On Fuzzy Pairwise α - Continuous and Fuzzy Pairwise α- open mapping

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Abstract :

We introduce the concept of fuzzy pairwise α - Continuous maps and fuzzy pairwise α - open maps and investigate the relation between these concepts from side and several properties of such maps .

المستخلص :

في هذا البحث سنقدم مفهومي (دوال αالمستمرة الضبابية) ودوال α- المفتوحة الضبابية

والعلاقة بينهما والخواص المهمة لهذه الدوال .

Introduction

Fuzzy topological spaces were introduced by C.L. Chang[2] who studies a number of basic concepts including fuzzy continuous maps and compactness fuzzy topological spaces are natural generalization of topological spaces .A bitopological space is a collection of two topologies defined on non-empty set X , or if (X ,τ_1) and (X,τ_2) are two topologies defined on X then the bitopology X is denoted by (X,τ_1,τ_2) . In (1965) O.Njastad [ ] introduced a concepts of α - open set and α- continuous mapping also in (1983) S .Mashhour, I.A. Hasanein and S.N. EL-Deeb introduced an α-open mapping and they were also studied an α -continuous mapping . In this research we shall study a fuzzy pairwise α-continuous in bitopological spaces and also we shall give anew definitions of a fuzzy concept of an α- open mapping in bitopological spaces .

A mapping $f:(X,\tau)\rightarrow(Y,\rho)$ is called α- continuous (briefly. α-cont.) iff the inverse image of each open set of Y is α-open in X where X and Y are topological spaces [4] . A mapping $f:(X,\tau)\rightarrow(Y,\rho)$ is called α-open iff $f(A)$ is α-open in Y for each open set in X [4].

1- Some basic definitions

Definition (1.1) [3]

A fuzzy topology τ on X is a collection of subsets of I^X such that

1. $0,1 \in \tau$
2. if $\lambda,\mu\in\tau$ then $\lambda\wedge\mu\in\tau$
3. if $\lambda_i\in\tau$ for all $i\in J$,then $\vee\lambda_i\in\tau$.

And (X , τ) is called fuzzy topological space , members of τ are called fuzzy open sets in (X , τ) and its complement is called fuzzy closed set .

Definition (1.2)

A fuzzy bitopological space (briefly , fbts) is a non– empty set with two fuzzy topological spaces τ_1 and τ_2 define on X ,in other word is the triple (X,τ_1,τ_2) where (X , τ_1) and (X , τ_2) are two fuzzy topological spaces .

Definition (1.3) [5]

Let $f:X\rightarrow Y$ be a mapping from X into Y , if λ is a fuzzy set in X and μ is a fuzzy set in Y then we define $f(\lambda)$ and $f^{-1}(\mu)$ as follows

$$f(\lambda)(y)=\begin{cases}\displaystyle\sup_{x\in f^{-1}(y)}\{\lambda(x)\} & \text{iff } f^{-1}(y) \text{ is non-empty}\\ 0 & \text{Other wise}\end{cases}$$

And $f^{-1}(\mu)(x)=\mu(f(x))$ for all $x\in X$.

Definition (1.4) [4]

For a fuzzy set λ of X , the τ_i - closure and τ_i - interior are defined respectively , as follows

τ_i-cl (λ) = inf { $v : v \geq \lambda$, $1-v \in \tau_i$ }

τ_i-int (λ) = sup { $v : v \leq \lambda$, $v \in \tau_i$ }

2- Fuzzy pairwise α - Continuity

Definition (2.1)

A subset λ of a fuzzy topological space (X , τ) is called fuzzy α-set if $\lambda \leq$ τ_i- int (cl (int (λ))) .

Definition (2.2)

A mapping $f:(X,\tau)\rightarrow(Y,\rho)$ is said to be fuzzy α-continuous (briefly f α- cont.) iff the inverse image of each fuzzy open set of Y is an fuzzy α- set .

Definition (2.3)

A mapping $f:(X,\tau_1,\tau_2)\rightarrow(Y,\rho_1,\rho_2)$ is said to be fuzzy pairwise α- continuous (briefly f P α- cont.) iff the mapping $g:(X,\tau_1)\rightarrow(Y,\rho_1)$ and $h:(X,\tau_2)\rightarrow(Y,\rho_2)$ are fuzzy α-continuous .

Theorem (2.4)

Let $f:(X,\tau_1,\tau_2)\rightarrow(Y,\rho_1,\rho_2)$ be a mapping from fuzzy bitopological space X into fuzzy bitopological space Y , then the following statement are equivalent :

i) f is fuzzy pairwise α- continuous .
ii) For each $x \in X$ and each fuzzy open set v in Y containing $f(x)$, there exists $x \in X$ such that , $w \in X$, $f(w) < v$.
iii) the inverse image of each fuzzy closed set in Y is fuzzy α-closed .

before a prove of this theorem above ,we have to know that .

Remark (2.5) [4]

Every open set is an α - set .

Remark (2.6) [3]

Every fuzzy open set is a fuzzy α - set .

Theorem (2.7)

Any fuzzy pairwise continuous mapping $f:(X,\tau_1,\tau_2)\rightarrow(Y,\rho_1,\rho_2)$ is a fuzzy pairwise α - continuous .

Proof :

Let $f:(X,\tau_1,\tau_2)\rightarrow(Y,\rho_1,\rho_2)$ is fuzzy pairwise continuous mapping , hence $g:(X,\tau_1)\rightarrow(Y,\rho_1)$ and $h:(X,\tau_2)\rightarrow(Y,\rho_2)$ are fuzzy continuous . Now let λ is an open in (Y,ρ_1) and μ is an open in (Y,ρ_2) ,thus $g^{-1}(\lambda)$ and $h^{-1}(\mu)$ are two fuzzy open in (X,τ_1) and (X,τ_2) respectively , and by Remark (2.6) we have $g^{-1}(\lambda)$ and $h^{-1}(\mu)$ are two fuzzy α - set in (X,τ_1) and (X,τ_2) respectively which means that $f:(X,\tau_1,\tau_2)\rightarrow(Y,\rho_1,\rho_2)$ is fuzzy pairwise α - continuous .

Now , proof of theorem (2.4)

i $\Leftrightarrow$ ii , it is easy by theorem (2.7) and definition of fuzzy pairwise continuity .

ii $\Leftrightarrow$ iii , it is easy if we take the complement of theorem (2.7) .

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