Inelastic magnetic electron scattering form factors of the $^{26}$Mg nucleus

KHALID S JASSIM$^{1,*}$, RAAD A RADHI$^{2}$ and NAJLLA M HUSSAIN$^{1}$

$^{1}$Department of Physics, College of Education for Pure Science, University of Babylon, P.O. Box 4, Hilla-Babylon, Iraq
$^{2}$Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq
$^*$Corresponding author. E-mail: khalidsj@uobabylon.edu.iq

MS received 14 July 2014; revised 11 September 2014; accepted 27 October 2014

Abstract. Magnetic electron scattering ($M_3$) form factors with core polarization effects, energy levels and $B(M_3)$ values to $3^+$ states of the $^{26}$Mg nucleus have been studied using shell model calculations. The universal $sd$ of the Wildenthal interaction, universal $sd$-shell interaction $A$, universal $sd$-shell interaction $B$, are used for the $sd$-shell orbits. Core polarization effects according to microscopic theory are taken into account by the excitations of nucleons from the $(1s_{1/2} 1p_{3/2} 1p_{1/2})$ core and also from valence $1d_{5/2} 2s_{1/2} 1d_{3/2}$ orbits into higher shells, with $4\hbar\omega$ excitation. In form factor calculations, the universal $sd$-shell interaction $B$ for the $sd$-shell is used with the Michigan three-range Yakawa effective NN interaction as a residual interaction for the core polarization calculations. The wave functions of the radial single-particle matrix elements have been calculated using harmonic oscillator potentials. The level schemes are compared with the experimental data up to 9.902 MeV. In this study, very good agreements are obtained for all nuclei. Results from $M_3$ form factor calculations with the inclusion of core polarization and new $g$-factors give good agreement with the experimental data.

Keywords. sd-shell nuclei; magnetic form factors; Nushell; energy levels.

PACS Nos 25.30.Bf; 25.30.Dh; 21.60.Cs; 27.30.+t (20≤A≤38)

1. Introduction

The shell-model assumption states that one can separate the system into a core part and a valence part, and describe the interaction between the core and the valence particle, and that among the valence particles. The interaction between them will excite the core. This process, known as core excitation, will then give rise to an effective force between the valence particles as the two of them will have shifted their state as a consequence of their interaction with the core, while the core would have returned to the original state. This process can be described as the polarization of the core by one of the valence particles. The core polarization effect on the form factor is based on
a microscopic theory, which combines shell-model wave functions and configurations with higher energy as first-order perturbations; these are called ‘core polarization (CP) effects’.

The scattering of electrons from the nucleon and nuclei at high energies has provided important information about the size of the nucleus. The electron energies are in the region of 100 MeV and higher, such that the de Broglie wavelength associated with the electron is in the range of the nuclear forces. At these energies the electron acts as a probe for measuring the size of the nucleus which is expected to be of dimension of the order of a few Fermis [1].

The universal sd-shell interaction (USD) Hamiltonian [2] has provided realistic sd-shell (0d5/2, 0d3/2, 1s1/2) wave functions for use in nuclear structure models, nuclear spectroscopy and nuclear astrophysics for over two decades. It is also an important part of the Hamiltonian used for the p–sd [3] and sd–pf [4–6] model spaces [7]. The USD Hamiltonian is based on 63 two-body matrix elements (TBME) and three single-particle energies (SPE) given in table I of [8]. The universal sd-shell interaction A (USDA) and universal sd-shell interaction B (USDB) [7] are new USD-type Hamiltonians based on 66 parameters to fit 608 energy data in sd-shell nuclei (A = 16–40) with a root mean square (rms) deviation of 130 and 170 keV, respectively [7,9]. These new interactions have clearly resolved the fluorine problem as well as all the oxygen isotopes. The SPEs for the 0d3/2, 0d5/2 and 1s1/2 orbitals are (in MeV) −1.9798, −3.9436 and −3.0612 for the USDA interaction and −2.1117, −3.9257 and −3.2079 for the USDB interaction. Richter and Brown [9] have made a comparison between the experimental and theoretical results of the corresponding levels in the 26Mg levels based on energies and electron scattering form factors. Results are based on the new USDA and USDB sd-shell interactions. The enlarged model space (0p3/2, 0p1/2, 1s1/2 and 0d3/2) have been used to calculate electron scattering form factors for some p-shell nuclei. Results improve the agreement data remarkably well and provide an essential role for electromagnetic transitions and electron scattering form factors.

Shell model calculations have been used to calculate electron scattering form factors [11] for different states of 10B, 32Sc and 48Ca nuclei. The results with the inclusion of CP effects modify the form factors markedly and describe the experimental data very well in the range of the momentum transfer (q) values. The form factor calculations using the shell model code, Nushell, take into account the collective modes in nuclei and the CP effects are evaluated by adopting the purely empirical Tassie Model [12] together with the calculated ground state charge density distribution obtained for the low mass 1s–0d shell nuclei using the occupation number of the states where the sub-shell 1s is included with an occupation number of protons. Inelastic electron scattering form factors in some odd-A sd-shell nuclei (17O, 27Al and 39K) [13] were calculated by taking into account higher energy configurations outside the sd-shell model space (CP effects). The sd-shell model calculations fail to describe the data in both the transition strength and the form factor; the inclusion of CP effects modifies the form factors markedly and describes the experimental data very well in both the absolute strength and the momentum transfer dependence. Recently, Jassim et al [14] have studied the nuclear structure of 23Na, 25Mg, 27Al and 41Ca nuclei using shell model calculations. A set of two-body interactions are used in this paper. The shell model calculations are performed using two shell model methods. The CP effects are calculated using microscopic theory with CPM3Y code.
The one-body density matrix (OBDM) elements are calculated using Nushell code. The wave functions of radial single-particle matrix elements were calculated using harmonic oscillator (HO). Very good agreements are obtained for all nuclei in this study.

The aim of this paper is to test the CP effects on electron scattering form factors which combines shell model wave functions and highly excited states for ten $3^+$ states of $^{26}\text{Mg}$. In form factor calculations, the USDB effective interaction for the $sd$-shell model space is used with the realistic Michigan three Yakawas (M3Y) effective interactions which are used for the CP matrix elements calculations. The following nucleon $g$ factors are used: $g_s (p) = 5.0, g_s (n) = -3.5, g_l (p) = 1.174$, and $g_l (n) = -0.106$ [15].

2. Theory

The reduced matrix elements of the magnetic operator $\hat{T}_\Lambda^m$ consist of two parts, one for the model space (MS) matrix elements and the other for the CP matrix elements [16].

$$\langle X_f || \hat{T}_\Lambda^m || X_i \rangle = \langle X_f || \hat{T}_\Lambda^m || X_i \rangle_{\text{MS}} + \langle X_f || \delta \hat{T}_\Lambda^m || X_i \rangle_{\text{CP}},$$

where the states $|X_i\rangle$ and $|X_f\rangle$ are described by the MS wave functions. Greek symbols are used to denote quantum numbers in coordinate space and isospin, i.e., $X_i \equiv J_iT_i$, and $\Lambda \equiv JT$.

The MS matrix elements are expressed as the sum of the product of the one-body density matrix elements (OBDM) times the single-particle matrix elements, which is given by

$$\langle X_f || \hat{T}_\Lambda^m || X_i \rangle_{\text{MS}} = \sum_{\alpha,\beta} O_{X_i,X_f}^X (\alpha, \beta) \langle \alpha || \hat{T}_\Lambda^m || \beta \rangle_{\text{MS}},$$

where $\alpha$ and $\beta$ denote the final and initial single-particle states, respectively (isospin included) for the MS. $O_{X_i,X_f}^X$ are the OBDM elements.

Similarly, the CP matrix element can be written as

$$\langle X_f || \delta \hat{T}_\Lambda^m || X_i \rangle_{\text{CP}} = \sum_{\alpha,\beta} O_{X_i,X_f}^X (\alpha, \beta) \langle \alpha || \delta \hat{T}_\Lambda^m || \beta \rangle_{\text{CP}}.$$

By using the first-order perturbation theory, the single-particle matrix element for the higher-energy configurations outside the core and MS is given by [17]

$$\langle \alpha || \delta \hat{T}_\Lambda^m || \beta \rangle = \langle \alpha || V_{12} \frac{P}{E_i - H^{(0)}} \hat{T}_\Lambda^m || \beta \rangle + \langle \alpha \hat{T}_\Lambda^m \frac{P}{E_f - H^{(0)}} || V_{12} || \beta \rangle,$$

where $P$ is the projection operator onto the space outside the MS and $V_{12}$ is adopted as a residual two-body interaction. $E_i$ and $E_f$ are the energies of the initial and final states, respectively. $H^{(0)}$ is the unperturbed Hamiltonian. Equation (4) is written as [17]

$$\langle \alpha || \delta \hat{T}_\Lambda^m || \beta \rangle = \sum_{\alpha,\beta} \frac{(-1)^{\beta+\alpha} + X}{e_\beta - e_\alpha - e_{\alpha_1} + e_{\alpha_2}} \times \langle \alpha || V_{12} || \beta \rangle \times \langle \alpha_2 || \hat{T}_\Lambda^m || \alpha_1 \rangle \times \sqrt{(1 + \delta_{\alpha_1\alpha})(1 + \delta_{\beta\beta}) + A},$$

where $X = J_i$, $\delta_{\alpha_1\alpha}$ and $\delta_{\beta\beta}$ are the isospin factors.
where $A$ represents the additional terms with $\alpha_1$ and $\alpha_2$ exchanged with an overall minus sign. The indices $\alpha_1$ and $\alpha_2$ run over particle and hole states, respectively, and $e$ is the single-particle energy. The CP parts allow particle–hole excitations from the core and MS into higher orbits. These excitations are taken up to $4\hbar\omega$.

The SPEs $(e)$ are given by \[ e_{nlj} = \left(2n + l - \frac{1}{2}\right)\hbar\omega + \begin{cases} -\frac{1}{2}(l + 1)(f(r))_{nl}, & \text{for } j = l - \frac{1}{2} \\ \frac{1}{2}l(f(r))_{nl}, & \text{for } j = l + \frac{1}{2} \end{cases} \] (6)

with \[
(f(r))_{nl} \approx 20A^{-2/3} \text{ MeV}, \quad \hbar\omega = 45A^{-1/3} - 25A^{-2/3}.
\] (7)

The reduced single-particle matrix element becomes \[
\langle \alpha_2 | \| \hat{T}_{JT}^m \| | \beta_1 \rangle = \sqrt{\frac{2T + 1}{2}} \sum_{t_z} I_T(t_z) \langle \alpha_2 | \| \hat{T}_{JT}^\eta \| | \alpha_1 \rangle,
\] (8)

where \[
I_T(t_z) = \begin{cases} 1, & \text{for } T = 0 \\ (-1)^{\frac{1}{2}-t_z}, & \text{for } T = 1 \end{cases}
\] (9)

and $t_z = 1/2$ and $-1/2$ for the protons and neutrons, respectively.

The single-particle matrix element of the magnetic operator \[
\langle \alpha_2 | \| \hat{T}_{JT}^m \| | \alpha_1 \rangle = i\mu_N \left[ g_l(t_z)O_m^J(a, b) + g_s(t_z)S_m^J(a, b) \right],
\] (10)

where $\mu_N = (\epsilon h/2m_p c) = 0.1051$ e.fm, with $m_p$ the proton mass and $\vec{\sigma}$ the Pauli spin matrices, $g_l$ and $g_s$ are the orbital and spin-free nucleon $g$-factors, respectively.

\[
O_m^J(a, b) = P_J(m, l_a, l_b)C_J(j_a, j_b)A_J(j_a, j_b)
\times \sqrt{J(J + 1)} \frac{q}{2J + 1} \left\{ n_{al_a} \right| j_J(qr) \left| n_{bl_b} \right\}
\] (11)

and \[
S_m^J(a, b) = \frac{1}{2} \left[ J(J + 1) \right]^{1/2} C_J(j_a, j_b)P_J(m, l_a, l_b)
\times \{ B(j_a, j_b)[n_{al_a} j_J(qr)d/dr|n_{bl_b}] + [n_{bl_b} j_J(qr)d/dr|n_{al_a}] \}
+ \{ B(j_a, j_b) - J(J + 1) \} \left\{ n_{al_a} \right| \frac{1}{r} j_J(qr) \frac{d}{dr} \left| n_{bl_b} \right\}
\] (12)

where $C(j_a, j_b)$ is the coefficient of the electric parity-selection rule.

\[
P_J(m, l_a, l_b) = \frac{1}{2} \left[ 1 + (-1)^{l_a + l_b + J + 1} \right],
\] (13)

\[
A_J(a, b) = \left[ 1 + \frac{B(a, b)}{J} \right] \left[ 1 - \frac{B(a, b)}{J + 1} \right],
\] (14)
Magnetic electron scattering form factors of the $^{26}$Mg nucleus

$$B(a, b) = 2 + D(j_a, l_a) + D(j_b, l_b),$$

(15)

$$D(j, l) = j (j + 1) - l(l + 1) - 3/4.$$  

(16)

Elastic and inelastic electron scattering form factors in terms of angular momentum $J$ and momentum transfer $q$, between the initial and final states of spin $J_{i,f}$ and isospin $T_{i,f}$, are given by [18]

$$|F_{m}^{J}(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left\{ \sum_{T=0,1} (-1)^{T_f-T_i} \left( \begin{array}{ccc} T_f & T & T_i \\ T & T_f & T_i \end{array} \right) \langle X_i || T_{m}^{J,T}(q) || X_f \rangle \right\}$$

$$\times |F_{cm}(q)|^2 \times |F_{fs}(q)|^2,$$

where $T_z$ is the projection along the z-axis of the initial and final isospin states. $F_{fs}(q) = \exp(-0.43q^2/4)$ is the nucleon finite size (fs) form factor and $F_{cm}(q) = \exp(q^2b^2/4A)$ is

Figure 1. Energy levels of the $^{26}$Mg nucleus from the experimental data in [21] are compared to the shell model results with three effective interactions USD, USDA and USDB.
3. Results and discussion

Calculations are presented for ten $3^+$ states of the $^{26}\text{Mg}$ nucleus, using USD, USDA, and USDB effective interactions for the $sd$-shell model to generate the OBDM elements $O_{X_i,X_f}^X(\alpha, \beta)$ given in eq. (2). These matrix elements are calculated using the shell model Nushell code [19]. The many-particle matrix elements that include both the $sd$ MS and the CP effects are calculated using eqs (2) and (5). Finally, the nuclear form factor can be obtained from eq. (15) with the inclusion of CP effects according to microscopic theory. The radial parts of the single-particle wave function are those of the HO potential [20]. According to the conventional $sd$-shell model, the nucleus is described as an inert core of $^{16}\text{O}$ plus ten nucleons distributed over the $sd$-shell. Figure 1 shows the energy levels calculated by three different interactions (USD, USDA and USDB) with the experimental data [21]. The very good agreement between the calculated results and the experimental data for states $3_1^+, 3_2^+, 3_3^+, 3_4^+$ and $3_5^+$ are used for all effective interactions. The calculated $3_6^+$ state is at 8.004 MeV for USD, 8.18 MeV for USDA and 8.22 MeV for USDB interaction.

![Figure 2](image-url)

**Figure 2.** The transverse $M3$ form factors for the transition to $3_1^+$ state in $^{26}\text{Mg}$ using the USDB effective interaction, with (MS+CP) and without (MS) CP effects.
Magnetic electron scattering form factors of the $^{26}\text{Mg}$ nucleus compared to the experimental value of 8.248 MeV. The USD, USDA and USDB interactions predict the $3^+_1$ state at 9.576, 9.608 and 9.584 MeV respectively, less than the experimental value of 9.902 MeV.

The comparison between the theoretical and experimental $M3$ form factors for the transition to $3^+$ (3.941 MeV) are shown in figure 2. The solid and dashed curves represent the results with and without CP effects, respectively. From this figure, one can see that the theoretical results with the inclusion of CP effect are in excellent agreement with the experimental data in all regions of the momentum transfers $q$. Figure 3 shows a comparison between the calculated form factors with the inclusion of CP effects (solid curve) and those of $sd$-shell model calculation (dashed curve). The $sd$-shell model calculations without the inclusion of the CP effects give a remarkably good agreement with the experimental data in all regions of the momentum transfers $q$, while the inclusion of CP effects cause reduced electron scattering form factors in the second peak for this state. The dependence of the transverse $M3$ form factor on the momentum transfer $q$ for $3^+_4$ states is plotted in figure 4. In this transition, the electron excites the nucleus from the ground state ($J^T_i = 0^++$) to the fourth excited ($J^T_f = 3^+_4$) state at an excitation energy of 7.296 MeV. It is noticed from figure 4, that the MS fails to describe the experimental data [22], while the inclusion of the CP effect gives a very

![Figure 3](image)

**Figure 3.** The transverse $M3$ form factors for the transition to $3^+_2$ state in $^{26}\text{Mg}$, with (MS+CP) and without (MS) CP effects.

Figure 4. The transverse $M3$ form factors for the transition to $3^+_4$ state in $^{26}$Mg, with (MS+CP) and without (MS) CP effects.

good agreement in the entire momentum transfer region compared to that without the CP effect. Figure 5 shows the transverse $M3$ form factors as a function of momentum transfer $q$ for the $3^+_6$ state with an excitation energy of 8.248 MeV. From this figure, we notice that the inclusion of CP effects enhanced the $M3$ form factor in the first maximum region and reduced results by a factor of 2. The inclusion of CP effects does not significantly affect the calculation of the form factor for the second maximum, and experimental data are well described in this region. The $B(M3)$ values for six $3^+$ states using USD, USDA and USDB interactions in $^{26}$Mg are compared with the experimental data [22] in table 1. The $B(M3)$ values for $3^+_1$ and $3^+_2$ transitions of the $^{26}$Mg nucleus show closeness to the experimental values for all effective interactions. In the present calculations, the $B(M3)\downarrow(3^+_3)$ transition probabilities for the $^{26}$Mg nucleus are predicted as 150, 116 and 118 e$^2$ fm$^4$ for the USD, USDA and USDB interactions, respectively, whereas the experimental value is 190 e$^2$ fm$^4$. In the present calculations, $B(E2)3^+_4$ values are predicted as 224, 402 and 281 e$^2$ fm$^4$ for USD, USDA and USDB, respectively, compared to the experimental value as 117 e$^2$ fm$^4$. The values of $B(E2)$ transition probabilities for the $3^+_5$ transitions are 109, 46 and 50 e$^2$ fm$^4$ for USD, USDA and USDB, respectively. These values show deviation from the experimental value of 230 (7) e$^2$ fm$^4$. 

Magnetic electron scattering form factors of the $^{26}\text{Mg}$ nucleus

Figure 5. The transverse $M_3$ form factors for the transition to $3_6^+$ state in $^{26}\text{Mg}$ with and without CP effects.

Table 1. Experimental [21] and theoretical reduced transition probabilities, $B(M3)$, for the positive-parity states in the $^{26}\text{Mg}$ nucleus. The theoretical $B(M3)$ values were calculated using the Nushell code.

<table>
<thead>
<tr>
<th>State ($J^n_π$)</th>
<th>$B(M3)$ ($e^2 \text{ fm}^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp.</td>
</tr>
<tr>
<td>$3_1^+$</td>
<td>340 (11)</td>
</tr>
<tr>
<td>$3_2^+$</td>
<td>700 (30)</td>
</tr>
<tr>
<td>$3_3^+$</td>
<td>190 (4)</td>
</tr>
<tr>
<td>$3_4^+$</td>
<td>117 (20)</td>
</tr>
<tr>
<td>$3_5^+$</td>
<td>230 (7)</td>
</tr>
</tbody>
</table>

4. Conclusion

The nuclear structure (energy levels, transverse $M3$ form factor and $B(M3)$ values) for six $3^+$ states in $^{26}\text{Mg}$ were calculated by taking into account the higher energy configurations outside the $sd$-shell MS with $4\hbar\omega$ excitation which are called core polarization (CP) effects. The USD and USDB interactions for the $sd$-shell are used with the M3Y effective

NN interaction as a residual interaction for the CP calculations. The effect of CP is found essential in both the transition strengths and momentum transfer dependence, and gives a good description of the data with new g-factor values: $g_s (p) = 5.0$, $g_s (n) = -3.5$, $g_l (p) = 1.175$ and $g_l (n) = -0.106$. The use of a modern interaction may give a better description of the form factors.

Acknowledgements

The authors thank the University of Babylon, College of Education for Pure Science, Department of Physics for supporting this work.

References
