Variation of Probability Values Respect to Exponential Distribution with Fuzzy Logic

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Abstract

In spite of there is relationship between fuzzy logic principles and a probability principles in calculate values for computations. There are some distinct from values different probability distributions a values of a membership functions .This led to finding a variation in these computations. Some continuous distributions and continuous membership functions has several command properties .In this paper we choose an exponential distribution and find a corresponding calculation in fuzzy logic through membership functions.

1. Introduction:

The difference between the degree of membership in the set (of some member), and a probability of being in that set.

The point of difference is, the probability involves a crisp set theory (probability of it belongs to class or not), and don't allow for an element to be a partial member in a class (or a set, as in fuzzy logic[1]).

Probability is an indicator of frequency or likelihood that an element is in a class, while fuzzy set theory deals with the similarity of an element to a class that is between elements in a class. Anyone who doesn't know and haven't study fuzzy logic and fuzzy sets think, that fuzziness is just a clever disguise for probability, which is never true .

Although fuzzy logic is known latterly ,it has been communicated with many other sciences for its benefits in practical applications(applicable branches).Since 1991, fuzzy logic is used in technology as an industrial tool to be fuzzy control, but the theoretical side stay requisite .

Probability theory and fuzzy set theory have been communicated since they were depend on same range to be in closed interval $[0,1]$,also membership function(MF) that characterize the fuzzy set depend on some parameters(time_ verify parameters)and its values chosen from parameter space(real number).While probability distribution also depend on parameters describe the distribution and determine its values and shape ,which chosen from parameter space, many ways used to locate these parameter values.

Also the values of MF constraints are as $0 \le \mu \le 1$, while probability has a main condition as $\sum p(u) = 1$.

To shed light on such a relationship, a probability distribution used to compare the values that computed by Exponential distribution function with that values computed by Gaussian MF (both were continuous functions) on tables for values of dependent variable(s) applied for both functions and values for parameters that be in each function .

2. Chosen Membership Function MF :

A MF that chosen for this work is Gaussian MF is defined over an infinite support , that is infinite set from space of variable values into real values at $[0,1]$ not exceed *0* to a negative values ,with single maxima since it is convex function ,shape of Gaussian MF is give positive values, this function is symmetric since it an Exponential Function over squared value to get positive values(in spite variable values are negative or positive) .

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[1] A fuzzy logic is basically a multi_valued logic that allows intermediate values to be defined between conventional evaluations like; true/false ,black/white ,yes/no ,high/ low (see any reference on fuzzy logic).

Gaussian MF is in form(For Use with MATLAB User's Guide) as ;

$$\mu(x,s,t) = e^{\frac{-(x-t)^2}{2s^2}} \qquad \ldots \quad ()$$

That its symmetric depends on width parameter s and center parameter t.

For special case when $s=1$ and $t=0$ as:

$$\mu(x) = e^{-x^2/2} \qquad \ldots \quad ()$$

That were used in this paper for standard properties .

Gaussian function do not led to negative value ,this restrict values smaller than 1 and over than 0 , that is exactly what we want from a MF .

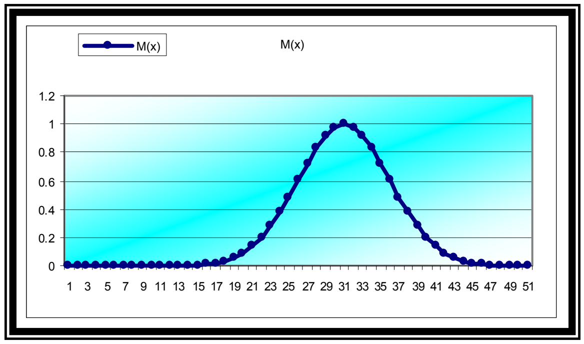


Figure (1): The Gaussian MF .

3.The Exponential Distribution

The exponential distribution is a simple distribution also commonly used in reliability engineering. Mathematically, it use in inappropriate situations. It is used to model the behavior of units that have a constant failure rate (or units that do not degrade with time or wear out), this means that the population has no wear-out or infancy problems.

The pdf of this distribution is defined by variable x and parameter λ ;

$$f(x,\lambda) = \lambda e^{-\lambda x} \qquad \text{.... (1)}$$

Also it is not very useful in modeling data in the real world. Electronics will have a decreasing failure rate . In Weibull distribution a special case where $\beta = 1$ and $\lambda = 0$.

Since x is random variable and λ is parameter of the distribution. There are many ways to estimate values of the parameters in pdf , but in this work we suppose values occurred randomly in all tables of the listed experiments since λ scale $f(x)$ differently , that depend on x values .

3.1 The Exponential Failure Rate Function:

The failure rate function for exponential distribution is:

$$\lambda(x)=\frac{f(x)}{R(x)}=\frac{\lambda e^{-\lambda(x)}}{e^{-\lambda(x)}}$$
$$=\lambda$$

The exponential failure rate function is: $\lambda(x)=\dfrac{f(x)}{R(x)}=\dfrac{\lambda e^{-\lambda(x-\gamma)}}{e^{-\lambda(x-\gamma)}}$

$$=\lambda$$

Where λ is constant.

Once again, note that the constant failure rate is a characteristic of the exponential distribution, and special cases of other distributions only. Most other distributions have failure rates that are functions of time. The shape parameter, as it named refer, define shape of a distribution. Exponential distribution does not have a shape parameter since it has a predefined shape that does not change. γ is location parameter that be some in supposition too small or +ve or -ve used to determine origin and shift of distribution. Probability Distributions can have any number of parameters, which determine the shape and location of graphic of distribution function effect is reflected in the shapes of the pdf and the failure rate function.

4. *Probability Values for Fuzzy logic with Exponential Distribution*

Both fuzzy logic and probability are valid approaches to the classification problem(Knapp,1998),for example ,if we were to classify "*old* " ,fuzzy membership make much more sense that probability since in probability each (person)either "*old* " has probability or not has probability ,that is *probability*=0 .

Also in another way a person who is dying of thirst in the desert is given two bottles of fluid ,one bottle's label says that it has a 0.9 membership degree in the class of fluid known as nonpoisonous drinking water(or sea water, swamp water, cola,…,etc) .The other bottle's label states that it has a 90% probability of being pure drinking water and a10% probability of being poison ,Which bottle(if you where there) choose?

A fuzzy bottle contains (swamp water ,as example) cola ,this also makes sense since cola would have a 0.9 membership in the class of nonpoisonous fluids .

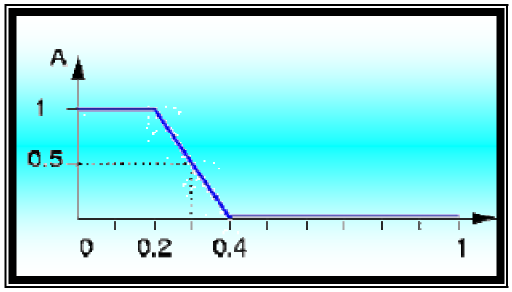
This example was given by *Bezdek* see reference(Knapp,1998) as a good example to demonstrate the conceptual difference for statistical and fuzzy classification .The degree of certainty(somewhere) sounds like a probability (perhaps subjective probability),but it is not quit the same .Hot and cold can have 0.6 and 0.5 as their membership degrees in these fuzzy sets(a fuzzy values),but not as probabilities (which could not) (Rao & et al,1993).

It is become clear that both operate over the same numeric range ,and have similar values as; 0 representing False(or non membership ,in fuzzy) ,and 1 representing True(or full membership in a fuzzy) .

Let us take for instance a possible interferometer coherence g values to be the set X of all real numbers between 0 and1,from this set X as a subset A can be defined as(all values $0 \le g \le 0.2$),that is(Garibaldi & et al) ;

$$A=\{g:0\le g\le 0.2\} \qquad \dots \quad (4)$$

Since g starts at 0 ,the lower range of this set ought to be clear ,the upper range on the other hand ,is rather hard to define. The MF operating in this case on the fuzzy set of interferometer coherence g returns a value between 0.0 and 1.0 ,for example ,an interferometer coherence g of 0.3 has a membership of 0.5 to the set low coherence , see figure(2) .



Figure(2): Characteristic of a fuzzy set

The probabilistic approach yield the natural language statement *"there is an 50% chance that g is low"*, the probability view suppose that g is or not low it is just that we only have an 50% chance of knowing which set it is in .By contrast, fuzzy terminology supposes that g is "*more or less low" ,*or in some other term corresponding to value of 0.50 .

The comparison cleared through helpful properties for Weibull distribution and Gaussian Membership Function, that both functions are real valued functions on t to range [0,1]on domain with real infinite variable values, and both are continuous functions with every where positive and with single maxima .All previous properties led to a comparison through numerical values for functions that not exceed than 1 and not less than 0 ; $0 \leq F(t) \leq 1$,which also satisfied for $\mu(t)$.

Also the comparison depended on parameters values for taking as; $a = \sigma$, $b = m$,or other form at be with extension to a parameters space is the same for both cases ,which will be a real space for this work .An understanding of the rate may provide insight as to what is causing the failures :

- A decreasing failure rate would suggest "infant mortality". That is, defective items fail early and the failure rate decreases over time as they fall out of the population.
- A constant failure rate suggests that items are failing from random events.
- An increasing failure rate suggests "wear out" - parts are more likely to fail as time goes on.

The result for this all will be explained and graphics through practical examples with numerical values .

5. Calculated the Resulted Probability Values and Discussion:

Here the variable x is supposed as a time t ,parameter values and its effect unction distribution values with numerical results given in the tables () that list all values that were supposed and computed, which represent time, values of parameters that where chosen in real space .

We show that the influence of the time, the shape and scale parameters on the exponential distribution function, Gaussian function and failure rate. For different values of times and the parameters under given conditions for each function .Each table followed by three graphics represent the results (at that table) a polar graph in surface and polar curve and curvature, which all help us to describe and discuss the difference in values in more accurate .

5.1 Influence of Variable

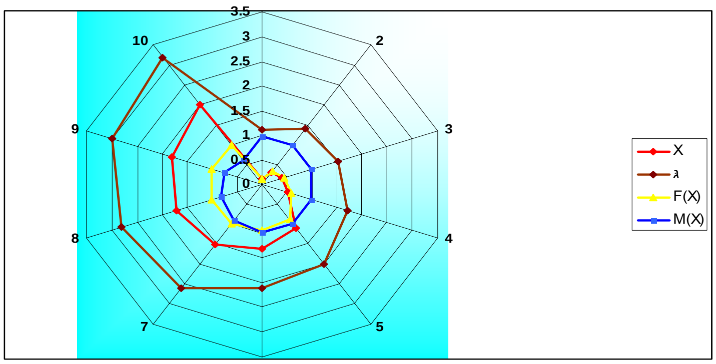
Some times the variable x is supposed as a time t

5.1.1 First Experiment: The data of this attempt were shown in table(1) ,in which different values were supposed for the variable x and parameter λ .the values were within [0.1,2] for x and within [1.1,3.2] which all real numbers .In front values of PDF $F(x)$ for exponential distribution were calculated and then the membership function $\mu(x)$ calculate membership degree for each value of x as below;

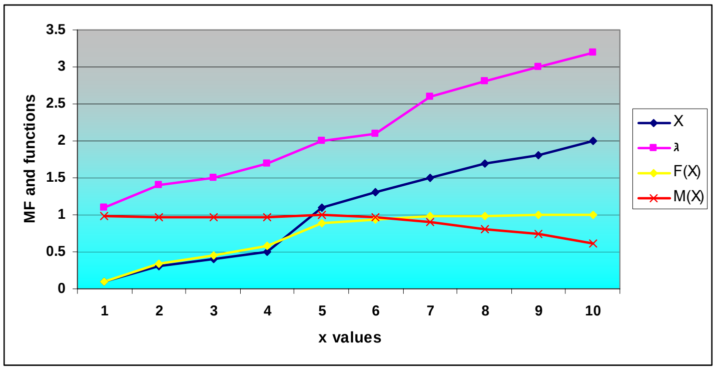
Table (1):Calculated values for PDF $F(x)$ and membership function $\mu(x)$

for x and λ

x	λ	*f(x)*	*M(x)*
0.1	1.1	0.104166	0.979954
0.3	1.4	0.342953	0.963917
0.4	1.5	0.451188	0.96464
0.5	1.7	0.572585	0.969233
1.1	2	0.889197	0.997254
1.3	2.1	0.934781	0.971174
1.5	2.6	0.979758	0.91051
1.7	2.8	0.991434	0.812004
1.8	3	0.995483	0.749762
2	3.2	0.998338	0.606531



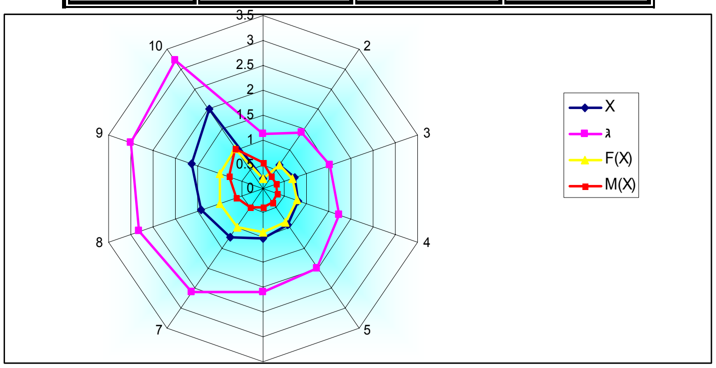
*Figure(3):Values of **M(X)** and **F(X)** for λ values in [1.1,3.2] .*



*Figure(4):*Pointer figure for ***M(X)*** *and* ***F(X)*** *for* **λ** *values*

Table(2):Influence of **λ** *with different values of x in [0.2,2]*

x	λ	*F(x)*	*M(x)*
0.2	*1.1*	*0.197481*	*0.523091*
0.6	*1.4*	*0.568289*	*0.30851*
0.7	*1.5*	*0.650062*	*0.306358*
0.8	*1.7*	*0.743339*	*0.316004*
0.9	*2*	*0.834701*	*0.336553*
1	*2.1*	*0.877544*	*0.367879*
1.2	*2.6*	*0.955843*	*0.46394*
1.4	*2.8*	*0.980159*	*0.604109*
1.6	*3*	*0.99177*	*0.774142*
2	*3.2*	*0.998338*	*1*



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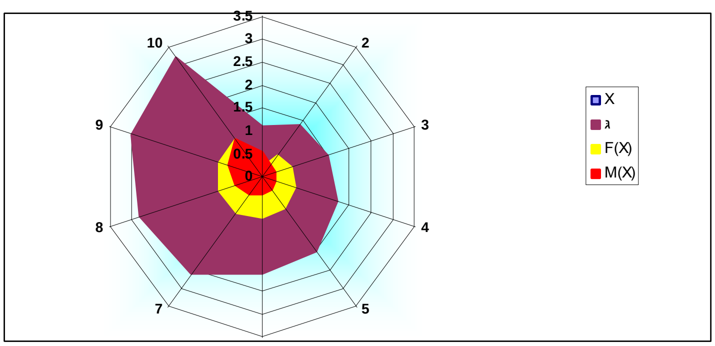

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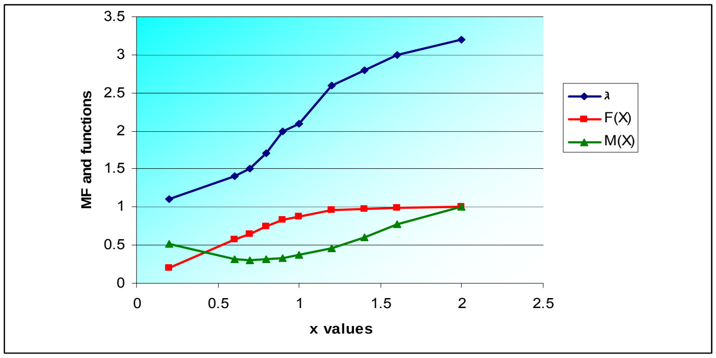


*Figure(7): Pointer figure for **M(X)** and **f(X)** for **λ** values with different values of x in [0.2,2] .*

5.1.2 Second Experiment: This table the effect of parameter λ through take single value at $\lambda = 0.1$ in return for values of variable x in closed interval [0,2], see in figure() the values of functions that were calculated for the supposed variable values .

Considering constant value for the parameter λ as λ =0.1 for all supposed values of *x* in [0,2] with step 0.1 for accuracy ,as it shown in table(3) below with corresponding values of functions *F(x)and M(x)* .

Table (3):Calculated values for PDF $F(x)$ and membership function $\mu(x)$

for x and $\lambda = 0.1$

x	λ	*F(x)*	*M(x)*
0	0.1	0	1
0.1	0.1	0.00995	0.979954
0.2	0.1	0.019801	0.968507
0.3	0.1	0.029554	0.963917
0.4	0.1	0.039211	0.96464
0.5	0.1	0.048771	0.969233
0.6	0.1	0.058235	0.976286
0.7	0.1	0.067606	0.984373
0.8	0.1	0.076884	0.992032
0.9	0.1	0.086069	0.997753
1	0.1	0.095163	1
1.1	0.1	0.104166	0.997254
1.2	0.1	0.11308	0.988072
1.3	0.1	0.121905	0.971174
1.4	0.1	0.130642	0.945539
1.5	0.1	0.139292	0.91051
1.6	0.1	0.147856	0.865888
1.7	0.1	0.156335	0.812004
1.8	0.1	0.16473	0.749762
1.9	0.1	0.173041	0.680621

2	0.1	0.181269	0.606531

The function *M(x)* reach maximum value which is 1 at two values of *x*; in 0 and 1 which give the function features to be with a two maxima ,every where positive and not symmetric ,while *F(x)* has just single maxima (0.181269) at 2 ,the figures(8,9,10) below showing the graphic of these function

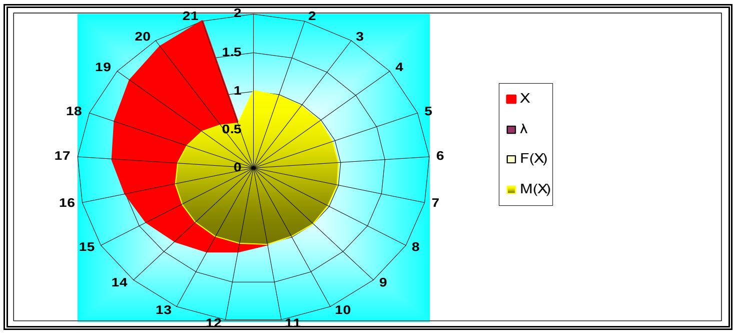
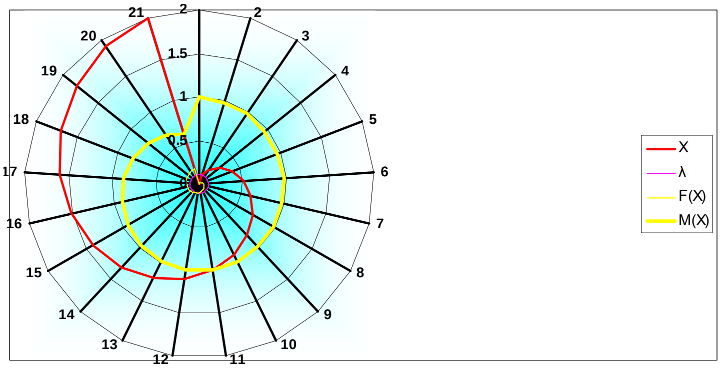


Figure (): .



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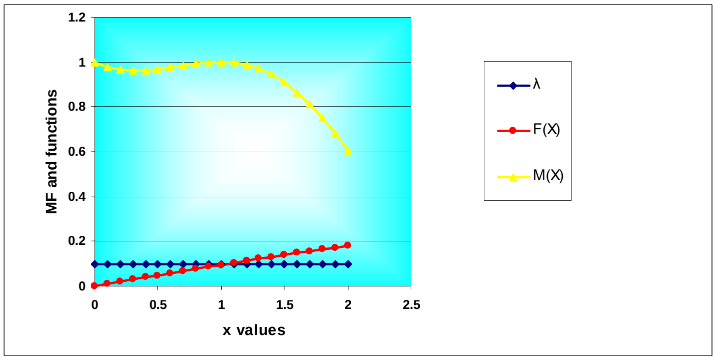


Figure (): .

5.1.3 ***Third Experiment:*** Considering constant large value for the parameter λ such as λ =1 for all supposed values of *x* in previous table(3) ,as it will shown in table(4), below with corresponding values of functions *F(x)and M(x)* .

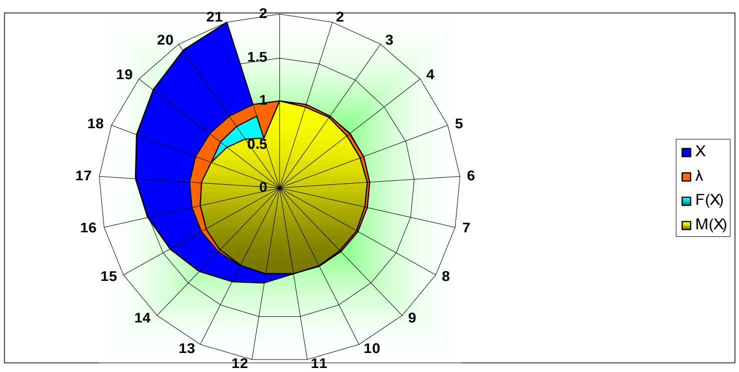
Table(4):Values of $\lambda=1$ *with values of x in [0,2]with step 0.1 .*

Table ():Calculated values for PDF $F(x)$ *and membership function* $\mu(x)$

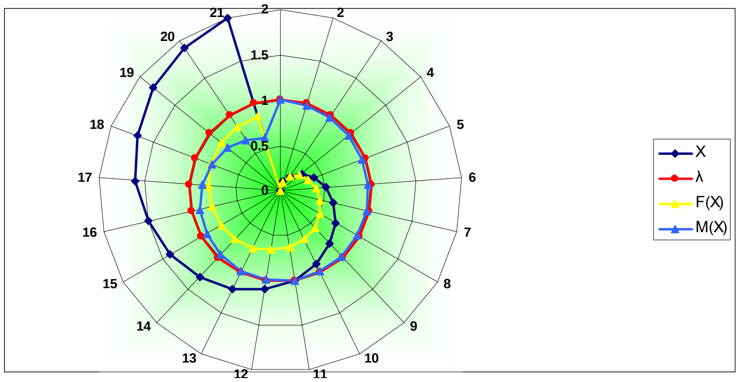
$for\ x\ and\ \lambda = 1$

x	λ	*F(x)*	*M(x)*
0	1	0	1
0.1	1	0.095163	0.979954
0.2	1	0.181269	0.968507
0.3	1	0.259182	0.963917
0.4	1	0.32968	0.96464
0.5	1	0.393469	0.969233
0.6	1	0.451188	0.976286
0.7	1	0.503415	0.984373
0.8	1	0.550671	0.992032
0.9	1	0.59343	0.997753
1	1	0.632121	1
1.1	1	0.667129	0.997254
1.2	1	0.698806	0.988072
1.3	1	0.727468	0.971174
1.4	1	0.753403	0.945539
1.5	1	0.77687	0.91051
1.6	1	0.798103	0.865888
1.7	1	0.817316	0.812004
1.8	1	0.834701	0.749762
1.9	1	0.850431	0.680621
2	1	0.864665	0.606531

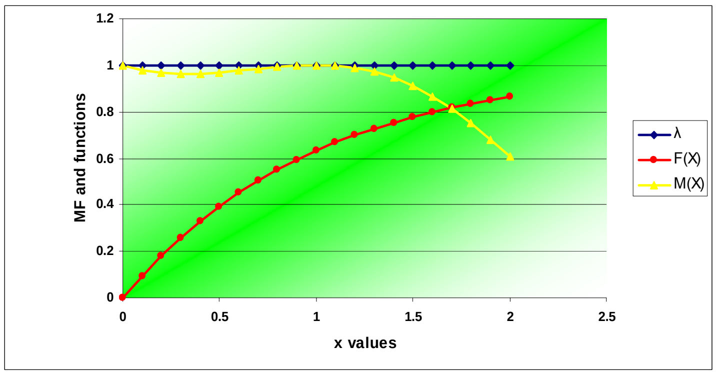
The function *M(x)* reach maximum value which is 1 at two values of *x*; in 0 and 1 also which give the function features to be with a two maxima ,every where positive and not symmetric ,while *F(x)* has just single maxima (0.864665) at2 ,the figures(11,12,13) show graphic of these functions.



Figure(11): Grayed surfacing figure for table (4)



*Figure(12):*Grayed figure for functions table (4)



Figure(13): Pointer figure for M(X) and F(X) for constant value for λ with different *values of x in [0,2]* .

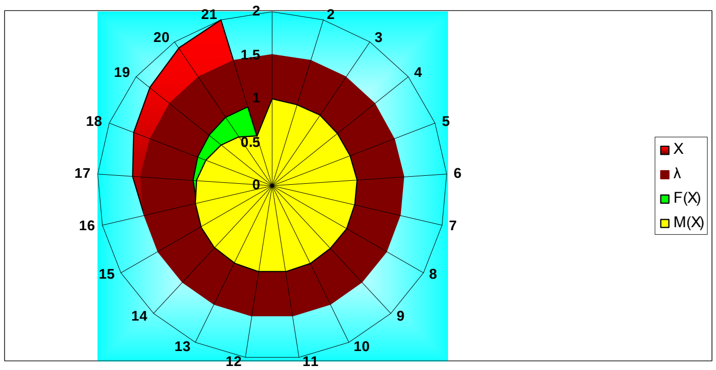
5.1.4 Fourth Experiment: Considering constant large value for the parameter λ such as λ =1.5 for all supposed values of *x* in table(3) ,as it will shown in table(5), below with corresponding values of functions *F(x)and M(x)* .

Table ():Calculated values for PDF $F(x)$ *and membership function* $\mu(x)$ *for x and* λ=1.5 with values of *x* in [0,2]with step 0.1

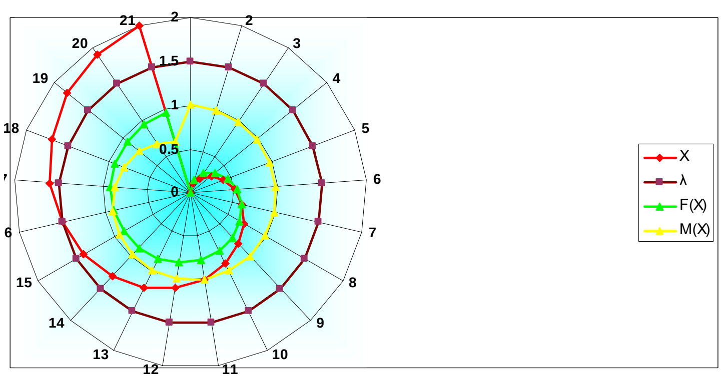
x	λ	*F(x)*	*M(x)*
0	*1.5*	*0*	*1*
0.1	*1.5*	*0.139292*	*0.979954*
0.2	*1.5*	*0.259182*	*0.968507*
0.3	*1.5*	*0.362372*	*0.963917*

0.4	1.5	0.451188	0.96464
0.5	1.5	0.527633	0.969233
0.6	1.5	0.59343	0.976286
0.7	1.5	0.650062	0.984373
0.8	1.5	0.698806	0.992032
0.9	1.5	0.74076	0.997753
1	1.5	0.77687	1
1.1	1.5	0.80795	0.997254
1.2	1.5	0.834701	0.988072
1.3	1.5	0.857726	0.971174
1.4	1.5	0.877544	0.945539
1.5	1.5	0.894601	0.91051
1.6	1.5	0.909282	0.865888
1.7	1.5	0.921918	0.812004
1.8	1.5	0.932794	0.749762
1.9	1.5	0.942156	0.680621
2	1.5	0.950213	0.606531

The function *M(x)* reach maximum value which is 1 at two values of *x*; in 0 and 1 also which give the function features to be with a two maxima ,every where positive and not symmetric ,while *F(x)*has just single maxima (0.950213) at 2 which is large than previous values in tables(3,4) ,the figures(17,18,19) show graphic of these functions.



Figure(17): Grayed surfacing figure for table (5)



Figure(18): Grayed figure for table (5)

5.2 Different values for λ and x;

1. Considering different values for the parameter λ in [1,4] , for supposing values of *x* in [0.1,2] ,as it shown in table(6) below with corresponding values of functions *F(x)and M(x)* .The function *M(x)* reach maximum value which is 0.997254 at 1.1 value of *x* at λ=2.; also which give the function features to be with a single maxima ,every where positive and not symmetric ,while *F(x)*has just single maxima (0.999665) at 2 which is large than previous tables ,the figures(20,21) that show graphic of these functions.

Table (6):Calculated values for PDF $F(x)$ *and membership function* $\mu(x)$

for x in [0.1,2] *and* λ in [1,4]

x	λ	*F(x)*	*M(x)*
0.1	1	0.095163	0.696979
0.3	1.2	0.302324	0.42021
0.4	1.3	0.405479	0.359155
0.5	2.1	0.650062	0.324652
1.1	2.2	0.911078	0.410245
1.3	2.4	0.955843	0.528877
1.5	3.1	0.990438	0.687289
1.7	3.3	0.996339	0.85813
1.8	3.5	0.998164	0.930531
2	4	0.999665	1

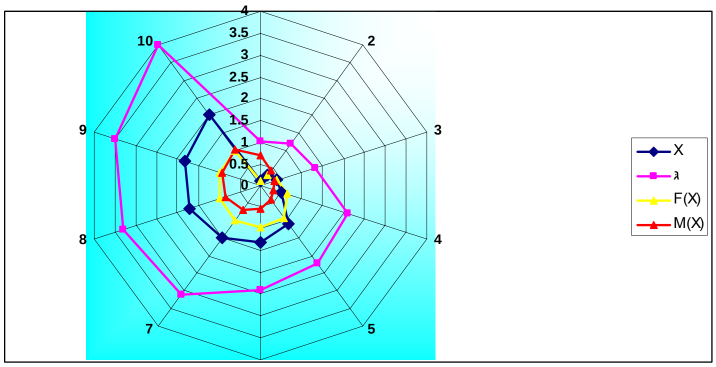
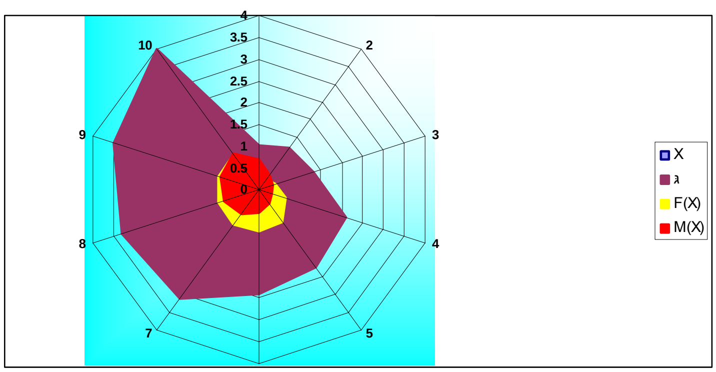
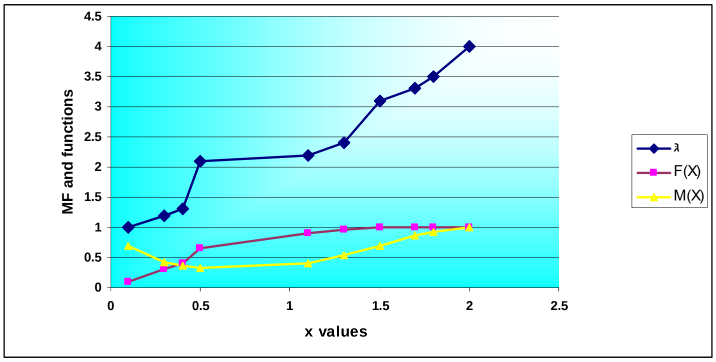


Figure ():Grayed figure for functions intable () .



Figure(17): Grayed surfacing figure for table



Pointer figure for M(X) and F(X) for constant value for λ with different

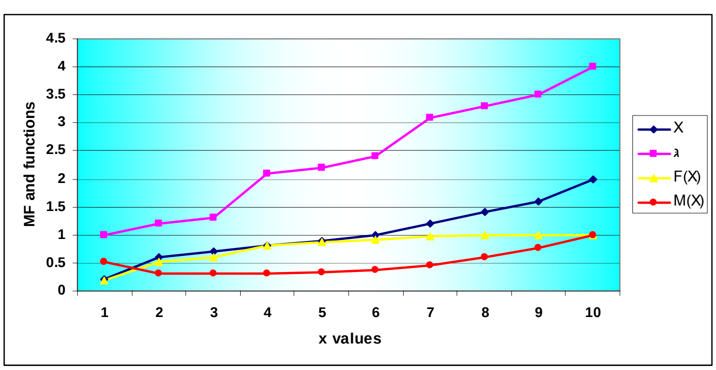
values of x

2. Considering different values for the parameter λ in [1,4] , for supposing values of *x* in interval [0.2,2],see the table .

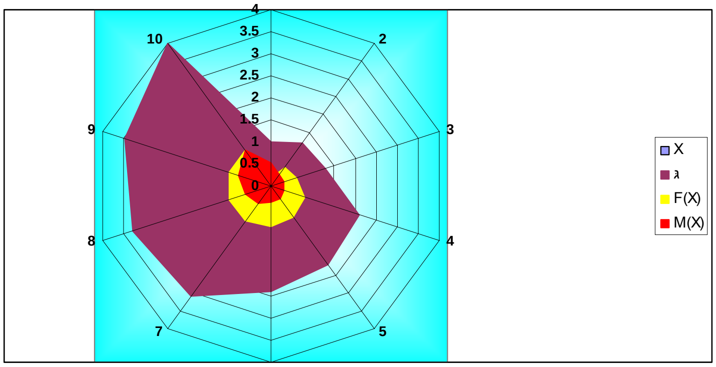
Table (6):Calculated values for PDF $F(x)$ *and membership function* $\mu(x)$

for x in [0.2,2] *and* λ in [1,4]

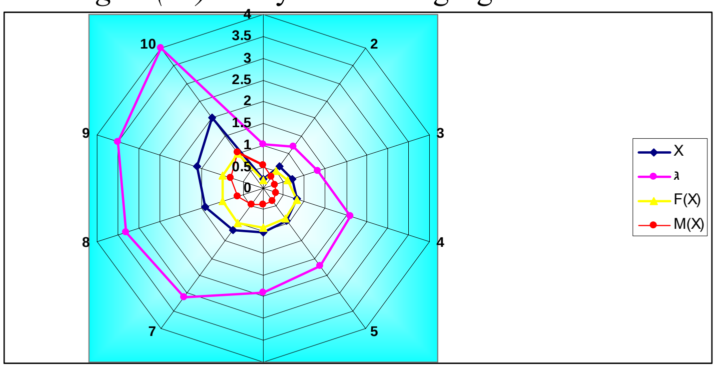
x	λ	*F(x)*	*M(x)*
0.2	*1*	*0.181269*	*0.523091*
0.6	*1.2*	*0.513248*	*0.30851*
0.7	*1.3*	*0.597476*	*0.306358*
0.8	*2.1*	*0.813626*	*0.316004*
0.9	*2.2*	*0.861931*	*0.336553*
1	*2.4*	*0.909282*	*0.367879*
1.2	*3.1*	*0.975766*	*0.46394*
1.4	*3.3*	*0.990147*	*0.604109*
1.6	*3.5*	*0.996302*	*0.774142*
2	*4*	*0.999665*	*1*



Pointer figure for M(X) and F(X) for constant value for λ with different
values of x



Figure(17): Grayed surfacing figure for table

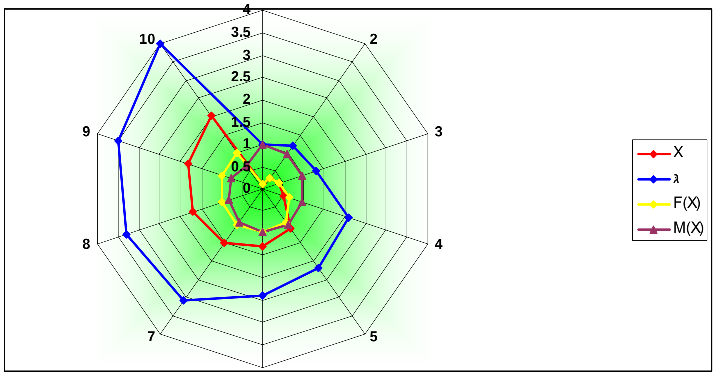


Figure(): Grayed figure for table:.

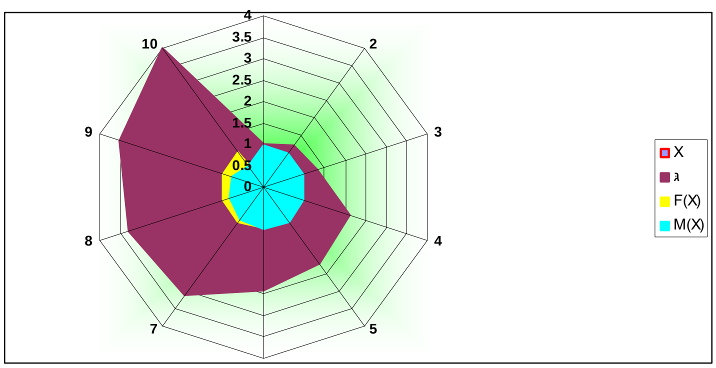
3. Considering different values for the parameter λ in [1,4] , for supposing values of x in [0.1,2]

Table ():Calculated values for PDF $F(x)$ *and membership function* $\mu(x)$ *for* x λ with values of x

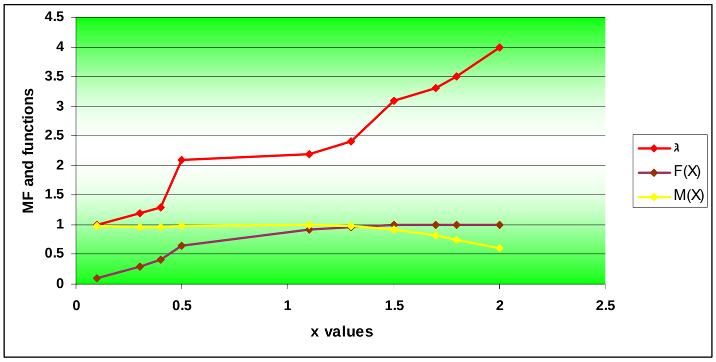
x	λ	*F(x)*	*M(x)*
0.1	***1***	***0.095163***	***0.979954***
0.3	***1.2***	***0.302324***	***0.963917***
0.4	***1.3***	***0.405479***	***0.96464***
0.5	***2.1***	***0.650062***	***0.969233***
1.1	***2.2***	***0.911078***	***0.997254***
1.3	***2.4***	***0.955843***	***0.971174***
1.5	***3.1***	***0.990438***	***0.91051***
1.7	***3.3***	***0.996339***	***0.812004***
1.8	***3.5***	***0.998164***	***0.749762***
2	***4***	***0.999665***	***0.606531***



Figure(): Grayed figure for table:.



Figure(17): Grayed surfacing figure for table



Pointer figure for M(X) and F(X) for constant value for λ with different

values of x

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