

Shell-Model Study of Neutron Rich Even-Even $^{52,54,56}\text{Cr}$ Isotopes

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Abstract: Large-scale shell model calculations were performed to study the low lying energies, binding energy and the reduced transition probabilities between energy levels for even-even $^{52,54,56}\text{Cr}$ isotopes by employing effective interactions GXPF1, GXPF1A, FPD6 and KB3G near the closed core ^{48}Ca with model space (HO). The results of calculation were compared with the recent experimental data and good global agreements were obtained for all nuclei under study and the best results achieved by using the effective interaction GXPF1A.

Keywords: Gamma transitions, excitation energies, Shell model

1. Introduction

The neutron-rich fp -shell nuclei far from the valley of stability are of particular interest in recent experimental and theoretical studies [1]. To provide a satisfactory description of these nuclei, the challenge remains to understand what mechanisms cause changes in nuclear shell structure as neutron number increases in nuclear systems. Theoretical calculations have questioned the persistence of the traditional magic numbers, which have been known to exist in stable nuclei [2].

Right from the time the shell model (SM) was established as the most basic model in nuclear structure, the nuclei in the $2p$ $1f$ region have been the focus of attention for various theoretical investigations based on the practical assumption of a Ca inert core within the domain of the shell model. The complexities arising due to the large number of valence particles outside the Ca core have hindered most of such theoretical attempts.

Only very recently some calculations in the framework of the projected Hartree-Fock (PHF), ν band mixing, and the highly truncated shell model have been reported for some of the nuclei in this region [1].

The nuclear shell model, a sophisticated theory based on the configuration interaction method, has been one of the central theoretical tools for understanding a wealth of data from nuclear structure experiments. Due to the rapid growth in the dimensionality of the Hilbert space with increasing degrees of freedom, we have to work within a reduced Hilbert space, the so-called model space. Accordingly, we use an effective interaction which is tailored to the chosen model space. This effective interaction forms an essential input to all shell-model studies. Equipped with modern sophisticated effective interactions, the shell model has successfully described many properties of nuclei [3].

The aim of the present work is to study the nuclear structure of even-even $^{52,54,56}\text{Cr}$ isotopes by means of large-scale shell model calculations by employing GXPF1, GXPF1A, FPD6 and KB3G effective interactions and compare the theoretical results with the most recent experimental data.

2. Shell Model Calculation

The independent-particle Hamiltonian of an A -particle system can be written in terms two-particle interactions as [4],

$$H = \sum_{k=1}^A T_k + \sum_{k=1}^A \sum_{l=k+1}^A W(\vec{r}_k, \vec{r}_l) \quad (1)$$

where $W(\vec{r}_k, \vec{r}_l)$ is the two-body interaction between the k^{th} and l^{th} nucleons. Choosing an average potential $U(r_k)$, the Hamiltonian becomes [4],

$$H = \sum_{k=1}^A [T_k + U(r_k)] + \sum_{k=1}^A \sum_{l=k+1}^A W(\vec{r}_k, \vec{r}_l) - \sum_{k=1}^A U(r_k) \quad (2)$$

where the first term is identical to the independent-particle Hamiltonian, and the second and third account for the deviation from independent particle motion, known as the residual interaction. Separating the summations into core and valence contributions, eqn.(2) can be re-written [4],

$H = H_{core} + H_1 + H_2 + V(\vec{r}_1, \vec{r}_2)$ (3) In the above equation, H_{core} contains all of the interactions of nucleons making up the core, H_1 and H_2 are the single-particle contributions from particles 1 and 2, and $V(\vec{r}_1, \vec{r}_2)$ is the residual interaction describing all interactions between particles 1 and 2 as well as any interaction with core nucleons. Inserting this form of the Hamiltonian into the Schrödinger equation yields an analogous expression for the energy [4],

$$E = E_{core} + E_1 + E_2 + \langle \Phi_{J,\tau} | V(\vec{r}_1, \vec{r}_2) | \Phi_{J,\tau} \rangle \quad (4)$$

Here, E_{core} is the binding energy of the core nucleus, E_1 and E_2 are defined as the single-particle energies of orbitals outside the core, and $\langle \Phi_{J,\tau} | V(\vec{r}_1, \vec{r}_2) | \Phi_{J,\tau} \rangle$ is the residual interaction which needs to be defined by theory. It is important to note that the energy given by eqn. (4) is for pure configurations only. In principle, any close-lying state with the same total angular momentum J and total isospin τ will mix. The mixed eigen states are given by linear combinations of the unperturbed wave functions [4],

$$(\Psi_{J,\tau})_p = \sum_{k=1}^g a_{kp} (\Phi_{J,\tau})_p \quad (5)$$

where g is the number of configurations that mix and the label p =1,2,..., g. The coefficients a_{kp} fulfill the condition [4],

$$\sum_{k=1}^g |a_{kp}|^2 = 1 \quad (6)$$

Inserting eqn. (5) into the schrödinger equation gives,

$$H(\Psi_{J,\tau})_p = E_p(\Psi_{J,\tau})_p \quad (7)$$

which leads to a system of linear equations [4].

3. Results and Discussions

3.1 Excitation Energies

The core are taken at ⁴⁸Ca for all nuclei under study with 4,6 and 8 valence nucleons for ⁵²Cr, ⁵⁴Cr and ⁵⁶Cr respectively. Figure 1 present the comparison between our theoretical work and the experimental for ⁵²Cr isotope. Our theoretical work predicts the 2⁺ state at 1.483 MeV, 1.286 MeV, 1.381 MeV and 0.830 MeV by employing the effective interactions GXPF1, GXPF1A, FPD6 and KB3G respectively. By comparing them with experimental value 1.434 MeV we obtained the best agreement at GXPF1 interaction. For 4⁺ the theoretical results are 2.277 MeV, 2.081 MeV, 2.125 MeV and 1.430 MeV by employing the same effective interactions respectively. We can notice from the figure that the experiment value at 4⁺ 2.369MeV has a good agreement with GXPF1 interaction like 2⁺ state.

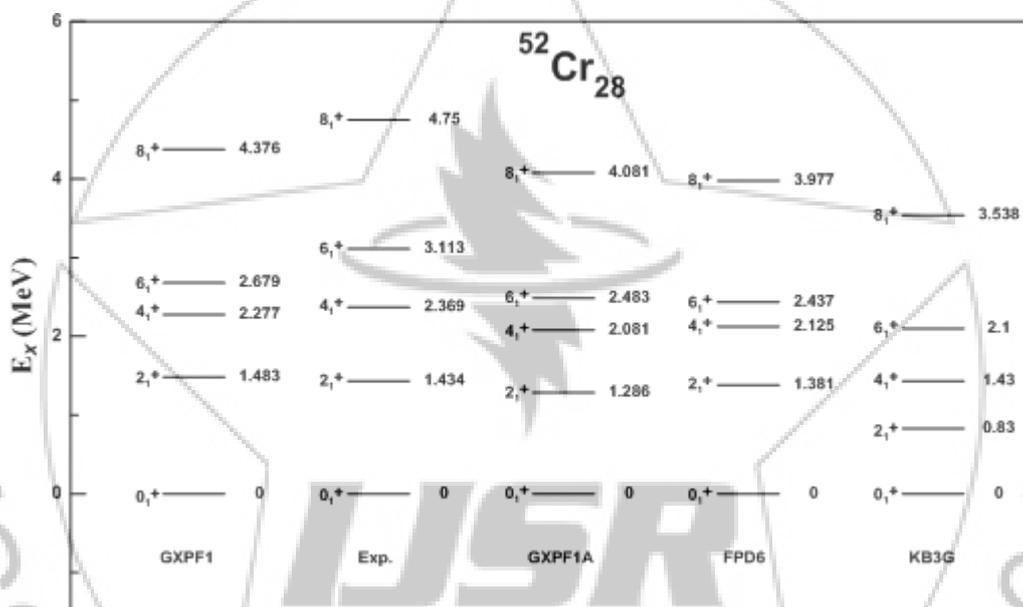


Figure 1: Comparison of the experimental excitation energies taken from Ref [5] with the present theoretical work using GXPF1, GXPF1A, FPD6 and KB3G effective interactions.

Figure 2 presents the calculated excitation energy levels for ⁵⁴Cr with 6 particles outside core. Our calculations predicts the values 1.075 MeV, 0.939 MeV, 0.973 MeV and 0.667 MeV for 2⁺ state by employing GXPF1, GXPF1A, FPD6 and KB3G effective interactions respectively which compared with experimental value at 0.834 MeV. The 4⁺ state is predicted with values 2.128 MeV, 1.885 MeV, 1.941 MeV

and 1.431 MeV by employing the same effective interactions respectively and compared with experimental value 1.823MeV. The best agreement with experimental value is achieved at GXPF1A interaction for both 2⁺ and 4⁺ states.

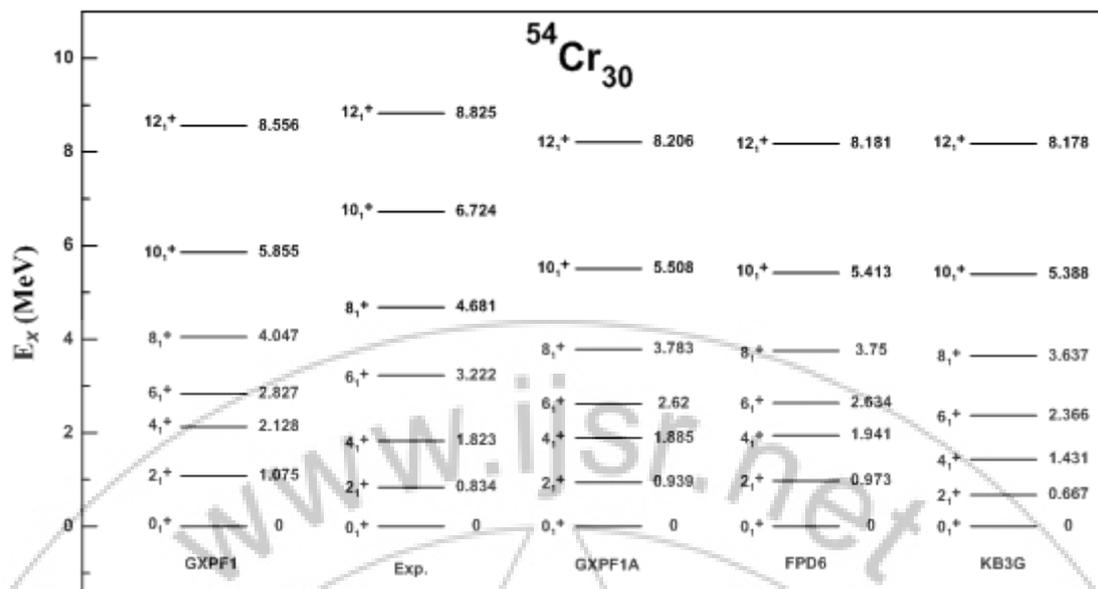


Figure 2: Comparison of the experimental excitation energies taken from Ref [5] with the present theoretical work using GXPF1, GXPF1A, FPD6 and KB3G effective interactions.

The effective interactions GXPF1, GXPF1A, FPD6 and KB3G were employed for ^{56}Cr with 8 particles outside core ^{48}Ca and model space (HO) as shown in Figure 3. The results of the excitation energies for 2_1^+ are 1.283 MeV, 1.134 MeV, 1.139 MeV and 0.842 MeV respectively and compared with the experimental value 1.006 MeV. The

excellent agreement was achieved at GXPF1A interaction. The 4_1^+ we obtained the values 2.262 MeV, 2.011 MeV, 2.014 MeV and 1.542 MeV by employing the same effective interactions respectively and compared with experimental data 2.681 MeV which has best agreement with GXPF1 interaction.

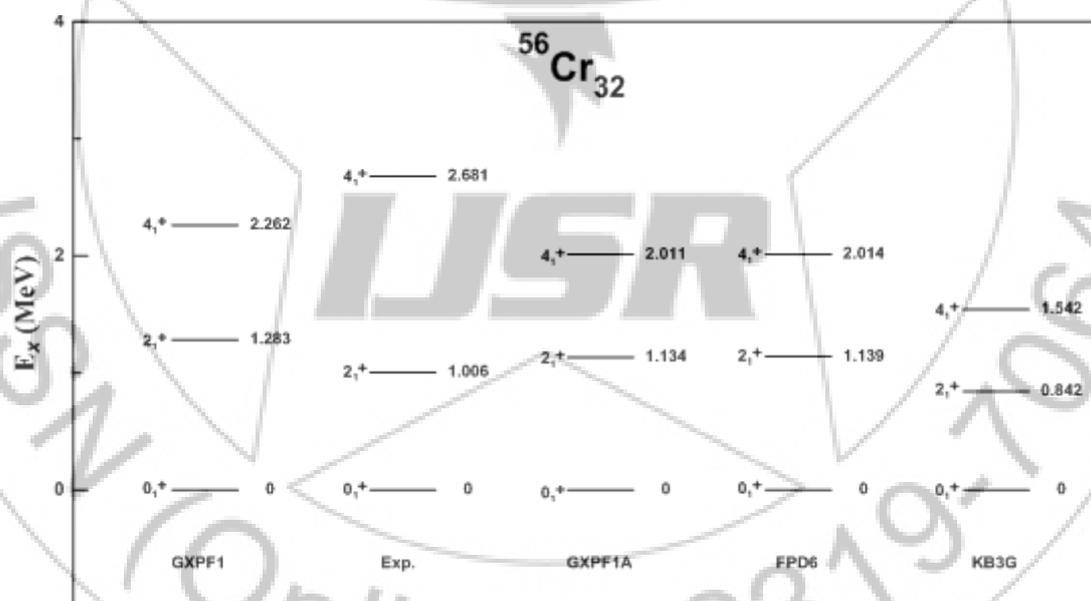


Figure 3: Comparison of the experimental excitation energies taken from Ref. [5] with the present theoretical work using GXPF1, GXPF1A, FPD6 and KB3G effective interactions.

3.2 Reduced Transition Probabilities

The electromagnetic transition probability $B(E2; 0_{g.s}^+ \rightarrow 2_1^+)$ values calculated for both model spaces and interactions are compared with experimental data for Cr isotopes. The effective charges for proton and neutron are evaluated by taking the average value of the effective charge for proton and neutron from the fitting with the experimental values for all Cr isotopes studied in the present work. Therefore for GXPF1A interaction the effective charges are $e_p^{eff} = 0.68e$

for proton and $e_n^{eff} = 0.49e$ for neutron. For FPD6 effective interaction the values are $e_p^{eff} = 0.68e$ for proton and $e_n^{eff} = 0.49e$ for neutron. From Fig.4 we can see although these effective interacting are successful in describing the energy levels for the Cr isotopes are less successful in describing the reduced transition probabilities and instead the core polarization effect should be included from the closed core which might improve the calculations.

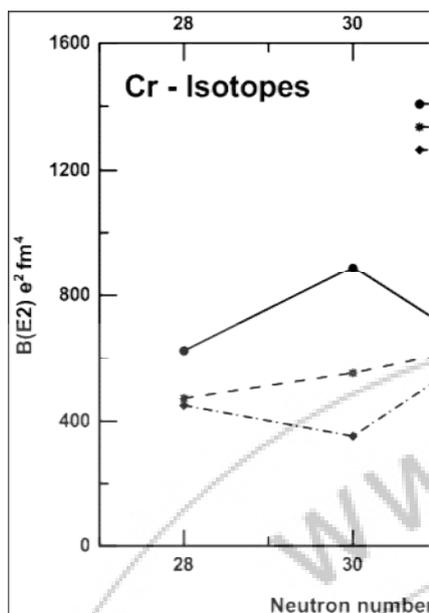


Figure 4: Comparison of the large-scale shell model calculations using FPD6 (stars) and GXPF1 (diamonds) effective interactions with the experimental $B(E2; 0_{g.s}^+ \rightarrow 2_1^+)$ strengths (closed circles) for the chain of even-even Cr isotopes. Experimental data are taken from Ref.[6].

3.3 Binding Energies

For the chain of Cr isotopes we obtained binding energies by employing the best interactions FPD6 and GXPF1A as shown in Fig.5. We can see that both interactions have excellent agreement with the experimental data for all isotopes of Cr.

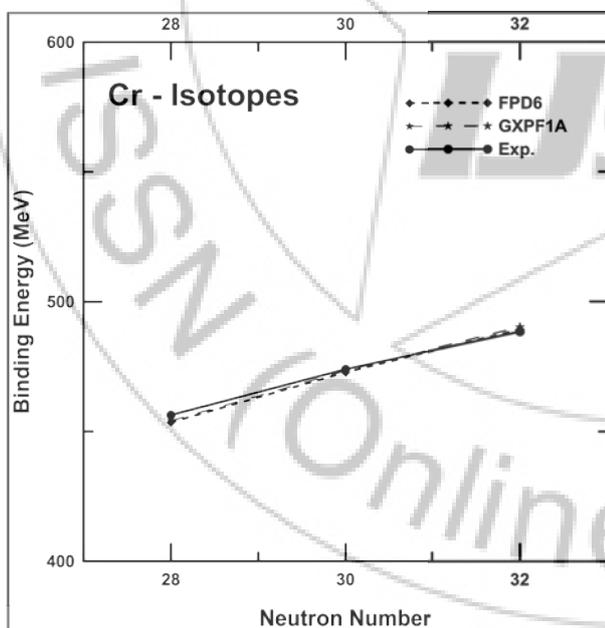


Figure 5: Comparison between calculated binding energy (in units of MeV) with their corresponding experimental data for even-even Cr isotopes. Experimental data taken from Ref.[7].

4. Conclusions

In the present work large scale shell model calculations have been performed for neutron rich even-even Cr isotopes with $A=52,54,56$ in full fp space without truncation by employing GXPF1, GXPF1A, FPD6 and KB3G effective interactions. The facts that core-polarization contribution is considered through the effective charges are not always an adequate choice for the calculation for the reduced transition probabilities and the core polarization should be included through a microscopic theory. The systematic study of the low-lying 2_1^+ and 4_1^+ when compared with experiment our results exhibit an agreement that may be considered quite well. The experimental binding energy are very well reproduced by the current shell model calculation.

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