i = 0.3, 0.5, 0.8, \text{and } 1.0$) are shown as solid curves on each plot. For $\varepsilon_{\text{max}} E/P_i < 15$, the thickness ratio decreases significantly with increasing $\lambda_i$. For $\varepsilon_{\text{max}} E/P_i > 40$ the thickness ratio remains fairly constant and approaches a minimum for each inner shape.

Given an application and material, tube shapes which satisfy a particular strain failure criterion can be identified as follows. The value of $\varepsilon_{\text{max}} E/P_i$ is defined by the pressure requirements of the application, the material modulus, and the strain failure limit. A vertical line passing through this ratio intersects each $\lambda_i$ curve for a selected value of $\lambda_o$. For the selected $\lambda_i$, a corresponding value of $\lambda_o$ is shown on the ordinate. Thus, the sets of dimensionless geometry parameters $\lambda_o$, $\lambda_i$, and $\lambda_r$, which satisfy the strain limit for a particular application, can be identified.

To illustrate the use of plots, consider an application in which the strain limit is 0.05 and $E= P_i$ is 600 ($\varepsilon_{\text{max}} E/P_i = 30$). Then, the results shown in Fig. 3 can be plotted on a single graph (Fig. 4). The curves correspond to outer elliptical shapes of $\lambda_o = 0.3, 0.5, \text{and } 0.8$. Regardless of the outer shape, $\lambda_i$ decreases with increasing $\lambda_o$. Each curve represents combinations of thickness and inner shape that meet the pressure and strain limit requirements. For a particular outer shape ($\lambda_o$), there are many combinations of thickness and inner shape from which to choose. As expected, the circular tube requires the thinnest wall. However, the circular tube produces the greatest form drag and may not provide the maximum heat transfer.
Fig. 3 Selection of the thickness ratio $\lambda_t$ for the specified values of $\varepsilon_{\text{max}} E/P_i$ for elliptical tubes with $\lambda_o = 0.3, 0.5, 0.8,$ and 1.0 at (a) $\lambda_o = 0.3$, (b) $\lambda_o = 0.5$, and (c) $\lambda_o = 0.8$
The effect of tube shape on deformation is illustrated in Fig. 5 for $E/P = 600$. Fig. 5(a) to (e) are tube designs based on a strain limit of 0.05 and Figs 5(f) and (g) are tube designs based on a strain limit of 0.10. The outer elliptical shapes shown are for $\lambda_o = 0.5$ and 0.8, and the inner shapes are for $\lambda_i = 0.5$ and 1.0.

The outer elliptical shapes shown are for $\lambda_o = 0.5$ and 0.8, and the inner shapes are for $\lambda_i = 0.5$ and 1.0. In contrast with a circular tube [Fig. 5(c)], when the outer shape is an ellipse, the deformation is not uniform and is greatest at the minor axis, $\theta = \pm 90^\circ$. The effect of the strain limit
on the outer profile is illustrated by comparing the deformation of an elliptical tube at 0.05 strain with that at 0.10 strain. Comparing Fig. 5(a) with Fig. 5(f), which both have \( \lambda_o = 0.5 \) and \( \lambda_i = 1.0 \), it is shown that, when the strain limit is 0.05, the streamlined shape is maintained. When the strain limit is 0.10, the deformation at the minor axis is significant, creating a sharp protrusion at \( \theta = \pm 90^\circ \) that is expected to increase form drag.

4- Case study

The final selection of tube geometry depends on the required thermal duty of the tubes and the operating conditions. Case studies are presented for an automotive charge air cooler and radiator made of nylon tubes. In this exercise, the tube is assumed to have \( \lambda_o = 0.5 \) with a fixed outer radius \( r_o = 2\text{mm} \), representative of nylon circular tubes made by a major polymer manufacturer. The inner shape is selected with the criteria of 0.05 strain and maximum heat transfer rate. Two tubes are considered. Tube A has a circular inner flow passage (\( \lambda_i = 1 \)), and tube B has an elliptical inner flow passage with \( \lambda_i = 0.5 \).

Nominal operating conditions for the two heat exchangers are provided in Table 2. For these conditions, \( \varepsilon_{\text{max}} E / P_i \) for 0.05 maximum strain is 30 for the charge air cooler. The module of nylon are assumed to be 182 MPa in air. Table 3 provides the required \( \lambda_i \), determined from Fig. 3(b), together with the specified geometric parameters.

| Table 2 Nominal operating conditions of an automotive charge air cooler [6] |
|-----------------|--------|---------|--------|--------|--------|
| **Automotive application** | **Surface** | **Fluid** | **\( T_{\text{outer}} \) (°K)** | **\( T_{\text{inner}} \) (°K)** | **U (m/s)** | **\( P_i \) (MPa)** |
| Charge air cooler | Outer | Air | 298 | 400 | 8 | 0.3 |

| Table 3 Charge air cooler and radiator tube geometry based on a strain limit of 0.05. Tube A has a circular inner flow passage (\( \lambda_i = 1 \)) and tube B has an elliptical inner flow passage (\( \lambda_i = 0.5 \)) |
|-----------------|--------|--------|--------|--------|--------|--------|--------|
| **Application** | **Tube** | \( r_o \) (mm) | \( \lambda_o \) | \( \lambda_i \) | \( \lambda_i \) | \( L_o \) (mm) | \( r_i \) (mm) | \( L_i \) (mm) | \( A_i \) (mm²) |
| Charge air cooler | A | 2 | 0.5 | 1 | 0.084 | 4 | 1.83 | 1.83 | 10.52 |
| Charge air cooler | B | 2 | 0.5 | 0.5 | 0.4 | 4 | 1.2 | 2.4 | 9.04 |

The heat transfer rate is determined numerically using the commercial software ANSYS. The temperature distribution is computed by solving the two-dimensional conduction problem across the elliptical tube wall. The heat transfer rate per unit length tube is calculated by integrating the heat flux over the surface of the tube according to

\[
Q = \int_{\theta=0}^{2\pi} q R d\theta
\]  
(2a)

where \( q \) is the heat flux calculated from the temperature gradient on the tube wall surface

\[
q = -k_w \left( \frac{\partial T}{\partial n} \right)_{\text{wall}}
\]  
(2b)
and $k_w$ is the thermal conductivity of the material. The thermal conductivity of the nylon is assumed to be 0.24 W/mK. Fluid properties are evaluated at the inlet temperature and pressure. Triangular meshes were refined until the heat transfer rate varied less than 0.01. Heat transfer coefficients, will calculate from equations (3a), (3b), and (4), are applied as boundary conditions at the outer and inner surface of the tubes.

The heat transfer coefficients are assumed to be uniform along the outer and inner heat transfer surfaces. The outer overall convective heat transfer coefficient $h_o$ is determined from a Nusselt number correlation for a circular and an elliptical tube [12] according to

$$Nu_{2L_o} = 0.27 Re_{2L_o}^{0.6} Pr^{0.37} \left( \frac{Pr}{Pr_w} \right)^{0.25}$$

$$2000 \leq Re_{2L_o} \leq 2 \times 10^4$$

(3a)

and

$$h_o = \frac{Nu_{2L_o} k_f}{2L_o}$$

(3b)

The overall convective heat transfer coefficient in the tubes is determined assuming a uniform heat flux boundary. For these tubes and operating conditions, flow in the tube is laminar. For fully developed laminar flow in an elliptical channel [20],

$$Nu_{D_h} = \frac{6}{11} \left( \frac{D_h}{r_i} \right)^2 (1 + \lambda_i^2)$$

(4a)

In this expression, the hydraulic diameter $D_h$ equal to

$$D_h = \frac{4A_i}{P_{inner}} = \frac{2r_i L_i}{\sqrt{0.5(r_i^2 + L_i^2)}}$$

(4b)

Heat transfer rates expressed per unit length of tube are presented in Fig. 6. For the charge air cooler, the tube with an elliptical inner flow passage provides slightly improved performance compared with the tube with the circular flow passage.

**Fig. 6** Heat transfer rate of an elliptical tube with $\lambda_i = 0.5$ with a circular inner channel (A) and an elliptical inner channel (B) with $\lambda_i = 0.5$ in an automotive charge air cooler ($E/P_i = 600$).

In general, it may be concluded that, if conduction across the polymer wall poses the dominant thermal resistance, optimum thermal performance for a specified strain limit and outer shape is obtained by selecting the tube geometry which minimizes the amount of polymeric material; i.e. the shape of the inner flow passage with the greatest cross sectional area should be selected. On the other hand, if the dominant thermal resistance is the convective heat transfer inside the tube,
optimum performance is achieved by selecting the inner shape that provides the maximum heat transfer surface area; i.e. the shape with the greatest perimeter should be selected. To help guide the selection of inner tube geometry for thermal design, Fig 7 show plots of shaped tube efficiency for shaped elliptical tubes with $\lambda_o = 0.3, 0.5, 0.8, \text{ and } 1.0$ at (a) $\lambda_i = 0.3$, (b) $\lambda_i = 0.5$, and (c) $\lambda_i = 0.8$.
5- Conclusions

A method for analyzing the mechanical and thermal performance of streamlined tubes intended for use in polymer heat exchangers is presented. The mechanical analysis considers the case in which the outer tube shape must remained streamlined (i.e. deformation is limited) and the inner flow passage is designed for optimal thermal performance. This combination of requirements is a particular challenge for polymer materials. The key to the mechanical analysis is use of dimensionless parameters including two dimensionless length scales, which characterize the tube geometry, and the ratio $\varepsilon_{\text{max}} E/P_i$, which captures the effect of material stiffness and loading. Using this approach, a set of design curves can be generated from which combinations of tube geometry and materials can be selected that satisfy the deformation constraint. Once specific geometries that satisfy the mechanical constraint are identified, thermal performance such as the shape tube efficiency can be evaluated.

The method was demonstrated for elliptical tubes of non-uniform wall thickness. A finite element solution for the strain as a function of the tube material and geometry was determined for several geometries and a family of design curves for elliptical tubes was created.

References


**APPENDIX**

**Notation**

- $A_i$ cross-sectional area of the inner flow passage ($m^2$)
- $D_h$ hydraulic diameter of an elliptical tube ($m$)
- $E$ Young’s modulus (Pa)
- $h_o$ overall outer heat transfer coefficient (W/m$^2$ K)
- $k_w$ thermal conductivity (W/m K)
- $L_i$ length of the semi major axis of the inner surface of the tube (m)
- $L_o$ length of the semi major axis of the outer surface of the tube (m)
- $n$ vector normal to the wall
- $Nu$ Nusselt number
- $Pi$ internal fluid pressure (Pa)
- $P_{inner}$ inner perimeter ($m$)
- $Pr$ Prandtl number evaluated at the fluid bulk temperature
- $Pr_{tw}$ Prandtl number evaluated at the tube wall surface temperature
- $q$ heat flux (W/m$^2$)
- $Q$ heat transfer rate per unit length tube (W)
- $r$ radial coordinate ($m$)
- $r_i$ length of the semi minor axis of the inner surface of the tube (m)
- $r_o$ length of the semi minor axis of the outer surface of the tube (m)
- $Re$ Reynolds number
- $t_{90}$ wall thickness at $\theta = 90^\circ$ ($m$)
\( T_{\text{inner}} \) fluid temperature inside the tube (K)
\( T_{\text{outer}} \) fluid temperature outside the tube (K)
\( U \) outer cross-flow air velocity (m/s)

\( \varepsilon \) von Mises strain
\( \eta \) shaped tube efficiency
\( \theta \) angular coordinate of a point on the ellipse (deg)
\( \lambda_i \) length ratio of the minor axis to the major axis of the inner surface = \( r_i/L_i \)
\( \lambda_o \) length ratio of the minor axis to the major axis of the outer surface = \( r_o/L_o \)
\( \lambda_t \) length ratio of the wall thickness \( t_{90} \) to the outer semi minor axis = \( t_{90}/r_o \)
\( v \) Poisson’s ratio
\( \sigma \) von Mises stress (Pa)

**Subscripts**

f fluid
max maximum value
w wall material
wall tube wall