Natural Convection Analysis around a Hot Solid Circular Cylinder Embedded Inside an Octagonal Enclosure at Various Orientation Locations

Salam Hadi Hussain and Ahmed Kadhim Hussein

Abstract—The problem of the natural convection in a two-dimensional octagonal enclosure is investigated numerically. A circular hot solid cylinder is placed at the center of the enclosure while all the eight walls of the enclosure is kept at a cold isothermal temperature. The cylinder moves by an equal amount (R/2 and R) in the horizontal, vertical and diagonal directions. The development mathematical model is governed by the coupled equations of continuity, momentum and energy and is solved by using the finite volume method. The computations are carried out for wide ranges of the governing parameters such as Rayleigh number (Ra) and internal volume method. The results explained that the cylinder location has a significant effect on the flow and thermal fields in the circular cylinder location. Also, it is found that the average Nusselt number for various orientation locations.

Keywords—Heat transfer, Natural convection, Cylinder, Octagonal enclosure, Cylinder location.

I. INTRODUCTION

Investigation of natural convection heat transfer in enclosures with irregular geometries has been performed in numerous studies due to its relevance to practical fluid flow devices, such as heat exchanger systems, solar collectors, electronic cooling modules and wet clutches [1-2]. Two types of natural convection phenomena can be noticed in the nature. The first is called external natural convection that is caused by the heat transfer interaction between a single wall and a very large fluid reservoir close to the wall. The second is called internal natural convection which occurs inside the enclosure walls. In comparison with uniform geometry enclosures, a very limited papers were investigated the natural convection in irregular geometries. For example, Talabi and Nwabuko [3] studied numerically the natural convection in parabolic enclosure. It was found that in case of isothermal hot wall, the heat transfer rate to the cold wall increased when both Grashof and Prandtl numbers increased. Lewandowski et al. [4] studied numerically the natural convection inside enclosures of different hemispherical convex or concave shapes. They concluded that the existence of the hemispherical bottom shape had a significant influence on natural convection heat transfer. Chen and Cheng [5] studied numerically the buoyancy-induced flow and convective heat transfer inside an inclined arc-shaped enclosure, while Mahmud [6] investigated the free convection inside an L-shaped enclosure. Dagtekin et al. [7] predicted the entropy generation for natural convection in a T-shaped enclosure. Heat removal was achieved from cooler vertical sides while the horizontal walls were perfectly insulated. Mahmoodi [8] investigated numerically free convection in L-shaped cavities filled with nanofluid. He found that effect of presence of nanoparticles on heat transfer enhancement was more apparent for narrow L-shaped cavities. Hussein and Hussain [9] performed a numerical modeling of steady laminar natural convection flow in a modified air – filled square enclosure with an inclined triangular top wall. They concluded that the flow and thermal fields had uniform patterns when the Rayleigh number was low for various values of wave amplitude and undulations number. Some other papers can be found in [10-12]. From the other side, natural convection in enclosures containing cylinders had received much attention as an important method of heat transfer enhancement. Lacroix [143] performed a numerical study of natural convection heat transfer around two heated horizontal cylinders inside a rectangular cavity cooled from above. Fu et al. [14] investigated the enhancement of natural convection in an enclosure by a rotating circular cylinder inside it. Braga and Lemos [15] studied numerically natural convection within a square cavity filled with a fixed amount of conducting solid material consisting of either circular or square obstacles. They showed that the average Nusselt number for cylindrical rods was slightly lower than those for square rods. Additional references can be found in [16-17]. According to this literature review, there is no attempt highlighting the natural convection in an octagonal enclosure containing a hot circular solid cylinder at its center and moving at various orientation locations. This attempt represents the first original contribution of the present work to deal with the octagonal enclosure.

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II. PROBLEM FORMULATION

A. The Physical Model and Governing Equations

The geometry considered in the present problem is shown in Fig. 1. It consists of an octagonal enclosure with an equal side length (L), where all the enclosure walls are kept at an isothermal cold temperature (T_c). The flow is assumed to be Newtonian, two-dimensional, steady and incompressible. The enclosure is filled with air (Pr = 0.71). In order to create the natural convection flow, a circular solid cylinder of radius (R= 0.2) is put inside the enclosure and kept at a temperature (T_h) higher than the enclosure walls where (T_h > T_c). The flow and the heat transfer phenomena in the enclosure are investigated for Rayleigh number (Ra) varying from 10^3 to 10^7 and various internal circular cylinder location (δ). The cylinder moves along the octagonal enclosure centerline in the horizontal (right and left) and vertical (upward and downward) directions by an equal amount (R/2 and R). Moreover, it moves along the two left and right diagonals of the octagonal enclosure with the same amount (R/2 and R). Using the Boussinesq approximation and neglecting the viscous dissipation effect the dimensionless governing equations can be written as follows:

\[ \nabla \cdot \mathbf{U} = 0 \]  
(1)

\[ U \nabla U = -\nabla P + Pr \nabla^2 U + Ra Pr \theta \]  
(2)

\[ U \nabla \theta = \nabla^2 \theta \]  
(3)

The above non-dimensional governing equations were obtained by utilizing the following dimensionless sets as follows:

\[ X, Y = \frac{x, y}{L}, U = \left(\frac{u, v}{V}\right) \frac{L}{a}, P = \frac{p L^2}{\rho a^2}, \theta = \frac{T - T_c}{T_h - T_c} \]  
(4)

The local and average Nusselt numbers are given by:

\[ Nu = \left. \frac{\partial \theta}{\partial n} \right|_W, \overline{Nu} = \frac{1}{L} \int_0^L Nu \, ds \]  
(5)

B. Problem Boundary Conditions, Numerical Implementation

The problem non-dimensional boundary conditions are represented by:

1. No-slip boundary conditions are applied at all enclosure walls, i.e., U = V = 0.
2. All the eight walls of the octagonal enclosure is maintained at a uniform cold temperature (i.e. \( \theta = 0 \)).
3. The surface of the internal circular solid cylinder is maintained at a uniform hot temperature (i.e. \( \theta = 1 \)).

The dimensionless governing equations are solved by using the finite volume method with a staggered grid system. The full details of the method of solution can be found in [18]. The SIMPLE algorithm has been adopted for the pressure-velocity coupling. A non-uniform grid meshes which are clustered adjacent the octagonal enclosure sharp corners to increase the results accuracy. A convergence criterion of \( 10^{-8} \) was chosen for all dependent variables while the value of (0.25) was taken as an under-relaxation parameter.

III. RESULTS AND DISCUSSION

A. Rayleigh Number Effect

The predicted streamlines and isotherms in the octagonal enclosure are shown in Figs. (2-5). The natural convection characteristics are small when the Rayleigh number is small (\( Ra = 10^4 \)). In general, the hot fluid around the heated circular cylinder is moved upward and replaced by the cold one. This cyclic process produces the well-known convection cells. The main convection cells fill the entire octagonal enclosure, while some minor cells can be observed adjacent the enclosure corners. In addition, a symmetry of the streamlines and isotherms can be seen along the mid-line of the enclosure when (\( Ra = 10^4 \)). Also, the thermal boundary layer begins to develop slowly around the heated circular cylinder and the heat transfer is essentially dominated by the heat conduction. As the Rayleigh number increases to (\( Ra = 10^7 \)) a large strength of flow circulation occurs which indicates that the natural convection heat transfer enhances inside the octagonal enclosure. As a result, the streamlines and isotherms begin to confuse especially adjacent the heated circular cylinder. Therefore, isotherms are converted from parallel linear lines when (\( Ra = 10^7 \)) to non-homogenous one and the non-linearity in isotherms can be seen clearly. This behavior is due to the increase of the heat transfer from the heated circular cylinder to the fluid inside the enclosure as a result of the flow circulation increasing. Also, the isotherms show a large temperature gradient near the heated circular cylinder.
Fig. 2 Distribution of streamlines for different orientation locations of circular cylinder at $Ra = 10^4$

Fig. 3 Distribution of isotherms for different orientation locations of circular cylinder at $Ra = 10^4$

Fig. 4 Distribution of streamlines for different orientation locations of circular cylinder at $Ra = 10^7$

Fig. 5 Distribution of isotherms for different orientation locations of circular cylinder at $Ra = 10^7$
Table I

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From the other hand, it can be noticed that the flow circulation is slow down near the enclosure walls. This is because the flow has a large area due to the octagonal shape while passing from the heated circular cylinder to the top region of the enclosure and then passing along the octagonal surface which leading to decrease the flow circulation.

B. Circular Cylinder Effect

The effect of the circular hot cylinder embedded in the octagonal enclosure on the streamlines and isotherms is shown in Figs. 2-5. When the natural convection effect is slight (i.e., Ra = 10^7), the streamlines around the circular hot cylinder is smooth and uniform. The same observation can be noticed with respect to isotherms which appear to be parallel to the octagonal enclosure cold sidewalls. The reason of this observation is due to the weak effect of the natural convection inside the enclosure. But, as the natural convection effect increases significantly (i.e., Ra = 10^12), the circulation of the fluid inside the enclosure improves and becomes strong enough to change the heat transfer from conduction in the previous case to convection in the present case. Consequently, the isotherms change its shape to semi-curved and approximately parallel to the circular hot cylinder. Furthermore, the flow field inside the octagonal enclosure is separated in to two re-circulating vortices around the cylinder.

C. Cylinder Orientation Locations Effect

The location of the circular hot cylinder has an important effect on the streamlines and isotherms as shown in Figs. 2-5. When the cylinder moves in the horizontal direction (to the right [φ = 0°] or to the left [φ = 180°]) from the center of the octagonal enclosure (where a two symmetrical vortices can be seen around the cylinder), the cylinder pushes the vortex in front of it which causing to increase the flow convection area behind it. This enlarging in the flow circulation increases as the cylinder location increases from (δ = +R/2 or -R/2) to (δ = +R or -R). In this case, some minor vortices can be seen around the upper surface of the cylinder. The isotherms are approximately circular and it can be noticed that when the cylinder moves by the amount (δ = +R or -R), the thickness of thermal boundary layer adjacent the upper right (when the cylinder moves in the right direction) or upper left (when the cylinder moves in the left direction) surfaces of the cylinder increases significantly. When the cylinder moves in the vertical direction (to the upward [φ = 90°] or to the downward [φ = 270°]) from the center of the octagonal enclosure, a two symmetrical vortices can be observed around the cylinder and their size increase as the cylinder location increases from (δ = +R/2 or -R/2) to (δ = +R or -R). The same patterns of the isotherms can be seen in this case except that the thermal boundary layer can be seen when the cylinder moves by the amount (δ = +R or -R) in the upper zone (when the cylinder moves in the upward direction) or in the lower zone (when the cylinder moves in the downward direction). Now, when the cylinder moves along the right diagonal [i.e., φ = 45° or φ = 225°] or along the left diagonal [i.e., φ = 135° or φ = 315°] of the octagonal enclosure, it pushes the vortex in front of it diagonally and again the flow circulation area increases as the cylinder location increases from (δ = +R/2 or -R/2) to (δ = +R or -R) in a diagonal direction. Again, similar patterns of the isotherms can be noticed except that the thermal boundary layer can be seen in the upper or lower diagonal zones when the cylinder moves by the amount (δ = +R or -R).

D. Nusselt Number Results

Table I displays the total surfaces or overall average Nusselt number of the internal hot cylinder (Nu_C) and the surface average Nusselt number of the internal hot cylinder (Nu_O) along with (δ) for different Rayleigh numbers and orientation locations. As expected, when the Rayleigh number increases, the average Nusselt number increases due to the increase in the buoyancy force and the natural convection effect when the Rayleigh number is high. Moreover, it can be noticed that the
overall average Nusselt numbers (\(\text{Nu}_\text{o}\)) for various orientations are less than the corresponding values of the average Nusselt number of the internal hot cylinder (\(\text{Nu}_\text{C}\)). This difference increases as the Rayleigh number increases. The reason of this behavior is due to high temperature gradient adjacent the internal hot cylinder which causes to increase its average Nusselt number (\(\text{Nu}_\text{C}\)). Furthermore, it can be concluded from the results of Table I that as the \(\delta\) increases, both average Nusselt numbers increase for various orientation locations. This is because the increase of \(\delta\) causes to make the hot cylinder moves fastly towards the enclosure walls. As a result, the flow area between the hot cylinder and the enclosure walls decreases which causing to increase the temperature gradient for both the cylinder and enclosure walls and therefore the average Nusselt numbers increase. This behavior can be considered general except that when \([Ra = 10^6]\), the average Nusselt numbers decrease slightly as \(\delta\) increases. This is due to the flow separation at the edge of the turbulent flow when \([Ra = 10^7]\).

IV. CONCLUSIONS

The following conclusions can be drawn from the results of the present work.

1. For low Rayleigh number (\(Ra = 10^4\)), the isotherms are approximately parallel to the vertical cold sidewalls and the heat transfer is occurred via heat conduction.
2. The thermal boundary layer adjacent the heated circular cylinder increases as the Rayleigh number increases to \((Ra = 10^7)\). Also, the isotherms are distorted and the flow circulation increases.
3. High average Nusselt number is obtained when the Rayleigh number is high.
4. The internal circular solid cylinder has a significant effect on the flow and thermal fields inside the octagonal enclosure.
5. It is found that the average Nusselt number of the internal hot cylinder (\(\text{Nu}_\text{C}\)) is greater than the overall average Nusselt number for various orientation locations.
6. When the cylinder moves by the amount (\(\delta = +R\) or \(-R\)), the thermal boundary layer thickness adjacent the upper surface of the cylinder increases.
7. The cylinder location has a significant effect on the flow and thermal fields inside the octagonal enclosure and different behaviors are seen when the cylinder moves in the horizontal, vertical and diagonal directions.

NOMENCLATURE

- \(L\) Length of each side of the octagonal enclosure (m)
- \(\text{Nu}\) Average Nusselt number
- \(n\) The normal direction with respect to the enclosure left sidewall
- \(P\) Dimensionless pressure
- \(P\) Pressure (N/m²)
- \(Pr\) Prandtl number
- \(R\) Radius of internal solid circular cylinder (m)
- \(Ra\) Rayleigh number
- \(T\) Temperature (K)
- \(U\) Dimensionless velocity component in X-direction
- \(u\) Dimensional velocity component in x-direction (m/s)
- \(V\) Dimensionless velocity component in Y-direction
- \(v\) Dimensional velocity component in y-direction (m/s)
- \(x\) Non-dimensional coordinate in horizontal direction
- \(y\) Non-dimensional coordinate in vertical direction

GREEK SYMBOLS

- \(\alpha\) Thermal diffusivity (m²/s)
- \(\theta\) Non-dimensional temperature distribution
- \(\phi\) Cylinder orientation location along the enclosure diagonal (degree)
- \(\delta\) Circular cylinder location
- \(\rho\) Density of fluid (kg/m³)

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