Numerical Simulation of Forced Convective Flow and Heat Transfer in Gravitationally Inclined Circular Pipes with Equi-Spaced Internal Fins

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ABSTRACT

The paper investigated forced convective fluid flow and heat transfer in gravitationally oriented circular ducts with equi-spaced internal longitudinal fins for the purpose of augmenting heat transfer in heat exchangers of circular cross-section. Navier–Stokes equation of motion and energy transport equation were adopted as the governing equations for the fluid flowing in the geometry. The governing equations were non-dimensionalized using certain defined transformation parameters. A computational algorithm was developed based on the finite difference scheme for the normalized governing equations. A Quick-BASIC program was developed for the implementation of the scheme. The computed results show that between $0^\circ$-$15^\circ$ pipe inclinations, there is a rapid increase in heat transfer enhancement which slowly increases towards $75^\circ$ inclination and remains almost a plateau up to the vertical position of the pipe. It was discovered that although increased fin height above $H = 0.2$ increases the heat transfer surface area; parameters of fluid volume and flow velocity are adversely affected and therefore, any protrusion beyond $H = 0.2$ is a waste of material and resources. Reduction of fin half angle positively affects the heat augmentation. For an inclination range of $0^\circ \leq \omega \leq 8^\circ$, increase in the number of fins increases the mean Nusselt number. The effect of ratio of conductivity of solid to fluid on heat transfer was found to be significant only for high Prandtl number fluid flow. This investigation does not only provide consistent results with those previously computed via fluid flow and heat transfer in circular ducts with longitudinal internal fins but also emphasize critically the influence of pipe orientation on heat transfer particularly for piping network.

KEYWORDS: Fluid Flow, Heat Transfer, Gravitational orientation, Internal Fins, Simulation

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Area ($m^2$)</td>
</tr>
<tr>
<td>$A-A$</td>
<td>Area of cross-section ($m^2$)</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat capacity at constant pressure (J/kgK)</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter (m)</td>
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<tr>
<td>$D$</td>
<td>Normalized diameter</td>
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<tr>
<td>$dx$</td>
<td>Total derivative in the x-coordinate</td>
</tr>
<tr>
<td>$F$</td>
<td>Number of fins</td>
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<tr>
<td>$g$</td>
<td>Acceleration due to gravity ($m/s^2$)</td>
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<tr>
<td>$h$</td>
<td>Dimensional fin height (m)</td>
</tr>
<tr>
<td>$h'$</td>
<td>Heat transfer coefficient (W/m$^2$K)</td>
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<tr>
<td>$H$</td>
<td>Normalised fin height</td>
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<tr>
<td>$k$</td>
<td>Thermal conductivity (W/mK)</td>
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<tr>
<td>$k_*$</td>
<td>Ratio of thermal conductivities of solid to fluid</td>
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<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
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<tr>
<td>$p$</td>
<td>Pressure (N/m$^2$)</td>
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<tr>
<td>$P$</td>
<td>Wetted perimeter (m)</td>
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<tr>
<td>$P_e$</td>
<td>Peclet number</td>
</tr>
<tr>
<td>$P_e$</td>
<td>Peclet number modifier</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$q_m$</td>
<td>Apparent heat flux at the inner wall of tube (W/m$^2$)</td>
</tr>
<tr>
<td>$r$</td>
<td>Dimensional radius (m)</td>
</tr>
<tr>
<td>$R$</td>
<td>Normalised radius</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number</td>
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<td>$t$</td>
<td>Dimensional temperature (K)</td>
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<td>$t_e$</td>
<td>Uniform entering fluid temperature (K)</td>
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<td>$T$</td>
<td>Normalised temperature</td>
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<tr>
<td>$u$</td>
<td>Dimensional velocity (m/s)</td>
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<tr>
<td>$v$</td>
<td>Velocity (m/s)</td>
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<tr>
<td>$w$</td>
<td>Dimensional pipe thickness (m)</td>
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<tr>
<td>$W$</td>
<td>Normalised pipe thickness</td>
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<tr>
<td>$x$</td>
<td>Axial co-ordinate; horizontal axis</td>
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<tr>
<td>$X$</td>
<td>Normalised axial co-ordinate</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>Change in x</td>
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<tr>
<td>$y$</td>
<td>Vertical axis</td>
</tr>
<tr>
<td>$z$</td>
<td>Dimensional radial distance from the fin tip to entre of pipe (tube) (m)</td>
</tr>
<tr>
<td>$Z$</td>
<td>Normalised radial distance from the fin tip to centre of pipe</td>
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The analysis of flow and heat transfer enhancement in ducts of various geometries is of prime importance particularly to those who are interested in the design, development, augmentation and effective performance of heat transfer in heat exchangers as in the case of refrigeration, steam generators, forced hot water heating systems, split-system heat pumps, etc [1,2]. However, circular ducts have gotten wide applications in the iron and steel industry, air-conditioning and refrigeration industry, hydro-electric power generation and the oil and gas sector. These pipes with fluid flowing in them are basically used as heat exchangers (which are either shell or finned-tube type) for cooling or heating the systems and surroundings [3,4]. When heat transfer rate is less than necessary, one of such methods for heat transfer enhancement is using internally extended surface called fin [5]. The longitudinal fin of constant cross-sectional area is widely used in practice to enhance heat dissipation [6], which has found application in heat recovery steam generators, condensers, pre-cooler and evaporators, forced hot water heating systems, etc.

Though most equipment is designed for operation in the turbulent flow regime, laminar flow has to be considered for partial load operation [2]. Under these conditions, laminar forced convective heat transfer in a circular tube with internal longitudinal fins has been of interest in research for many years. This has been numerically analyzed by several researchers but most of these investigators concentrated on the effects of fin and tube conductivity. Some of these early works include studies by Aung and Worku [7] who employed dimensionless parameters to obtain a criterion for the existence of reversed flow under thermal boundary conditions of uniform heating on one wall while the other wall was thermally insulated. Yao [8] studied mixed convection in vertical channel between parallel plates with symmetric uniform temperature and symmetric uniform flux heating and conjectured that fully developed flow might consist of periodic reversed flow. Kettner et al. [9] carried out numerical study of laminar heat transfer in finned tubes by setting a thermal boundary condition at the outer wall. Rowley and Patankar [10] carried out research on analysis of laminar flow and heat transfer in tubes with internal circumferential fins. Eckert and Robert [11] submitted that when fins are advantageous, the heat transfer is increased by placing them as near to each other as practicable. Kuniyasu et al. [12] worked on numerical analysis on laminar forced convective heat transfer in thermal entrance region of circular tube with internal longitudinal fins. In their study, they concluded among other things that the elongation of fin height induces the velocity vortex. Orfi and Galanis [13] studied the effects of tube inclination, as well as that of the Grashof and Prandtl numbers, on the fully-developed ascending flow in uniformly heated tubes under combined forced and free convection with uniform heat flux at the wall. Prakash and Liu [14] in their work analyzed heat transfer of a laminar flow in the entrance region of an internally finned circular duct while Kettner et al [15] also looked into heat transfer by tubes that are internally finned. Zeitoun and Hegazy [16] in their work studied numerically laminar flow inside internally finned pipes. In the investigation a thermal boundary condition of uniform wall temperature was considered for different fin heights. Likewise is Soliman and Feingold [17] who undertook the study of flow in longitudinal internally finned tubes. They studied laminar fluid behaviour in the tubes. Rustum and Soliman [18, 19] studied both numerically and experimentally heat transfer in tubes with longitudinal internal fines. Dogan and Sivrioglu [20] in 2010 published a paper on experimental investigation of mixed convection heat transfer from longitudinal fins in a horizontal rectangular channel.

The present investigation considered the effects of varieties of variable parameters such as conduit’s orientation, fin’s heights, fin’s angle which is a function of fin’s thickness, walls thickness, number of fins, fluid properties, gravity of the flow, etc. The work was able to identify some critical parameters that could enhance the
design of heat exchangers and general pipe network. It also tracked the influence of flow regimes on flow behaviours and looked at the heat transfer coefficients at both the entrance and fully developed fluid flow regions.

2. PROBLEM FORMULATION

![Physical Model and the Co-ordinate System](image)

The model adopted for this study is a circular tube with internal longitudinal fins in cylindrical co-ordinate system \((r, \theta, x)\) inclined at an angle \((\omega)\) to the horizontal. The angle of inclination varied from 0\(^{\circ}\) to 90\(^{\circ}\) (vertical position). The effect of gravity is neglected except in the axial direction. The fins which are protuberances from the wall of the pipe form an integral part of the pipe material as depicted by Fig.1. The fins are truncated straight fins of triangular cross-section. The choice of this geometry is borne out of the fact that a reasonable percentage of weight is saved by using triangular fins over rectangular fins for equal heat flow [11]. It is assumed that the velocity profile is laminar forced convective and fully developed in the tube. It is also assumed that there is constant heat flux at the outer wall of the finned tube, internal longitudinal fins are symmetrically arranged along the axis inside the tube and the physical properties of the fluid and tube material are constant.

Though finned pipes had been looked into by many researchers [1-20], the present study intends to probe further into enhancing the design of pipe networking by establishing some critical flow and geometric parameters that can serve as design guide particularly for the inclined terrain.

2.1 Formulation of Governing Equations

With reference to [20], the primitive equations for continuity, momentum and energy transport are cast as follows:

2.1.1 Equation of continuity

\[
\frac{\partial v_r}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_x}{\partial x} + \frac{v_r}{r} = 0 \quad (1)
\]
The fluid flow in the x-direction is assumed totally predominant over \( r \) and \( \theta \), then;
\[
v_r = v_\theta = 0; \quad \frac{\partial p}{\partial r} = \frac{\partial p}{\partial \theta} = 0
\]  
(2)

2.1.2 Equation of motion
Following [21], the primitive equations of motion in \((r, \theta)\) directions is,
\[
\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = \frac{1}{\mu} \frac{\partial p}{\partial x} + \frac{\rho g}{\mu} \sin \omega.
\]  
(3)
Where, \( v_x = u \) and \( g_x = -g \sin \omega \).

2.1.3 The Energy Transport Equation
The energy transport equation is,
\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = \begin{cases} \frac{u}{\alpha} \frac{\partial T}{\partial x} & \text{(fluid)} \smallskip \frac{u}{\alpha} \frac{\partial T}{\partial x} & \text{(tube & fin)} \end{cases}
\]  
(4)
Where, \( \alpha_i = \frac{k}{\rho C_p} \).

2.2 Normalization Parameters
The following transformation parameters were used to non-dimensionalize equations (1-4):
\[
X = \frac{x}{r_{in} \rho \alpha}; \quad R = \frac{r}{r_{in}}; \quad H = \frac{h}{r_{in}}; \quad W = \frac{w}{r_{in}}; \quad k^* = \frac{k_x}{k_f}; \quad Pr = \frac{C_p \mu}{k}.
\]
\[
P_{t} = \frac{2 \mu u_{max} r_{in}}{\mu}; \quad P_{e} = \frac{2 r_{in} u_{max}}{\alpha_i}; \quad P_{k} = \frac{P_g}{p_{e} \left( \frac{dp}{dX} \right)}; \quad U = \frac{u}{r^2 / \mu \left( -dP/dx \right)};
\]
\[
T = \frac{t-t_{in}}{q_{in} r_{in} / k_f}; \quad D_h = \frac{d_h}{r_{in}}
\]

2.2.1 Normalized Momentum Transport Equation
\[
\frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} + \frac{1}{R^2} \frac{\partial^2 U}{\partial \theta^2} = -1 - Pr \sin \omega
\]  
(5)

2.2.2 Normalized Energy Transport Equation
\[
\frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} + \frac{1}{R^2} \frac{\partial^2 T}{\partial \theta^2} = \begin{cases} \frac{2U}{\alpha_i} \frac{\partial T}{\partial x} & \text{(for fluid)} \smallskip \frac{2U}{\alpha_i} \frac{\partial T}{\partial x} & \text{(for tube & fin)} \end{cases}
\]  
(6)

23. Applicable Boundary Conditions
The boundary conditions to be satisfied are:

a. Fluid region
i. at the centre, \( r = 0, u = u_{max} \).
The solution technique is based on the finite difference approximation of the normalized momentum transport equation. The boundary conditions are applicable at various surfaces, including the outer wall of the tube and the interface between solid and fluid. The energy transport equation is subject to the following boundary conditions:

### 3. SOLUTION TECHNIQUE

3.1 Finite Difference Analogue (FDA) of the Normalized Momentum Transport Equation

\[
U_{ij} = 0.5 \left[ \frac{1 + P \sin \theta + \frac{1}{(\Delta \theta)^2} + \frac{1}{2 \Delta R \Delta \theta}}{(\Delta \theta)^2 + (R \Delta \theta)^2} \right] U_{ij} + \frac{1}{(\Delta \theta)^2} \left[ U_{ij} + \frac{1}{2 \Delta R} \left( U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} + U_{i,j} \right) \right]
\]
3.2 FDA of the Normalized Energy Transport Equation

The finite difference analogue of the normalized energy transport equation is

$$T^n_{i,j} = \frac{0.5}{\left(\frac{1}{\Delta R} + \frac{1}{(R_i) \Delta \theta} + \frac{1}{\Delta X}\right)} \left\{ \frac{1}{\Delta R} \left[ \frac{1}{2} \left( \frac{1}{\Delta R} \right) T^n_{i+1,j} + \frac{1}{2} \left( \frac{1}{\Delta R} \right) T^n_{i-1,j} \right] \right\} + \frac{1}{(R_i) \Delta \theta} \left[ \frac{T^n_{i+1,j} + T^n_{i-1,j}}{2} \right] + \frac{U_{i,j}}{\Delta X} \left[ \frac{T^n_{i+1,j} - T^n_{i-1,j}}{2} \right]$$

(14)

$T^n_{i,j}$ is the known temperature at entrance while $T^n_{i+1,j}$, $T^n_{i-1,j}$ etc. are the unknown temperatures along the pipe and fin. For the advantage of data storage, accuracy and time economy, following Weltey[23], an iterative technique is selected for the solution of the problem. The velocity field was first evaluated followed by the temperature field. A first approximation for the interior values were evaluated by introducing the initial guess values. The procedures were repeated until convergence was attained subject to following condition:

$$\left| \Gamma^{m+1}_{i,j} - \Gamma^m_{i,j} \right| \leq 10^{-4}$$

where, $\Gamma$ is the damping variable which can represent $U$ and $T$. $m$ is the iterative counter.

4. EVALUATION OF THE MEAN NUSSELT NUMBERS AND VALIDATION OF RESULTS

4.1. Evaluation of the Mean Nusselt Numbers

The heat transfer coefficients and Nusselt numbers were defined based on average temperature at the surface of the inner wall of the finned tube and local heat flux. The circumferential local heat transfer coefficient and Nusselt number respectively are defined by [11,12] as:

$$Nu_{INT} = \left[ \frac{2}{T_w(X) - T_b(X)} \right]$$

(15)

Where, $T_b(X) = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{T_j UR \Delta R \Delta \theta}{UR \Delta R \Delta \theta}$; \hspace{1cm} $T_w(X) = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{TR \Delta \theta}{R \Delta \theta}$

The mean extended Nusselt number is given as:

$$Nu_{EXT} = \frac{D_h - \frac{dT}{dR}}{T_w - T_b}$$

(16)

The hydraulic diameter,

$$D_h = \frac{4A_c}{P}$$

(17)

and $A_c$ is the cross-sectional area of the geometry, and $P$ is the wetted perimeter.

4.2 Validation of the Numerical Scheme

The program for implementing the scheme was written in QB-45. The value of the average velocity is half the maximum velocity of flow ($U_{max} = 0.25$). This was obtained by running the program on a 161 x 12 grid in the (R-$\theta$) co-ordinate. Also, the Nusselt number for flow in a smooth tube subject to constant heat flux at its wall is 4.364 [21]. Upon running the finite difference analogue of the energy transport equation on a 25 x 26 grid in the (R-$\theta$) co-ordinate, an asymptotic value of 4.38 was obtained as the Nusselt number.

5. RESULTS AND DISCUSSION

5.1 Programme Validation
Fig. 2: Nusselt number along axial length of a finless pipe.

Fig. 3: Heat transfer along the pipe axis. \( F=8, W=0.2, H=0.5, k^*=500, \beta=3^\circ, Re=2000, Pr=0.7 \) at 15° inclination

Fig. 4: Graph of mean Nusselt number versus inclination for \( F=8, H=0.2, k^*=500, \beta=3^\circ, Re=700, W=0.2, Pr=0.7 \)
Fig. 5: Variation of average velocity with for various inclinations and for different fin heights. 
F=8, H=0.2, W=0.2, k*=500, β=3° Re=1000, Pr=0.7

Fig. 6: Plots of average velocity against inclinations for various Prandtl numbers. 
F=8, W=0.2, H=0.5, k*=500, β=3°, Re=2000

Fig. 7: Graph of extended Nusselt Number for various inclinations. 
H=0.5, Re=2000, F=8, W=0.2, k*=500, Pr=0.7

Fig. 2 is a plot of mean Nusselt number along the pipe axis for finless tube. The value of the mean Nusselt number decreases monotonically along the length of the pipe until it becomes invariant with X; thus \( (T_w - T_b) \) becomes a constant in the axial location \( \Delta X = 0.08 - 0.1 \). In this region, the flow is both hydro-dynamically and thermally fully developed. The obtained asymptotic value \( Nu = 4.38 \) is close to 4.364 - the well established value for thermally developed heat transfer in a circular duct with uniform-heat flux condition at the duct wall. This value also, is in good agreement with the work of Kuniyasu et al. [12]. Fig. 3 shows a comparison between the mean
extended Nusselt number and inner wall Nusselt number along the pipe axis. It is observed from the plot that there had been tremendous amount of heat transferred by the fin. The difference $(\text{Nu}_{\text{EXT}} - \text{Nu}_{\text{INT}})$ reduces along the axis of the pipe until it reaches the axial location $\Delta X = 0.08-0.1$ where it achieved nearly a constant value. The large difference in the axial location $\Delta X = 0.01-0.04$ may be attributed to the high potential of the flow at the entrance.

Fig. 4 presents the influence of orientation of the pipe on the mean Nusselt number. The result obtained depicts that the inclination enhances heat augmentation. Fig. 5 shows the effect of fin height and inclination on the velocity of flow. It was discovered from the plot that for all inclinations, average velocity of the flow reduces with increase fin length. This behaviour may be as a result of the reduction in the flow cross-section. Fig. 6 shows the variation of average velocity of fluid with tube inclination and for different Prandtl number. It’s worth noting that the higher the Prandtl number, the higher the velocity of flow. The influence of gravity certainly accounts for the increasing velocity as the Prandtl number increases. Fig. 7 shows the influence of pipe inclination and fin half angle on heat transfer from the extended surface. It was found that reduction in fin half angle enhances heat transfer. With reduction in this fin parameter, the cross sectional area of the flow increases which may result to an increase in the amount of fluid flow, and thus, more heat is been convicted.

Fig. 8: Variation of mean extended Nusselt Number with inclination for various Re. 
$H=0.5, F=8, W=0.2, k^*=500, \beta=3^\circ, Pr=0.7$

Fig. 9: Graph of mean extended Nusselt number with pipe inclinations for various Prandtl numbers. $F=8, W=0.2, k^*=500, \beta=3^\circ, Re=2000$

Fig. 10: Effects of Pipe orientation on mean extended Nusselt number for different W. $F=8, k^*=500, \beta=3^\circ, Re=2000, H=0.5, Pr=0.7$
Fig. 8 shows the variation of mean extended Nusselt number with varying Reynolds number over pipe inclination. It is observed that within an inclination range of $0^\circ \leq \omega \leq 15^\circ$ Reynolds number (Re) has no significant effect on the amount of heat transfer but the effect of Re becomes noticeable above this range. This is an indication that $\omega = 15^\circ$ is a critical orientation in pipe laying technology. Fig. 9 shows the variation of mean extended Nusselt number with inclination for different fluids. The plots show that there is proportionate increase in the mean Nusselt number with increase in Prandtl number. It worth noting that at $60^\circ$ inclination, the mean extended Nusselt number become invariant with inclination for fluids whose Prandtl falls between 0.7 and 1.0. These values of Prandtl number is a range within which the property of air falls. Fig. 10 shows a curve that depicts the effect of pipe thickness on mean extended Nusselt number. For the range of $k^*$ considered, it was observed that the effect of wall thickness on heat transfer is negligible. This is an indication that any fluid in that range behaves thermally identical, particularly when piping at $\omega > 60^\circ$.

Similarly, Figs. 11 and 12 show the effect of ratio of conductivities ($k^*$) on mean extended Nusselt number for fluid with low and high Prandtl numbers respectively. In Fig. 11 it is observed that although there is heat augmentation by inclination for low Prandtl number the heat transfer is invariant with $k^*$. In Fig. 12, heat transfer is equally enhanced by both the inclination and Prandtl number. The effect of $k^*$ becomes positively significant for $k^*$ value of the order of 1000. These results are in good agreement with the submissions made by Kettner et al. [9] in the literature.

6. CONCLUSION

A numerical simulation of fluid flow and heat transfer in gravitationally oriented circular pipes with equi-spaced internal fins had been investigated for the augmentation of heat transfer in heat exchangers of circular cross-section. A computer code in Quick-BASIC had also been developed for the implementation of the numerical scheme established for the configuration. The validity result of $\text{Nu} = 4.38$ and average velocity of 0.12506 obtained for finless circular geometry was found to compare favourably with works of Kuniyasu et al [12] ($\text{Nu} = 4.38$, Average velocity, $V_{\text{mean}} = 0.12496$) and Yuan [21] ($\text{Nu} = 4.364$, Average velocity, $V_{\text{mean}} = 0.125$) using the same set of parameters. Some concluding remarks which are also derivable from this study are as follows:

- Inclination enhances heat transfer.
• H = 0.2 is the optimum fin height for maximum heat augmentation in an inclined pipe; exceeding this value will amount to waste of materials and resources.
• For inclination range of $0^\circ \leq \omega \leq 15^\circ$, it is advisable for economic purposes to operate the forced convective mechanism at a low Reynolds number flow regime because in that range of inclination, Nusselt number does not show any remarkable change in numerical value for all the Reynolds numbers considered ($1000 \leq Re \leq 2000$).
• Though the average velocity of the fluid flow is highest at the vertical position of the pipe, the heat transfer does not noticeably enhanced by inclination beyond $75^\circ$.
• For the configuration considered, the effect of wall thickness on heat transfer is negligible.
• For low Prandtl number fluid like air, the ratio of thermal conductivity of the walls to that of the working fluid, $k^*$ has no significant effect on heat convected from the surface of the walls. However, the ratio becomes significant for high Prandtl number fluids, particularly when the value of $k^*$ is in the order of 1000.

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REFERENCES


