On Completely And Semi Completely Prime Ideal With Respect To An Element Of A Boolean $\alpha_1,\alpha_2$ Near-Ring

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Abstract: In this paper I generalize concepts of $\alpha_1,\alpha_2$ near-ring as well as of completely semi prime ideal with respect to an element x of a $\alpha_1,\alpha_2$ near-ring and the completely prime ideal of a $\alpha_1,\alpha_2$ near-ring with respect to an element x and the relationships between the completely prime ideal with respect to an element x of a $\alpha_1,\alpha_2$ near-ring and some other types of ideals, as well as I will valid if I is c.p.I of $\alpha_1,\alpha_2$ near-ring as well as if I is c.p.I of $\alpha_1,\alpha_2$ near-ring if I is a c.s.p.I of $\alpha_1,\alpha_2$ near-ring as well as I is e’-c.p.I of $\alpha_1,\alpha_2$ near-ring if it is a e’-c.s.p.I of $\alpha_1,\alpha_2$ near-ring and all its valid after put condition of Boolean a $\alpha_1,\alpha_2$ near-ring.

Keywords: $\alpha_1,\alpha_2$ near-ring, c.s.p.I, c.p.I, x-c.s.p.I, x-c.p.I

1 INTRODUCTION

In 1905 L.E Dickson began the study of near-ring and later 1930 Wieland has investigated it. Further, material about near ring can be found (cf.[2,5,9]). In 1977 G. Pilz, was introducing the notion of prime ideals of a near-ring (cf.[1,2,7]). In 1988 N.J.Groenewald was introducing the notions of completely (semi) prime ideals of a near-ring (cf.[6,10]). In 2011 H.H.abbass, S.M.Ibrahem introduced the concepts of a completely semi prime ideal with respect to an element of a near-ring[4]. In 2012 H.H.abbass, Mohanad Ali Mohammed introduce the concept of a completely semi prime ideals near-ring with respect to an element of a near-ring[3]. In 2010, S. Uma, R. Balakrishnan, T. Tamizh Chelvam introduce the concept of a $\alpha_1,\alpha_2$ near-ring near-ring[8].

1.1 Definition
A right near-ring is a set N together with two binary operations + and . such that

1. (N,+) is a group (not necessarily abelian)
2. (N,.) is a semigroup
3. $n_1(n_2+n_3)=n_1n_2+n_1n_3, \forall n_1,n_2,n_3 \in N$

1.2 Definition:
Let N be a right near-ring, if

1- for every a in N there exists x in N such that $a=xax$ then we say N is an $\alpha_1$ near-ring.

2- for every a in N-{0} there exists x in N-{0} such that $x=xax$ then we say N is an $\alpha_2$ near-ring.

1.3 Example:
1- the near-ring $(N,+,-)$ define on the klein's four group $N=\{0,a,b,c\}$ where addition and multiplication is defined as:

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Is an $\alpha_1$ near-ring (since $bab=ab=a$ , $bab=b$ , $ccc=c$ , $a0a=0$) its neither $\alpha_2$ near-ring (since there is no x in N-{0} such that $xax=x$) 2- the near-ring $(N,+,-)$ define on the klein's four group $N=\{0,a,b,c\}$, where addition and multiplication is defined as:

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Here (N,+) is an $\alpha_2$ near-ring it is neither $\alpha_1$ near-ring (since $xcx\neq c$ for any x in N).

1.4 Definition:
Let N be a $\alpha_1,\alpha_2$ near-ring, normal subgroup I of $(N,+,-)$ is called an right ideal of N if:

1- $NI \subseteq I$, where $NI=\{i \cdot n | n \in N, i \in I\}$

2- $\forall n, n_1 \in N$ and for all $i \in I, (n_1+i)n-n_1n \in I$

1.5 Example:
Consider the a N be a $\alpha_1,\alpha_2$ near-ring example(1.3) the normal subgroup $I=\{0,a\}$ is ideal of the near-ring N.

1.6 Remark
We will refer that all ideals in this paper are right ideals.
1.7 definition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, an ideal \( I \) of \( N \) is called a completely semi prime ideal (c.s.p.I) of \( N \) if \( x^2 \in I \) implies \( x \in I \) for all \( x \in N \).

1.8 example:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, the ideal \( I=\{0,a\} \) of \( N \) in example(1.3) is a completely semi prime ideal of \( N \), since \( \forall y \in N \) implies \( y \in I \).

1.9 definition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, an ideal of \( N \) is called a prime ideal if every ideal \( I_1, I_2 \) of \( N \) such that \( I_1, I_2 \subseteq I \) implies \( I_1 \subseteq I \) or \( I_2 \subseteq I \).

1.10 example:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, in example(1.3) the ideal \( I=\{0,a\} \) of the \( N \) is a prime ideal.

1.11 definition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, \( s \) be an ideal in \( N \) so \( s \) is semi prime if and only if for all ideals \( I \) of \( N \), \( I^2 \subseteq s \) implies \( I \subseteq s \).

1.12 definition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, \( I \) be an ideal of \( N \), then \( I \) is called a completely prime ideal of \( N \) if for all \( x, y \in N \) \( x.y \in I \) implies \( x \in I \) or \( y \in I \) denoted by (c.p.I) of \( N \).

1.13 example:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, in example(1.3) the ideal \( I=\{0,a\} \) is complete prime ideal of the \( N \).

1.14 proposition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, every c.p.I of \( N \) is a c.s.p.I of \( N \).

Proof:
Let for all \( y \in N \), such that \( y^2 \in I \) as \( I \) is c.p.I of \( N \)

So, \( y^2 = y.y \in I \)

then, \( y \in I \)

so, \( I \) is a c.s.p.I of \( N \).

1.15 definition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, is called Boolean if for all \( x \in N \), \( x^2 = x \).

1.16 example:
the near ring \((N,+,.)\) define on the klein’s four group \( N=\{0,a,b,c\} \), where addition and multiplication is defined as:

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Is a \( \alpha_1, \alpha_2 \) near-ring, as every Boolean near ring is \( \alpha_1 \) near-ring as well as \( \alpha_2 \) near-ring since \((00=00, 0a=a, a=b, bb=b, cc=c)\).

2 Completely and semi completely prime ideal with respect to an element of a \( \alpha_1, \alpha_2 \) near-ring

2.1 definition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring and \( x \in N \) \( I \) is called completely semi prime ideal with respect to an element \( x \) denoted by \((x-c.s.p.I)\) of \( N \) if for all \( y \in N \), \( x.y^2 \in I \) implies \( y \in I \).

2.2 example:
Consider the near ring \( N=\{0,a,b,c\} \), where addition and multiplication is defined as:

\[
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+ & 0 & a & b \\
0 & 0 & a & b \\
a & a & 0 & c \\
b & b & c & 0 \\
c & c & b & a \\
\end{array}
\]

is a \( \alpha_1, \alpha_2 \) near-ring, the ideal \( I=\{0,a\} \) is a c.s.p.I of \( N \).

2.3 definition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, \( I \) be an ideal of \( N \) and \( x \in N \), then it is called a completely prime ideal with respect to an element \( x \) denoted by \((x-c.p.I)\) of \( N \) if for all \( y, z \in N \), \( x.y.z \in I \) implies \( y \in I \) or \( z \in I \).

2.4 example:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring, in example(2.2) the ideal \( I=\{0,b\} \) is \( a-c.p.I \) of \( N \).

2.5 proposition:
Let \( N \) be a \( \alpha_1, \alpha_2 \) near-ring and \( x \in N \), then every completely prime ideal with respect to an element \( x \) of \( N \) is a completely semi prime ideal with respect to an element \( x \) of \( N \).

Proof:
let \( I \) be a \( x-c.p.I \) and if for all \( y \in N \) such that \( x.y^2 \in I \), \( x.y^2 = x.y.y \in I \)

implies \( y \in I \)

then, \( I \) is \( x-c.s.p.I \) of \( N \).

2.6 Remark:
The converse of the proposition (2.5) may be not true, just where put the condition of Boolean of \( N \) its valid (proposition(2.11)).

2.7 proposition:
Let \( N \) be a Boolean \( \alpha_1, \alpha_2 \) near-ring then \( I \) be c.p.I of \( N \) if \( I \) is an \( x-c.s.p.I \) for all \( x \in N \).
2.8 Proposition:
Let N be a Boolean $\alpha_1, \alpha_2$ near-ring, then I be c.p.I of N if I is an $x$-c.p.I for all $x \in N$.

Proof:
$(\Rightarrow)$ Let $x, y \in N$ such that $x, y^2 \in I$
as N is Boolean, then $x, y^2 \in I \Rightarrow x, y \in I$
so $y \in I$ as I is c.p.I of N
then, I is an $x$-c.s.p.I for all $x \in N$.

$(\Leftarrow)$ let $\forall y \in N$, $x, y \in I$
then, $x, y^2 \in I$ so that $y \in I$
then I is c.p.I of N.

2.9 Proposition:
Let N be a Boolean $\alpha_1, \alpha_2$ near-ring, then every $x$-c.p.I of N is c.s.p.I of N.

Proof:
As every $x$-c.p.I of N is c.p.I of N (by proposition (2.8)) and every c.p.I of N is c.s.p.I of N (by proposition (1.14)) then every $x$-c.p.I of N is c.s.p.I of N.

2.10 Proposition:
Let N be a Boolean $\alpha_1, \alpha_2$ near-ring, then every $x$-c.s.p.I of N is c.p.I of N.

Proof:
As every $x$-c.s.p.I of N is c.p.I of N (by proposition (2.7)) and every c.p.I of N is c.s.p.I of N (by proposition (1.14)) then every $x$-c.s.p.I of N is c.p.I of N.

2.11 Proposition:
Let N be a Boolean $\alpha_1, \alpha_2$ near-ring, then every $x$-c.s.p.I of N is c.p.I of N.

Proof:
As every $x$-c.s.p.I of N is c.p.I of N (by proposition (2.7)) and every c.p.I of N is c.s.p.I of N (by proposition (2.8)) then every $x$-c.s.p.I of N is c.p.I of N.

2.12 Remark:
The following diagram show us the relationships between these ideals.

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3 Completely and semi completely prime ideal $\alpha_1, \alpha_2$ near-ring with respect to an element

3.1 Definition:
Let N be a $\alpha_1, \alpha_2$ near-ring, is called a completely semi prime ideal near ring, denoted by (c.s.p.I $\alpha_1, \alpha_2$ near-ring) if every ideal of N are c.s.p.I of N.

3.2 Example:
Let N be a $\alpha_1, \alpha_2$ near-ring, in example(2.2) is a c.s.p.I of N since all its ideals $I_1=(0,a), I_2=(0,b), I_3=(0,a,b), I_4=(0), I_5=N$ are c.s.p.I of N.

3.3 Definition:
Let N be a $\alpha_1, \alpha_2$ near-ring, is called a completely prime ideal near ring, denoted by (c.p.I $\alpha_1, \alpha_2$ near-ring) if every ideal of N are c.p.I of N.

3.4 Example:
Let N be a $\alpha_1, \alpha_2$ near-ring, in example(2.2) is a c.p.I of N since all its ideals $I_1=(0,a), I_2=(0,b), I_3=(0,a,b), I_4=(0), I_5=N$ are c.p.I of N.

3.5 Proposition:
Let N be a $\alpha_1, \alpha_2$ near-ring, if N is a c.p.I $\alpha_1, \alpha_2$ near-ring, then N is a c.s.p.I $\alpha_1, \alpha_2$ near-ring.

Proof:
let N is a c.p.I $\alpha_1, \alpha_2$ near-ring so, every ideal of N is a c.p.I of N then, every ideal of N is a c.s.p.I of N (by proposition(1.14)) then, N is a c.s.p.I $\alpha_1, \alpha_2$ near-ring.

3.6 Definition:
Let N be a $\alpha_1, \alpha_2$ near-ring, is called a completely semi prime ideal $\alpha_1, \alpha_2$ near-ring, denoted by (x-c.s.p.I $\alpha_1, \alpha_2$ near-ring) if every ideal of N are x-c.s.p.I of N, where $x \in N$.

3.7 Example:
Let N be a $\alpha_1, \alpha_2$ near-ring, in example(2.2) is a (c.s.p.I of N) since all its ideals $I_1=(0,a), I_2=(0,b), I_3=(0,a,b), I_4=(0), I_5=N$ are c.c.s.p.I of N.

3.8 Definition:
Let N be a $\alpha_1, \alpha_2$ near-ring is called a completely prime ideal $\alpha_1, \alpha_2$ near-ring, denoted by (x-c.p.I $\alpha_1, \alpha_2$ near-ring) if every ideal of N are x-c.p.I of N, where $x \in N$.

3.9 Example:
Let N be a $\alpha_1, \alpha_2$ near-ring in example(2.2) is a (c.c.p.I of N) since all its ideals $I_1=(0,a), I_2=(0,b), I_3=(0,a,b), I_4=(0), I_5=N$ are c.c.p.I of N.
3.10 Proposition:
If N is a x-c.p.l \( \alpha_1, \alpha_2 \) near-ring ,then N is a x-c.s.p.l \( \alpha_1, \alpha_2 \) near-ring, where \( x \in N \).

Proof:
Let N is a x-c.p.l \( \alpha_1, \alpha_2 \) near-ring so, every ideal of N is a c.p.I of \( \alpha_1, \alpha_2 \) near-ring then, every ideal of N is a x-c.s.p.I of \( \alpha_1, \alpha_2 \) near-ring (by proposition(2.7)) then, N is a x-c.s.p.l \( \alpha_1, \alpha_2 \) near-ring.

3.11 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring then N be c.p.I iff N is an x-c.s.p.l for all \( x \in N \).

Proof:
Obvious by definitions above and proposition (2.8)

3.12 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring then N be c.p.I of N iff N is an x-c.p.l for all \( x \in N \).

Proof:
Obvious by definitions above and proposition (2.10)

3.13 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring if N is x-c.p.l then N is c.s.p.l

Proof:
Obvious by definitions above and proposition (2.9)

3.14 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring ,if N is x-c.s.p.l then N is c.s.p.l

Proof:
Obvious by definitions above and proposition (2.10)

3.15 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring ,if N is x-c.s.p.l then N is x-c.p.l.

Proof:
Obvious by definitions above and proposition (2.11)

3.16 Remark:
The following diagram show us the relationships between these a \( \alpha_1, \alpha_2 \) near-ring.

![Diagram](image)

4 Completely and semi completely prime ideal with respect to an multiplicative identity of \( \alpha_1, \alpha_2 \) near-ring

4.1 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring with multiplicative identity \( e' \) then I is e'-c.s.p.l of N iff it is a c.p.l of N.

4.2 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring with multiplicative identity \( e' \) then I is e'-c.p.l of N iff it is a c.p.l of N.

4.3 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring with multiplicative identity \( e' \) then I is e'-c.p.l of N iff it is a e'-c.s.p.l of N.

4.4 Example:
Let \( N=\{0,a,b,c\} \) be a \( \alpha_1, \alpha_2 \) near-ring ,with addition and multiplication defined as:

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the ideal I=(0,a) is b-c.p.l of N if it is a b- c.s.p.l of N but it is same example which clear that not all c.p.l of N are x-c.p.l for all \( x \in N \) since I=(0,a) is c.p.l of N but it is not a-c.p.l of N.

4.5 Proposition:
Let N be a Boolean \( \alpha_1, \alpha_2 \) near-ring with multiplicative identity \( e' \) then I is c.p.I of N if it is a e'-c.s.p.l of N.

4.6 Remark:
The following diagram show us the relationships between these ideals.
REFERENCES