Numerical Simulation of the Counterflow Regenerative Rotary Heat Exchanger

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ABSTRACT:

A mathematical model for the unsteady state heat and mass transfer in regenerative rotary heat exchanger is derived and solved numerically by using finite difference method. Triangular shape of the rotary matrix with silica gel (SiO₂) that used as desiccant material is taken here. The heat transfer coefficient and the mass transfer coefficient that used in the present study are suitable for different types of the matrix shapes. The unsteady state time effect, inlet air temperature, Number of Transfer Unit (NTUₜ), heat exchanger length, inlet air temperature and the desiccant material volume on the latent, sensible and total effectiveness are discussed. It has been notes that the increasing of the heat exchanger length, inlet air temperature, desiccant material volume and the (NTUₜ) will increase the sensible, latent and total effectiveness. The increasing of the inlet air mass flow rate will decrease the total, latent and sensible effectiveness. The steady state time for the latent effectiveness is lower than the steady state time for the sensible and total effectiveness.

Keywords: Heat Exchanger, Rotary, Enthalpy Exchange, Latent and Sensible Heat Transfer, Finite Difference, Regenerative, Counterflow.
INTRODUCTION

Energy recovery can substantially reduce the mechanical heating and cooling requirements associated with conditioning ventilation air in most locations and application. Heating and cooling systems can be downsized and down cost when energy recovery is used, because energy recovery systems reduce peak heating and cooling requirements, [ASHRAE, 2005].

Rotary regenerative heat exchangers have certain advantages over other types of heat exchangers for the recovery of heat from exhaust air [WorsÄe-Schmidt, 1991]. Two distinct advantages are that a considerably larger heat transfer area per unit volume and relatively less expensive, and this type of heat exchanger has higher effectiveness compared with other recovery heat exchanger. Effectiveness of plate-type heat exchangers is between 45% and 65%, as compared to 60% and 80% for rotary regenerators [Heinrich, and Franzke, 1993]. Besides these, the speed of the rotation of the matrix can be used to regulate the amount of the heat transferred. Thus, rotary regenerative heat exchangers combine both compactness and high performance [Skiepko, 1989]. Rotary regenerators find increasingly frequent use in air-conditioning systems and gas turbine engines. They have a long history as air pre-heaters in steam power plants [Romie, 1988].

In many researches considered, rotary heat exchangers that transfer sensible heat only and steady state operating conditions [Klein et al., 1990, Ciepliski et al., 1998, Simonson et al., 1999 a and b].

Kays and London, 1984, presented an “Effectiveness-NTU” (ε-NTU) method for predicting the effectiveness of sensible recuperative and rotary regenerative heat exchangers as a function of dimensionless numbers. They showed that the effectiveness depends on two dimensionless numbers for sensible heat exchangers (NTU and C_t) and four dimensionless numbers for rotary heat exchangers.
(NTU, C_r, C_r^*, h_a_0 / h_a_0). **Bowlen, 1993,** stated that the energy recovery in the cooling/dehumidifying mode can be up to 2.5 times greater than for an equally sized sensible heat exchanger and approximately 40% greater in the heating/humidifying mode. **Gunnar Stiesch, 1994,** investigated experimentally the performance of the rotary regenerative heat and mass exchangers for different types of desiccants material to allow energy savings in the heating and cooling of ventilated buildings by recovering energy from the exhaust air and transferring it to the supply air stream for different types of the desiccants. **JoAnna Christen Staton, 1998,** discussed different types of polymeric desiccant materials and the ultimate goal of his study was to develop a material that, when applied to an existing air-to-air heat exchanger would achieve the necessary heat and mass transfer in a single process, thus transforming a sensible heat exchanger into a total enthalpy exchanger. His study focused on the development and determination of appropriate polymeric desiccant materials for use in different heat and mass transfer applications. Various candidate materials were initially studied. **Buyukalaca, and Yilmaz, 2002,** presented a correction factor for the speed of the rotary heat exchanger effectiveness, this factor is used with traditional “Effectiveness-NTU” method to estimate corrected value of rotary heat exchanger effectiveness. **Yilmaz and Ukalaca, 2003,** presented a calculation method for the design of rotary regenerators having different flow channel geometries. This method is dependent to correct the tradition heat exchanger calculation method “Effectiveness-NTU” by introducing two factors for correction, one for rotational correction and other for the purge area. They showed this method is valid for rotational speeds between 0.05 and 7 rev/min. **Oyetope, 2005,** studied the energy wheel effectiveness by a selected international laboratory for many types and sizes of wheel. His study used a new, low-cost, transient method to predict the effectiveness using only data obtained from transient measurements. In his study, an analytical model was presented for predicting the effectiveness of rotating energy wheels using only the characteristics measured on the same non-rotating wheels exposed to a step change in temperature and humidity. A relationship between the step response and the periodic response of an energy wheel is developed using first order linear system design theory. **Wang et al., 2005,** developed a new experimental facility to study the transient response of sensors and equipment. They studied the transient response of a humidity sensor and used the same sensor to measure the relative humidity downstream of a non-rotating energy wheel following a step change in inlet air relative humidity.

In the present study, a mathematical model is derived to generate the governor equations for the unsteady state countercurrent flow regenerative heat exchanger with mass and energy conservation, because these equations are impossible to solved analytically, then a numerical techniques is used to solve these equations by using finite difference method. Triangular shape of the matrix and silica gel (SiO_2) is used as desiccant material in the present study because the silica gel is most popular desiccant material. It can selectively absorbs compounds of different molecular weights.

![Fig. (1), Cross Sections of Enthalpy Regenerative Heat Exchanger, [Reilly and Van Geet, 2003]](image)
MATHEMATICAL FORMULATION

The assumptions that will be used in the present study are (Fig. (1)):

1- The inlet and outlet state properties for the both fluids are uniform and not change with time.
2- The thermodynamic properties of the air and the matrix are not affected by the pressure drop in axial direction of the matrix which is small compared to the total pressure.
3- There is no mixing or carry out between the two fluid streams.
4- The fluids and matrix states are considered to be uniform in radial direction.
5- Angular and axial heat conduction and vapor diffusion due to temperature and concentration gradient respectively are neglected.
6- The matrix is considered to be a homogenous solid with constant matrix characteristics.
7- The convective heat and mass transfer coefficient between the air streams and the matrix are constant throughout the system.
8- The rotary regenerative heat exchanger operates adiabatically.
9- The working fluids (inlet and exhaust) are air.

The mathematical formulation of combined heat and mass transfer in unsteady state regenerative rotary heat exchanger can be derived by applying the mass and energy conservation on the rotary regenerative heat exchanger (Fig. (2)) compound with the previous assumptions, [JoAnna Christen Staton, 1998]:

\[ \sum (\text{the inlet air mass}) - \sum (\text{the outlet air mass}) = \text{mass change of the desiccant matrix material with time} \]

\[ M_{a,in,x} + M_{a,in,0} - M_{a,out,x} - M_{a,out,0} = \text{mass change of the desiccant matrix material with time} \] .......(1)

\[ M_{a,out,x} = M_{a,in,x} - \frac{\partial M_{a,in,x}}{\partial x} \Delta x \quad \text{and} \quad M_{a,out,0} = M_{a,in,0} - \frac{\partial M_{a,in,0}}{\partial \theta} \Delta \theta \] .............(2)

\[ - \dot{m}_a \frac{\partial w_a}{\partial x} \Delta x - M_a \frac{\partial w_a}{\partial \theta} \Delta \theta = M_m \frac{\partial w_m}{\partial \theta} \Delta \theta \] ........................................(3)

For the time interval \( \lambda_j \)
\[ -\dot{m}_a \frac{\partial w_a}{\partial x} + \Delta x - M_a \frac{\partial w_a}{\partial \theta} - \lambda_j = M_m \frac{\partial w_m}{\partial \theta} - \lambda_j \]  

\[ \dot{m}_a \frac{1}{L} = \dot{m}_{a,j} \frac{\Delta \theta}{\Delta x} \Rightarrow \dot{m}_a \frac{\Delta x}{\Delta \theta} = \dot{m}_{a,j} L \]  

Then, the final form can express as following:

\[ \dot{m}_{a,j} L \frac{\partial w_a}{\partial x} + M_a \frac{\partial w_a}{\partial \theta} + \lambda_j + M_m \frac{\partial w_m}{\partial \theta} = 0 \]  

For the same procedures mentioned previously. Applying the heat transfer balance for the system in order to get the energy equation, [JoAnna Christen Staton, 1998]:

The advective heat transfer in the air = heat transfer to the desiccant matrix material wheel

\[ \dot{m}_{a,j} L \frac{\partial H_a}{\partial x} + M_a \frac{\partial H_a}{\partial \theta} + \lambda_j + M_m \frac{\partial H_m}{\partial \theta} = 0 \]  

For the same procedures, applying the mass balance for the air and desiccant matrix material wheel, [Gunnar Stiesch, 1994]:

\[ \frac{\dot{m}_a}{\rho_a A} \frac{\partial w_m}{\partial \tau} = h_m (w_a - w_m) \Rightarrow \frac{\partial w_m}{\partial \tau} = \frac{h_m A}{\dot{m}_a / \rho_a} (w_a - w_m) \]  

\[ \frac{\partial H_m}{\partial \tau} = \frac{h A}{\dot{m}_a \rho_a} \frac{\partial H_a}{\partial T_a} (T_a - T_m) + \frac{h_m A}{\dot{m}_a / \rho_a} (w_a - w_m)H_m \]  

The final heat and mass transfer governing equations based on mass and energy conservation combined with the assumptions for the rotary regenerative heat exchanger as follow, [Gunnar Stiesch, 1994]:

1- Mass conservation:

\[ \frac{\partial w_a}{\partial z} + M_r \frac{\partial w_a}{\partial \tau} + \frac{\partial w_m}{\partial \tau} = 0 \]  

2- Energy conservation:

\[ \frac{\partial H_a}{\partial z} + M_r \frac{\partial H_a}{\partial \tau} + \frac{\partial H_m}{\partial \tau} = 0 \]  

3- Mass transfer rate:

\[ \frac{\partial w_m}{\partial \tau} = NTU_m (w_a - w_m) \]  

4- Thermal energy transfer rate:

\[ \frac{\partial H_m}{\partial \tau} = NTU_i \frac{\partial H_a}{\partial T_a} (T_a - T_m) + NTU_m (w_a - w_m)H_m \]  

The dimensionless parameters in Eqs. (10 -13) as following, [Gunnar Stiesch, 1994]:
\[
\tau = \frac{\theta}{\lambda_j \Gamma_j} \text{ for } 0 \leq \tau \leq \frac{1}{\Gamma_j}, \text{ where } \Gamma_j = \frac{M_m}{\lambda \dot{m}_{a,j}}, \text{ and } \tau = \frac{\theta}{\lambda_j \dot{m}_{a,j}} M_m
\]  

\[ z = \frac{x}{L}, \quad \text{NTU}_i = \frac{h_i A}{\dot{m}_a C_{pa}}, \quad \text{NTU}_m = \frac{h_m A}{\dot{m}_a / \rho_a}, \quad M_r = \frac{M_a}{M_m} \]  

The enthalpy of the moist air can be expressed as following, [Al-Judi, 1983]:

\[ H_a = H_{aa} + H_w = H_{aa} + w_a H_{sa} \]  

Where

\[ H_{aa} = \begin{cases} 
1.007 T_a - 0.026 & 0 < T_a < 60 \degree C \\ 
1.005 T_a & 0 > T_a > -10 \degree C 
\end{cases} \]  

\[ H_{sa} = 2501 + 1.84 T_a \]  

Then, Eq. (16) can be written as following:

\[ H_a = 1.007 T_a - 0.026 + w_a (2501 + 1.84 T_a) \]  

\[ T_a = \frac{H_a - 2501 w_a + 0.026}{1.007 + 1.84 w_a} \text{ and } w_a = \frac{H_a - 1.007 T_a + 0.026}{2501 + 1.84 T_a} \]  

The enthalpy of the matrix desiccant can be expressed as following, [JoAnna Christen Staton, 1998]:

\[ H_m = (C_{pm} + w_m \times 4.18 \text{ kJ/kg.K}) T_m \]  

The specific heat of desiccant (silica gel) known as a function of temperature, and the average value of the specific heat at the room temperature is [JoAnna Christen Staton, 1998].

\[ C_{pm} = 0.95645 \]  

\[ H_m = (0.95645 + 4180 w_m) T_m \]  

Due to the enthalpy and the temperature of the air are dependent on the longitudinal distance and the time, then:

\[ H_a = H_a(z, \tau) \text{ and } T_a = T_a(z, \tau) \]  

The chain rule is used to find the derivative of the air enthalpy with temperature, as following:

\[ \frac{\partial H_a}{\partial T_a} = \frac{\partial H_a}{\partial z} \frac{\partial z}{\partial T_a} + \frac{\partial H_a}{\partial \tau} \frac{\partial \tau}{\partial T_a} \]
FINITE DIFFERENCE TECHNIQUES:

Because of the analytical solution for the coupled partial equations with variable parameters is very complicated and difficult, the numerical solution is used. A finite difference technique is used to solve the Eqs. (10-13), as following, [Kreyszig, 1999]:

1- Mass conservation:
\[
\frac{w_{a}^{i,j} - w_{a}^{i+1,j}}{\Delta z} + M_r \frac{w_{a}^{i+1,j} - w_{a}^{i,j}}{\Delta \tau} + \frac{w_{m}^{i+1,j} - w_{m}^{i,j}}{\Delta \tau} = 0 \quad \text{(26)}
\]

2- Energy conservation:
\[
\frac{H_{a}^{i,j+1} - H_{a}^{i,j}}{\Delta z} + M_r \frac{H_{a}^{i+1,j+1} - H_{a}^{i,j+1}}{\Delta \tau} + \frac{H_{m}^{i+1,j} - H_{m}^{i,j}}{\Delta \tau} = 0 \quad \text{(27)}
\]

3- Mass transfer rate:
\[
\frac{w_{m}^{i+1,j} - w_{m}^{i,j}}{\Delta \tau} = NTU_m (w_{a}^{i,j} - w_{m}^{i,j}) \quad \text{(28)}
\]

4- Thermal energy transfer rate:
\[
\frac{H_{m}^{i+1,j} - H_{m}^{i,j}}{\Delta \tau} = NTU_m (T_{a}^{i,j} - T_{m}^{i,j}) \left( \frac{\partial H_{a}}{\partial z} \frac{\partial z}{\partial T_a} + \frac{\partial H_{a}}{\partial \tau} \frac{\partial \tau}{\partial T_a} \right) + NTU_m (w_{a}^{i,j} - w_{m}^{i,j}) H_{m}^{i,j} \quad \text{(29)}
\]

In order to solve the above equations, firstly determine the relative humidity for the air. Substitution the Eq. (28) in Eq. (26),
\[
\frac{w_{a}^{i,j} - w_{a}^{i+1,j}}{\Delta z} + M_r \frac{w_{a}^{i+1,j} - w_{a}^{i,j}}{\Delta \tau} + NTU_m (w_{a}^{i,j} - w_{m}^{i,j}) = 0 \quad \text{(30)}
\]

\[
w_{a}^{i+1,j} = \frac{\Delta \tau}{M_r} \left( NTU_m (w_{a}^{i,j} - w_{m}^{i,j}) + \frac{w_{a}^{i,j} - w_{a}^{i+1,j}}{\Delta z} \right) \quad \text{(31)}
\]

\[
w_{a}^{i,j+1} = \left( 1 - \frac{\Delta \tau}{M_r} NTU_m - \frac{\Delta \tau}{\Delta z M_r} \right) w_{a}^{i,j} + \frac{\Delta \tau}{M_r} NTU_m w_{m}^{i,j} + \frac{\Delta \tau}{\Delta z M_r} w_{a}^{i+1,j} \quad \text{(32)}
\]

Then from Eq.(28) determine the relative humidity of the desiccant matrix material.
\[
w_{m}^{i,j+1} = w_{m}^{i,j} + \Delta \tau NTU_m (w_{a}^{i,j} - w_{m}^{i,j}) \quad \text{(33)}
\]

\[
w_{m}^{i,j+1} = (1 - \Delta \tau NTU_m ) w_{m}^{i,j} + \Delta \tau NTU_m w_{a}^{i,j} \quad \text{(34)}
\]

For the enthalpy, to determine the air enthalpy, substitution Eq.(29) in Eq.(27),
\[
\left\{ \frac{H_{a}^{i,j+1} - H_{a}^{i,j}}{\Delta z} + M_r \frac{H_{a}^{i+1,j+1} - H_{a}^{i,j+1}}{\Delta \tau} + NTU_m (T_{a}^{i,j} - T_{m}^{i,j}) \left( \frac{H_{a}^{i+1,j} - H_{a}^{i,j}}{\Delta z} \frac{\Delta z}{T_{a}^{i+1,j} - T_{a}^{i,j}} + \frac{H_{a}^{i+1,j} - H_{a}^{i,j}}{\Delta \tau} \frac{T_{a}^{i+1,j} - T_{a}^{i,j}}{\Delta \tau} \right) \right\} = 0
\]

\[
\left\{ + NTU_m (w_{a}^{i,j} - w_{m}^{i,j}) H_{m}^{i,j} \right\} \quad \text{(35)}
\]
Where:

These equations are solved by using trail and error (firstly determined the $H_a$ from Eq. (40) at new time, then determined the $T_a$ from Eq. (41) at a new time and replay the procedures to get the approached values of $H_a$ and $T_a$ with good accuracy.

Finally, the desiccant matrix material enthalpy is determined as following:

$\frac{H_{m}^{i,j+1} - H_{m}^{i,j}}{\Delta t} = \text{NTU}_m \left( T_{a}^{i,j} - T_{m}^{i,j} \right) \left( \frac{H_{a}^{i,j+1} - H_{a}^{i,j}}{\Delta t} \frac{\Delta \tau}{T_{a}^{i,j+1} - T_{a}^{i,j}} + \frac{\Delta \tau}{T_{m}^{i,j} - T_{a}^{i,j}} \right) + \text{NTU}_m (w_{a}^{i,j} - w_{m}^{i,j}) H_{m}^{i,j}$ ..................................(43)

$\frac{H_{m}^{i,j} - H_{m}^{i,j-1}}{\Delta t} = \text{NTU}_m \left( T_{a}^{i,j} - T_{m}^{i,j} \right) \left( \frac{H_{a}^{i,j} - H_{a}^{i,j-1}}{\Delta \tau} \frac{\Delta \tau}{T_{a}^{i,j+1} - T_{a}^{i,j}} + \frac{\Delta \tau}{T_{m}^{i,j} - T_{a}^{i,j}} \right) + \text{NTU}_m (w_{a}^{i,j} - w_{m}^{i,j}) H_{m}^{i,j}$ ..................................(44)

$+ \text{NTU}_m (w_{a}^{i,j} - w_{m}^{i,j}) H_{m}^{i,j}$
\[ H_{m}^{i,j+1} = (1 + \Delta \tau NTU_m (w_{a}^{j,i} - w_{m}^{j,i}))H_{m}^{i,j} + \Delta \tau NTU_m (T_{a}^{i,j} - T_{m}^{i,j}) \left( \frac{H_{a}^{i,j} - H_{m}^{i,j}}{T_{a}^{i,j} - T_{m}^{i,j}} + \frac{H_{m}^{i,j+1} - H_{a}^{i,j}}{T_{m}^{i,j+1} - T_{a}^{i,j}} \right) \] .............(45)

The time interval that used in the present analysis to avoid the negative fluctuation term in the above equations is derived from the Eq. (32) and Eq. (34) as following:

\[ 1 - \frac{\Delta \tau}{M_r} - \frac{\Delta \tau}{\Delta z M_r} \geq 0 \quad \Rightarrow \quad \Delta \tau \leq \frac{M_r \Delta z}{NTU_m \Delta z + 1} \] ......................................................(46)

or

\[ 1 - \Delta \tau NTU_m \geq 0 \quad \Rightarrow \quad \Delta \tau \leq \frac{1}{NTU_m} \] ......................................................(47)

The lower value of the (\( \Delta \tau \)) from the Eq. (46) or Eq. (47) will be selected in the computer program. A Quick-Basic computer program is written to calculate the values of the enthalpy, temperature and moisture content for the air and the matrix.

**BOUNDARY AND INITIAL CONDITIONS**

The conditions in the following analysis are taken as the summer conditions because this condition is very common through the year for Arab countries, [JoAnna Christen Staton, 1998]:

**Table (1), Specifications of the Case Study**

<table>
<thead>
<tr>
<th>Standard</th>
<th>Supply</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>dry bulb temperature (°C)</td>
<td>35</td>
<td>21</td>
</tr>
<tr>
<td>wet bulb temperature (°C)</td>
<td>26</td>
<td>16</td>
</tr>
<tr>
<td>relative humidity (%)</td>
<td>47</td>
<td>60</td>
</tr>
<tr>
<td>humidity ratio (kg H₂O / kg Air)</td>
<td>0.017</td>
<td>0.009</td>
</tr>
<tr>
<td>mass flow rate (kg/s)</td>
<td>0.22</td>
<td>0.4</td>
</tr>
<tr>
<td>Silica gel density = 1200 kg/m³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminum foil density = 2700 kg/m³ and the specific heat (( C_{p,a} ))= 903 J/kg·K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotary heat exchange wheel length = 0.1 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotary heat exchange wheel diameter = 1 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The other needed air properties is determined from the psychometric chart</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The boundary and initial conditions that used during the solution process can be described as following. Two sets of boundary conditions were required to allow for different, independent air conditions of the supply and exhaust air streams, different supply and exhaust boundary conditions must be imposed. For most realistic applications the exiting supply and inlet exhaust conditions would be dependent in some manner.

When the passage of supply air over the surface of the desiccant channel is modeled, the solution satisfies the supply air boundary conditions:

\[ T_a(0, \tau) = T_{ai} , \quad \text{and} \quad w_a(0, \tau) = w_{ai} \] ......................................................(48)
When the passage of exhaust air over the surface of the desiccant channel is modeled, the solution satisfies the exhaust air boundary conditions:

\[ T_a (L, \tau) = T_{ac}, \quad \text{and} \quad w_a (L, \tau) = w_{ac} \] .........................................................(49)

The initial conditions applied to the governing equations, regardless of which air flow is being analyzed first, are:

\[ T_a (z,0) = T_{ai}, \quad \text{and} \quad w_a (z,0) = w_{ai} \] .........................................................(50)

The area of the hot and cold air flow are taken as following (hot air passage area =cold air passage area = (165°/360°) of the total area of the wheel, and the purge passage area = (30°/360°) of the total area of the wheel).

Then after applying the boundary and initial conditions, we calculate the humidity and enthalpy of the air and the matrix are calculated from the Eq.(32), Eq. (34), Eq. (40) and Eq. (45) , respectively.

**HEAT TRANSFER COEFFICIENT:**

Yılmaz and Cihan [1993] developed a method to calculate the convective heat transfer coefficient for flow in the channels of various cross-sections for a constant wall temperature boundary condition. The Nusselt number can be calculated by using the following equation:

\[ Nu_x = Nu_\infty \left[ 1 + \frac{4.212 \psi \Phi^3}{\gamma Nu_\infty^3} - 0.8 \left( \frac{\psi \Phi^3}{\gamma Nu_\infty^3} \right)^{2/3} \right]^{1/3} \] .........................................................(51)

In which \(Nu_\infty\) is the Nusselt number for the limiting case of \(Z \to \infty\) and can be obtained as follows:

\[ Nu_\infty = 3.657 \Phi \] .........................................................(52)

Where \(\Phi\) is a heat transfer factor taking the shape of the cross-section of the channel into account for developed velocity and temperature profiles at a constant wall temperature. It is given by [Yılmaz and Cihan, 1993]:

\[ \Phi = 1 + \frac{\Phi_{\infty} - 1}{1 + 1/(n-1)} + \Delta\Phi \] .........................................................(53)

The parameters \(\Phi_{\infty}\) and \(\Delta\Phi\) can be calculated by using the following equations:

\[ \Phi_{\infty} = 0.5155 \frac{d^*^2}{3 - d^*} \] .........................................................(54)

\[ \Delta\Phi = \Delta\Phi_{\text{max}} + \frac{0.95(n-1)^{0.5}}{1 + 0.038(n-1)^{1/3}} \] .........................................................(55)

Where

\[ \Delta\Phi_{\text{max}} = \frac{0.007d^*^8}{(1 + 10d^*^{-28})(1 + 64*10^{-8}d^*^{-28})^{1/2}} \] .........................................................(56)
The parameters \((d^*)\) and \((n)\) used in the equations above describe the shape of the channel [Yılmaz and Cihan, 1994]:

\[
d^* = \frac{d_{eq}}{d_{max}} \quad \text{and} \quad n = \frac{F}{F_{eq}} = \frac{P}{P_{eq}}
\]

Where \((F_{eq})\) and \((P_{eq})\) are, respectively, the cross-sectional area and periphery of the equivalent diameter of the flow channel. The corresponding values for \((d^*)\) and \((n)\) are given in Fig. (3). Equivalent diameter \((d_{eq})\) is defined as [Yılmaz and Cihan, 1994]:

\[
d_{eq} = \frac{4F}{P}
\]

Where \((d_{max})\) is the diameter of the largest circle that can be inscribed in the cross-section of the flow channel. The shape factor \((\psi)\), which is required for the calculation of Nusselt number, is given as [Yılmaz and Cihan, 1994]:

\[
\psi = 1 + \frac{\psi_x - 1}{1 + 0.33d^{*2.25}/(n - 1)}
\]

Where

\[
\psi_x = \frac{3}{8}d^*(3 - d^*)
\]

The other parameters that used in calculation of the Nusselt number as following [Yılmaz and Cihan, 1994]:

\[
\Phi = 1 + \frac{[3(d^*/2)^{7/8}/(1 + d^*)] - 1}{1 + 0.25/(n - 1)} \quad \text{and} \quad \gamma = \frac{L}{d_{eq} \cdot Re \cdot Pr}
\]

Where

\[
Re = \frac{ud_{eq}}{v}
\]

\[
Nu_a = \frac{h_a \cdot d_{eq}}{k_a}
\]

Fig. (3) , The Parameters \(d^*\) and \(n\) for Different Channel Shape [Yılmaz and Ukalaca, 2003]
MASS TRANSFER COEFFICIENT:

The common equation that used to calculate the mass transfer coefficient is, [Incoropera and DeWitt, 2002]:

\[
\text{Sh} = 0.023 \text{Re}^{0.83} \text{Sc}^{0.44}
\]

Where \( \text{Sh} \) is called Sherwood number (\( \text{Sh} = \frac{h_m d_{eq}}{D} \)), \( \text{Sc} \) is called Schmidt number (\( \text{Sc} = \frac{\nu}{\rho D} \)) and \( D \) is diffusion coefficient (\( \text{cm}^2/\text{s} \))

ROTARY REGENERATIVE HEAT EXCHANGER PERFORMANCE:

The performance of the rotary regenerative heat exchanger is calculated by three types of the effectiveness: the sensible (\( \varepsilon_s \)), latent (\( \varepsilon_L \)) and total (\( \varepsilon_t \)) effectiveness.

Energy from rotary heat exchanger is transferred both sensible and latent energy between the air streams due to the desiccant attached to the aluminium substrate of the wheel matrix. Sensible energy transfer is caused by the temperature differences between the two airstreams and latent energy transfer is caused by the water vapour pressure differences between the two airstreams. The total energy transfer rate in energy wheels is the sum of both sensible and latent energy transfer rates. The effectiveness is described as following:

\[
\varepsilon = \frac{\dot{m}_1 (\zeta_{\text{in}} - \zeta_{\text{out}})}{\dot{m}_{\text{min}} (\zeta_{\text{s,in}} - \zeta_{\text{s,exit}})}
\]

Where

\( \zeta = \) is equal either the dry bulb temperature, the humidity ratio, or the enthalpy of the moisture-carrying air stream. When temperature is used, a sensible effectiveness is obtained, reflecting the wheel’s ability to transfer heat. When humidity ratio is used, a latent effectiveness which reflects the wheel’s moisture transfer performance is obtained. When enthalpy is used, a total effectiveness is obtained, reflecting the wheel’s ability to transfer both heat and moisture.

\( \dot{m}_{\text{min}} = \) is the minimum value of either \( \dot{m}_s \) or \( \dot{m}_{\text{exit}} \)

Also, in the present analysis a dimensionless parameter used with the enthalpy heat exchanger is called Lewis number (\( \text{Le} \)), [Schultz, 1987], it is defined as the ratio of the number of transfer units for heat transfer to the number of transfer units for mass (humidity) transfer, as follows:

\[
\text{Le} = \frac{\text{NTU}_T}{\text{NTU}_w}
\]

In the rotary regenerative heat exchanger, Lewis number is very affected factor. The Lewis number depends on the type of desiccant used and the thickness of the desiccant coating. When the Lewis number greater than one, this means decreasing in the humidity transfer effectiveness compared with the thermal effectiveness for a fixed conditions of inlet states.

RESULTS AND DISCUSSION:

Before discussion the results, the accuracy of the Q. Basic computer program is checked with the other results [Gunnar Stiesch, 1994], for the data (\( \text{Le} =1, T_{s,\text{in}}=25^\circ \text{C}, T_{e,\text{in}}=35^\circ \text{C} \), triangular matrix
duct, matrix length =0.15 m, matrix diameter =2 m, Silica Gel). The comparison is plotted in Fig. (4),
the results of the current program for the sensible heat exchanger effectiveness are very acceptable with
the results of the Gunnar Stiesch, 1994.

The variation of the temperature and enthalpy of the inlet air against the dimensionless time is
plotted in Fig. (5). The increase of the dimensionless time will increase the temperature and enthalpy
due to increase the time for the energy transfer from the hot to cold fluid (air).

The effect of the dimensionless time on the rotary regenerative heat exchanger effectiveness for
(NTUt=4 and Le =2) is plotted in Fig. (6). The increase of the time will increase the sensible, latent and
total effectiveness. The sensible effectiveness is increased as the time increased due to transfer of the
sensible heat (temperature) from the hot fluid to cold fluid by the desiccant material. The latent
effectiveness is increased due to transfer of the relative humidity from the exhaust humid air to the inlet
fresh air and then reaches to steady state faster than the sensible heat effectiveness due to the mass
transfer by relative humidity which need less time compared with heat transfer by temperature to reach
the steady state value. The total effectiveness is also increases because it is the summation effect of the
sensible and latent effectiveness. The steady state time for the sensible and total effectiveness is
approximately identical and it’s greater than the steady state time for the latent effectiveness.

In Fig. (7), the variation of the sensible heat effectiveness is plotted against the dimensionless time
constant for different values of the Number of Transfer Unit (NTUt). The increase of the time will
increase the sensible effectiveness. The increase of the NTU value will increase the sensible
effectiveness due to increase of the quantity of the sensible energy that transferred from the hot fluid to
the cold fluid. The difference in the steady state sensible effectiveness value for larger values of the
NTUt is become not noticeable due to the system nearly reach to the steady state faster (higher
effectiveness) than when the NTUt value is small (lower effectiveness). The increasing of the NTUt
will increase the steady state value due to increase of the heat transfer between the hot fluid and cold
fluid. The same effect is appearing for the total effectiveness with dimensionless time for different
values of the NTUt (as shown Fig. (8)). Due to the steady state, the total effectiveness is directly
proportional to the steady state sensible effectiveness.

The effect of dimensionless time on the sensible effectiveness for different values of the inlet air
temperature is plotted in Fig. (9). The increasing of the inlet air temperature will increase the sensible
heat effectiveness value due to reduce of the maximum heat transfer and in same time increase the
energy of the inlet air. Also, the effect of the number of transfer unit on the sensible effectiveness for
different values of the inlet air temperature is plotted in Fig. (10). The increasing of the inlet
temperature and the number of the transfer unit increases the sensible effectiveness for the same
reasons mentioned before.

The effect of the heat exchanger length (matrix wheel length) against the steady state sensible,
latent and total effectiveness is plotted in the Fig. (11) for (NTUt=7 and Le =2). The increase of the
heat exchanger length will increase the steady state sensible, latent and total effectiveness due to
increase the exchange time for the energy and mass transfer between the fluids. The effect of the heat
exchanger length on the steady state value of the latent effectiveness is not noticeable compared with
the steady state value of the sensible and total effectiveness because the latent effectiveness needs less
time to reach the steady state value compared with sensible and total effectiveness.

The effect of the inlet air mass flow rate on the steady state sensible, latent and total effectiveness
is plotted in Fig.(12). The increase of the inlet air mass flow rate decreased the steady state sensible,
latent and total effectiveness due to decrease the number of transfer unit (NTU) for heat and mass
transfer. The increase of the mass flow rate will increase the inlet air storage energy and then the inlet
air needs more thermal and mass energy in order to rise its temperature and its relative humidity. The
effectiveness decreased firstly and this decreasing becomes sharply at high values of the inlet air mass
flow rate the reason is the maximum heat transfer and maximum mass transfer become large compared
with the actual mass and energy transferring.
The effect of the desiccants material mass on the sensible, latent and total effectiveness is plotted in Fig.(13). The increasing of the desiccants material mass (volume) will increase the energy storage inside the desiccant material (internal energy) and then increases the transferred energy from the hot fluid to the cold fluid through the desiccant material.

CONCLUSIONS:

The heat and mass transfer from a rotary regenerative heat exchanger is solved numerically by using finite difference techniques. The following conclusions that drawn from the results of the present work:

1- The increasing of the time will increase the sensible, latent and total effectiveness.
2- The steady state point for the latent effectiveness is less than the sensible and total effectiveness.
3- The increasing of the NTU will increase the effectiveness and reduce the time for reaching the steady state value.
4- The increasing of heat exchanger length will increase the sensible and total effectiveness sharply compared with latent heat effectiveness (the increase of the heat exchanger length not affected largely on the latent effectiveness).
5- The increasing of the inlet air flow rate will decrease the sensible, latent and total effectiveness.
6- The increase of the desiccant material mass (thickness) will increase the total, sensible and latent effectiveness.
7- Neglecting the latent heat transfer is not acceptable because the difference between the sensible and the latent effectiveness is noticeable.
8- The using of traditional NTU-Effectiveness method and the Log Mean Temperature Difference method for this system without any corrections may be lead to mistake in the calculations.

Fig.(4), Comparison between the present study and Gunnar Stiesch, 1994, (Le =1, T_{s,in}=25°C, T_{c,in}=35°C, triangular matrix duct, matrix length =0.15 m, matrix diameter =2 m, Silica Gel)

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Inlet Air Conditions ( Either Temperature (Ta) (°C) or Enthalpy (Ha)(kJ/kg.K))

<table>
<thead>
<tr>
<th>Inlet Air Temperature (°C)</th>
<th>Inlet Air Enthalpy (kJ/kg.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

The Dimensionless Time (τ)

The Heat Exchanger Effectiveness (ε)

**Fig.(5)**, The variation of the temperature and enthalpy of the inlet air against the dimensionless time

**Fig.(6)**, The total, sensible and latent effectiveness for the heat exchanger against the dimensionless time for (NTU = 4 and Le = 2)
The Dimensionless Time Constant ($\Gamma$)

The Sensible Effectiveness ($\varepsilon_s$)

Sensible Effectiveness

NTU$_t$ = 1
NTU$_t$ = 2
NTU$_t$ = 3
NTU$_t$ = 4
NTU$_t$ = 5
NTU$_t$ = 6
NTU$_t$ = 7
NTU$_t$ = 8
NTU$_t$ = 9
NTU$_t$ = 10

Fig.(7), The effect of the dimensionless time constant ($\Gamma$) on the sensible effectiveness for different values of the NTU$_t$

Fig.(8), The effect of the dimensionless time constant ($\Gamma$) on the total effectiveness for different values of the NTU$_t$
Fig. (9), The effect of dimensionless time ($\tau$) on the sensible effectiveness for different values of the inlet air temperature

Fig. (10), The effect of the number of transfer unit on the sensible effectiveness for different values of the inlet air temperature
Fig. (11), The effect of the heat exchanger length on the total, sensible and latent effectiveness (NTU \(_t\) = 7 and Le = 2)

Fig. (12), The effect of the inlet air mass flow rate on the sensible, latent and total effectiveness
Fig.(13), The effect of the desiccants material mass on the sensible, latent and total effectiveness

REFERENCES


