Euler line in graph theory

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Abstract:

In this paper an application of Euler line in hilla bridge and proof theorem in printor's problem / De Bruijn cycle are performance and the situation were r=3 and all possible of subsequence its means all Euler line in directed graph are taken.

1-Introduction:

The basic idea of graphs were introduced in 18^{th} centry by the great swiss mathematician . He used graphs to solved the famous königsberg bridge problem.

German city of königsberg(now it is Russian Kaliningrad) Was situated on the river pregel.it had a park situated on the banks of the river and two islands .mainland and islands were joined by seven bridges.A problem was whether it was possible to take a walk through the town in such a way as to cross over every bridge once, and only once.

A graph is a set of points (we call them vertices or nodes) connected by lines (edges or arcs).For more details see [1],[2] and [6].

2-:<u>Definition:[1][2][3]</u>

1-A graph G=(V, E) consist of two sets ,set V of vertices and E of edges such that each e Є G can be identified with a pair (u,v) of vertices in V. the vertices u and v are known as end points of E.

2-Awalk (w) in G is an alternative sequence of vertices and edges

Repeatation of vertices and edges are allowed awalk is said to be an open walk if initial and end vertices are different otherwise w is closed.

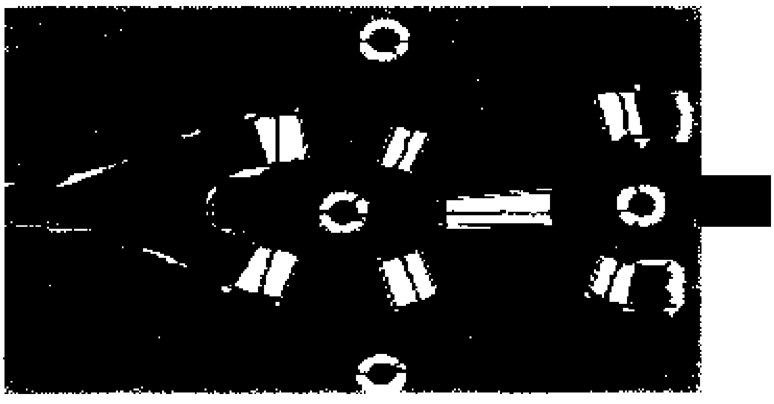
3-Atrail in G is an open walk without repeatation of edges .

4-A path in G is an open walk without repeatation of vertices .

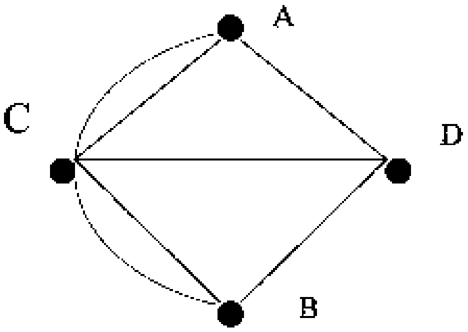
5-A graph G is said to be an Euler graph if there is aclosed trail which covers all the edge of G.
6-The degree of avertex is the number of edges incident on it . agraph is regular if all of its vertices have the same degree . i .e if $d(v) = k$, $\forall v \in V$ then G is a k – regular.
8-We say that G is connected if there is apath between each pair of vertices in G,if G is not connected it is disconnected graph .

3:<u>Königsberg bridge problem:[2][4][6]</u>

The Königsberg bridge problem is perhaps the best-know example in graph theory it was along-standing problem until solved by leonhard Euler (1707-1783) in 1736,by means of agraph Euler wrote the first paper ever in graph theory and thus became the originator of the theory of graph ,as well as of the rest of topology.the problem depicted in fig (3-1).
Two islands C and D.formed by the pregel river in Königsberg (then the capital of east Prussia but now renamed Kaliningrad and in west soviet Russia),were connected to each other and to the banks A and B with seven bridges ,as shown in fig(2-10).
The problem was to start at any of the four land areas of the city A,B,C or D, walk over each of the seven bridges exactly once. And return to the starting point (with out swimming across the river).
Euler represented this situation by means of a graph, as shown in fig(3-2). The vertices represent the land areas and the edges represent the bridges.
Euler proved that a solution for this problem does not exist .



(**3-1**)-_Königsberg bridge problem



(3-2)-graph of Königsberg bridge problem

4:Euler graphs:[2][4]

graph theory was born in 1736 with Euler famous paper in which he solved the Königsberg problem. In the same paper . Euler posed (and then solved) amore general problem .in what type of graph G is it possible to find a closed walk running through every edge of G exactly once? Such a walk is now called an Euler is called an Euler graph more formally:

If some closed walk in a graph contains all the of the graph ,then the walk is called an Euler line and the graph an Euler graph. By its very definition a walk is always connected since the Euler line (which is walk) contains all the edges of the graph an Euler graph is always connected, except for any isolated vertices the graph may have. since isolated vertices do not contribute any thing to the under standing of an Euler graphs do not have any isolated vertices and are there for connected.

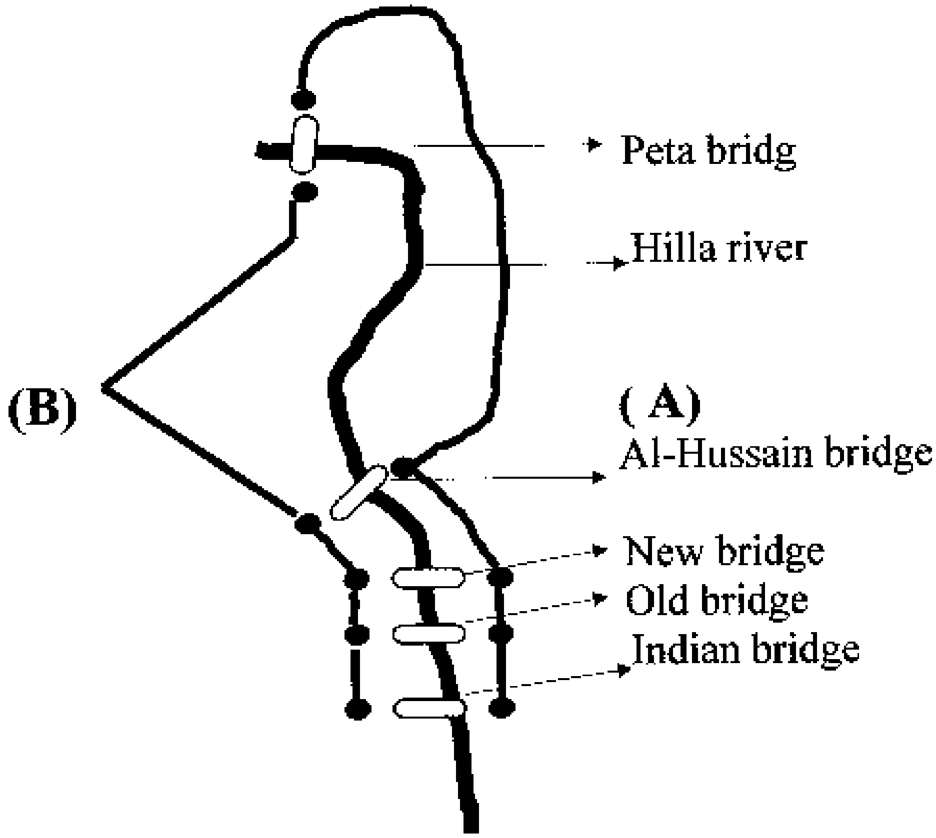
5:Theorem:[2][3]

A given connected graph G is an Euler graph iff vertices of G are of even degree .

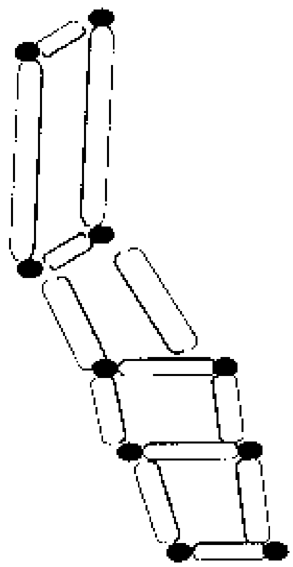
6:Hilla bridge:[5]

In this research we can explain the Hilla bridges are Euler graph. Now looking at the fig(6-1) and the fig(6-2).

The two areas of the city **A** and **B** were connected to each other and walk over each of the five bridges exactly twice and return to the starting point with out swimming across the river. Now we can see that there is a closed trail which covers all the edges of G and we can easy to see that the graph is connected and the vertices of the graph Hilla bridges are of even degree ,so by theorem or by definition it is Euler graph or it is have Euler line.



(6-1)-Hilla bridge



(**6-2**)-graph of hilla bridges

7:Theorem : - [1][2][3]

How long a cyclic sequence of digits o & 1 such that no subsequence (segment) of r bits of o & 1 will appear more than once in a cyclic sequence ?

Proof:

Tele printor's problem /De Bruijn cycle

We Know that there was 2^r possible subseq^n of r- bits so it is clear that the length of such a cyclic sequence Can not be $>2^r$ we want to construct a cyclic sequence of digit o & 1 of length 2^r such that no subseq $^{\underline{n}}$ (segment) of r bits of o&1 will repeat in the cycle .

We shall construct on Eular digraph as follows:

Let us consider a graph G=(V,E) such that all the (r-1) tuples are vertices of G . It is clear that , in G there are 2^{r-1} vertices .

$P= \alpha 1\ \alpha 2 \ldots\ldots \alpha r\text{-}1$ with $\alpha i = o$ or $1\ \forall i=1\ldots(r-1)$

Now, draw two edges from p to two vertices say q1&q2 & we label these edges as p0 & p1

i.e $\alpha 1 \alpha 2 \ldots\ldots\ldots \alpha r\text{-}1\ \ 0$ & $\alpha 1 \alpha 2 \ldots\ldots\ldots\ldots \alpha r\text{-}1\ \ 1$

let us take $q1=(\alpha 1 \alpha 2 \ldots\ldots\ldots \alpha r\text{-}1\ \ 0)$

and $q2=(\alpha 1 \alpha 2 \ldots\ldots\ldots \alpha r\text{-}1\ \ 1)$

Finally draw such two edges from each vertex in G Thus , $d^{+}(V)=2\ \forall\ v\in V$, Note that if p =(oo.....o) or (11.....1) then we get self – loop at p .

Similarly , for each $v \in V$, we get $d^{-}(V)=2$ thus , $d^{+}(v)$ for each $v\in V$, Also ,

$$\sum_{i=1}^{2^{r-1}} d^{+}(Vi)=e \Longrightarrow \sum_{i=1}^{2^{r-1}} 2=e \Longrightarrow e=2^r$$

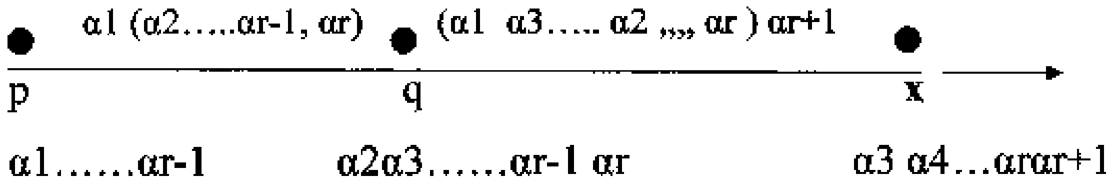
It is clear that the digraph G will be an Euler digraph .

Let T denote an Euler line in G & it contains

All 2^r edges .

Now , consider a seq^n by taking first digit from each edge comming consequtively in T then it will be the require cyclic seq^n of length 2^r .

Ex:

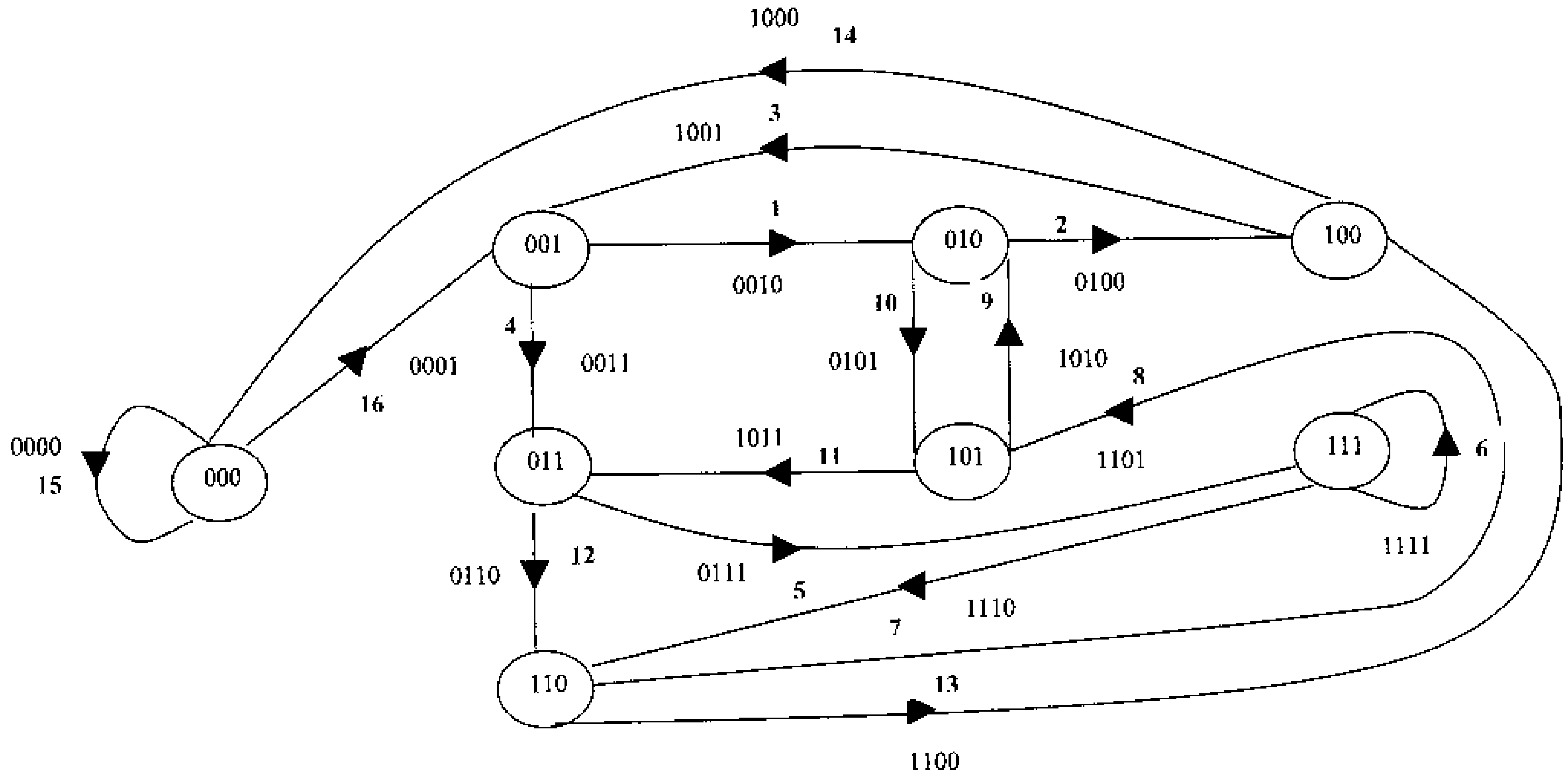


note that if p&q & x are as in the figure then the (r-1) tuple of the trail of first edges will be same as (r-1) tuple of head of the next edge in T. further note that any two edges have different labeling.

So,in acyclic seq^n any subseq^n of r-bits will not repeat.

Now for r=3 bit we can see that there is 16 Eular line
Such that it is not repeat.

1)-{15-16-4-5-6-7-8-9-2-3-1-10-11-12-13-14}.
2)-{15-16-4-12-8-9-2-3-1-10-11-5-6-7-13-14}.
3)-{15-16-4-12-13-3-1-10-11-5-6-7-8-9-2-14}.
4)-{15-16-4-12-8-11-5-6-7-13-3-1-10-9-2-14}.
5)-{15-16-4-5-6-7-8-9-10-11-12-13-3-1-2-14}.
6)-{15-16-4-5-6-7-8-11-12-13-3-1-10-9-2-14}.
7)-{15-16-4-5-6-7-13-3-1-10-11-12-8-9-2-14}.
8)-{15-16-4-12-8-9-10-11-5-6-7-13-3-1-2-14}.
9)-{15-16-1-10-11-5-6-7-8-9-2-3-4-12-13-14}.
10)-{15-16-1-2-3-4-12-8-9-10-11-5-6-7-13-14}.
11)-{15-16-1-10-11-5-6-7-8-9-2-3-4-12-13-14}.
13)-{15-16-1-10-9-2-3-4-5-6-7-8-11-12-13-14}.
14)-{15-16-1-2-3-4-5-6-7-8-9-10-11-12-13-14}.
15)-{15-16-1-10-11-12-8-9-2-3-4-5-6-7-13-14}.
16)-{15-16-1-10-11-12-13-3-4-5-6-7-8-9-2-14}.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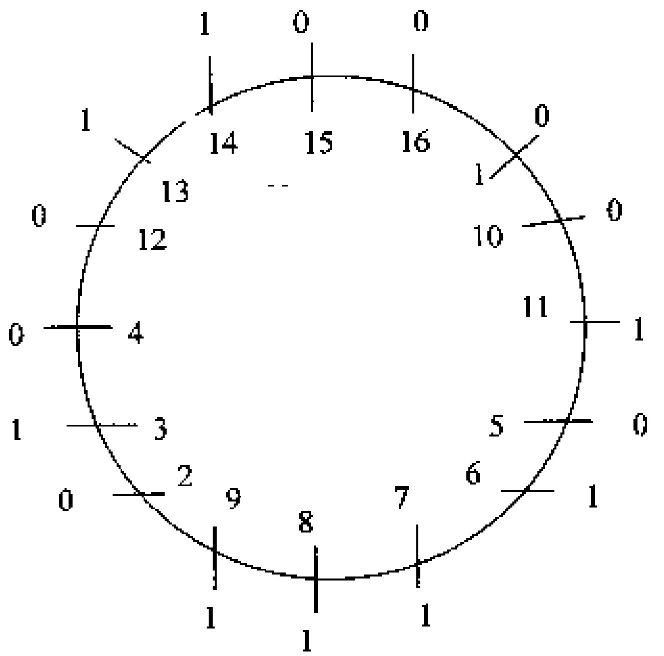
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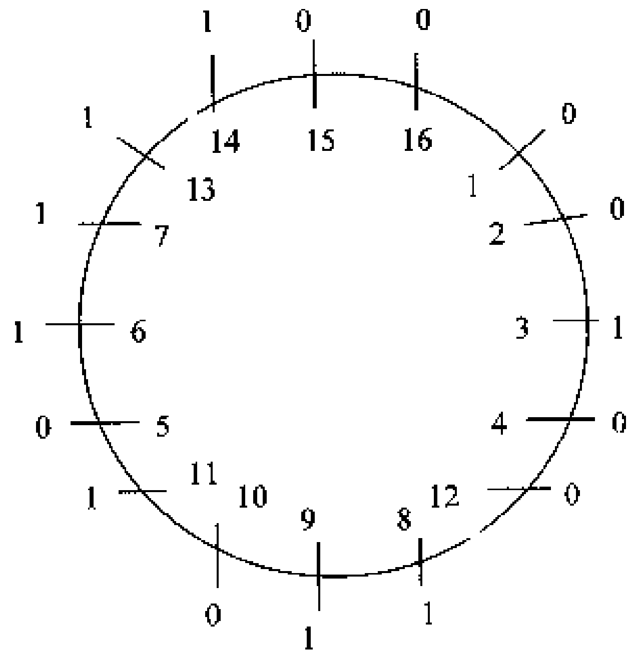
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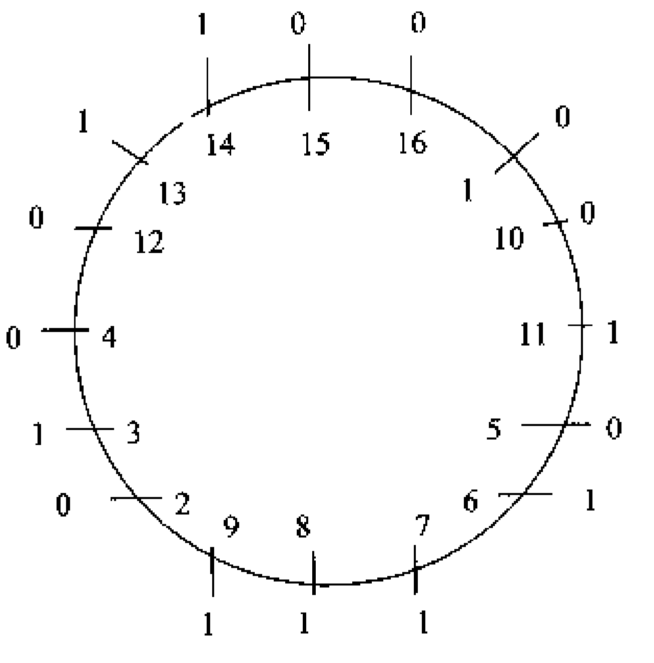
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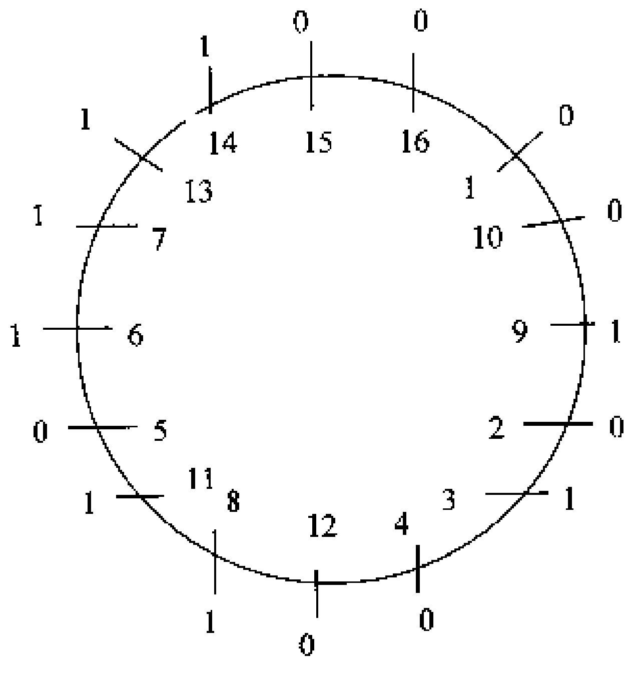
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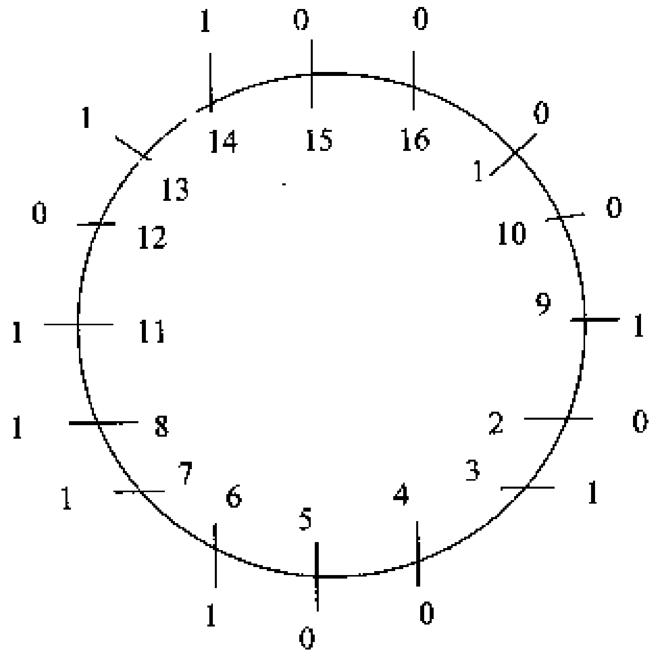
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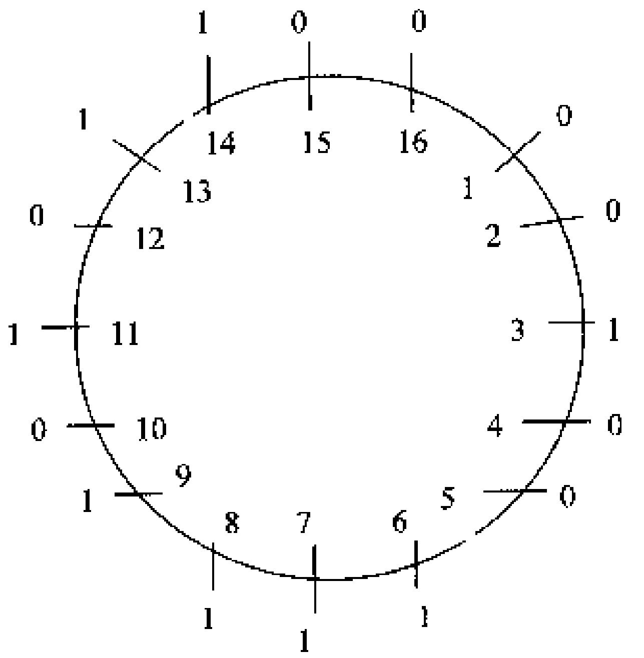
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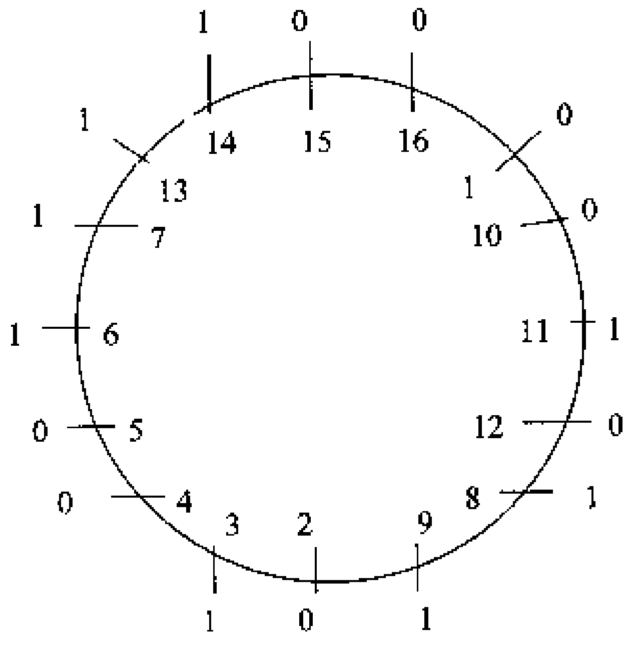
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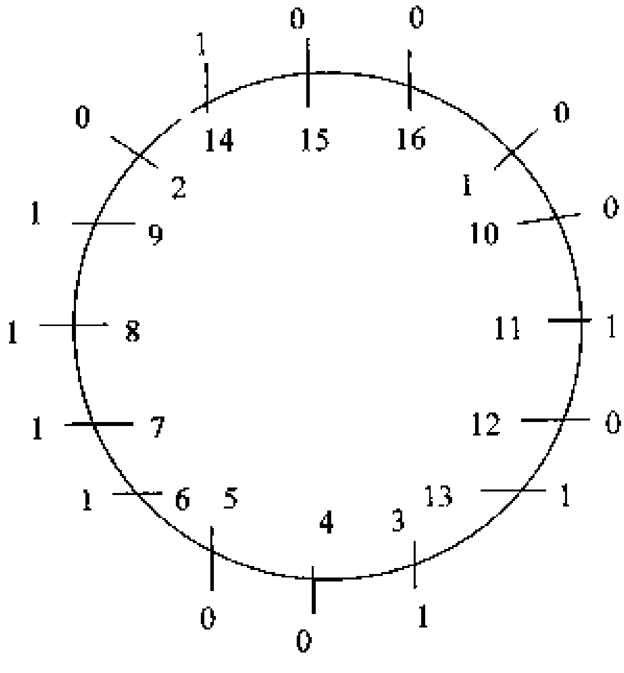
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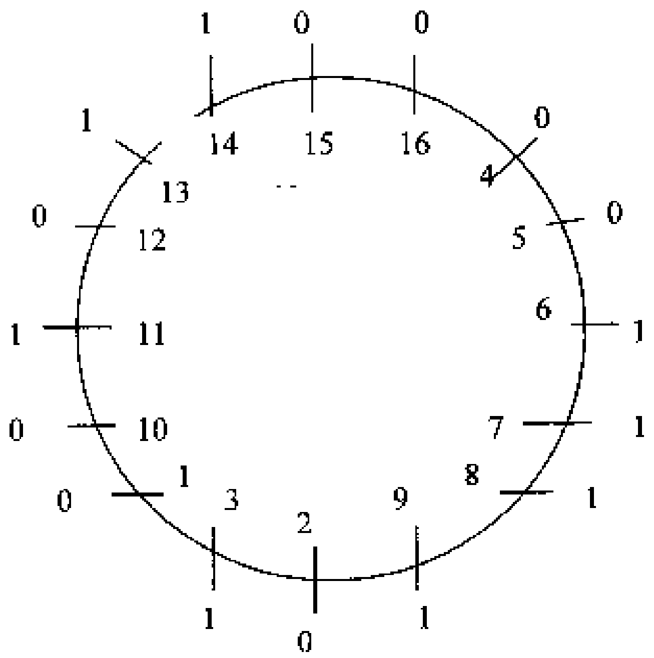
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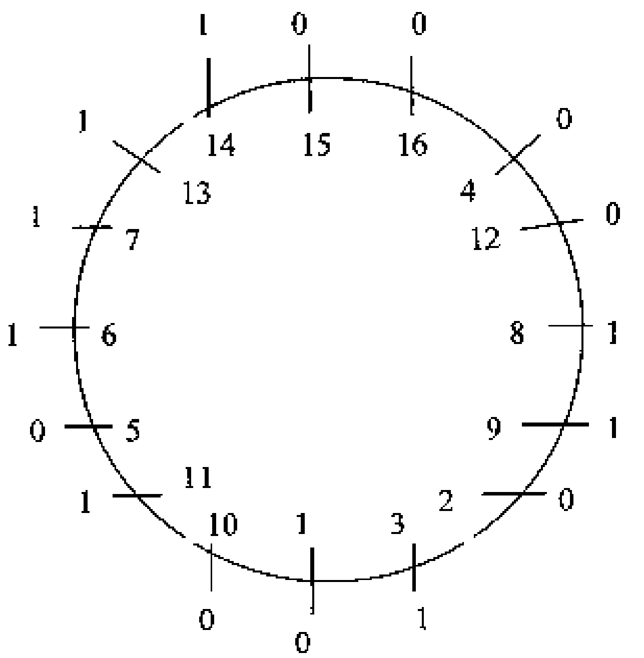
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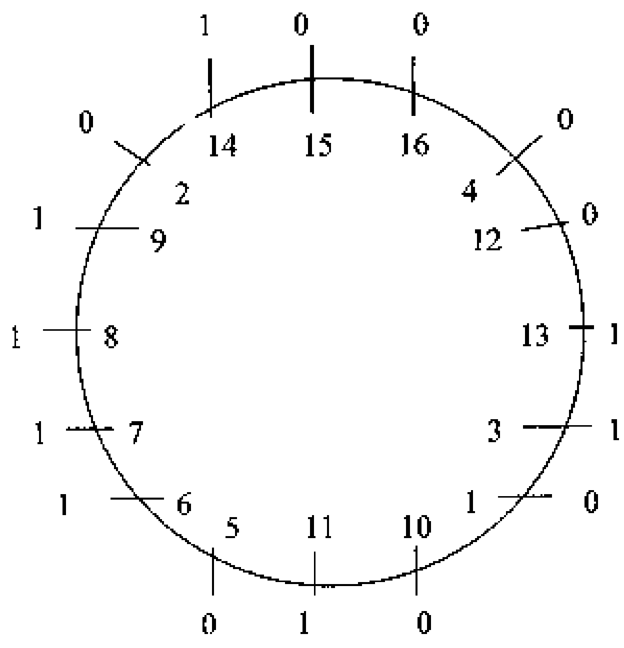
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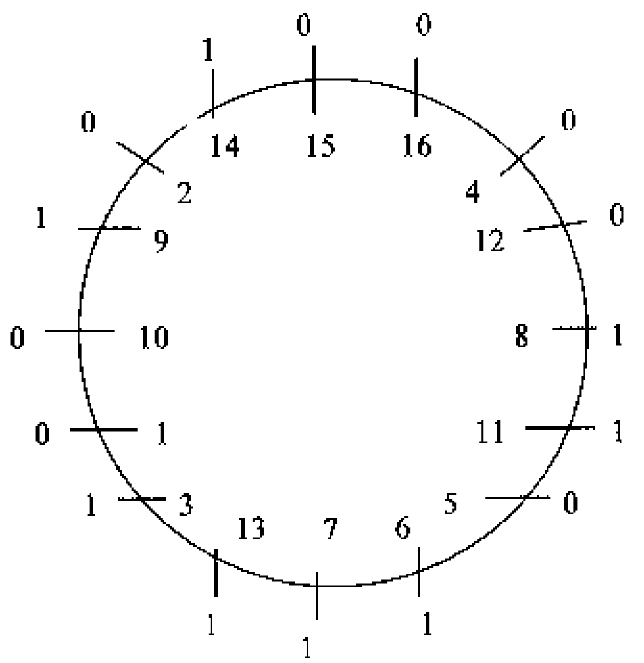
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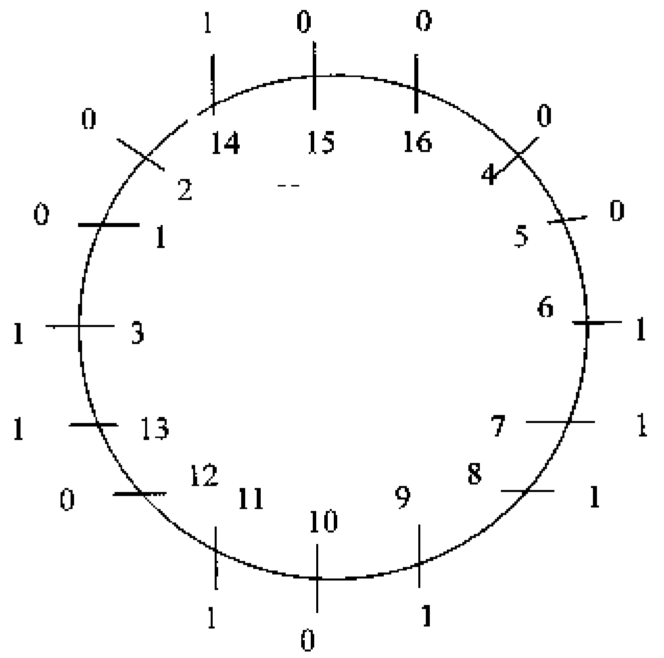
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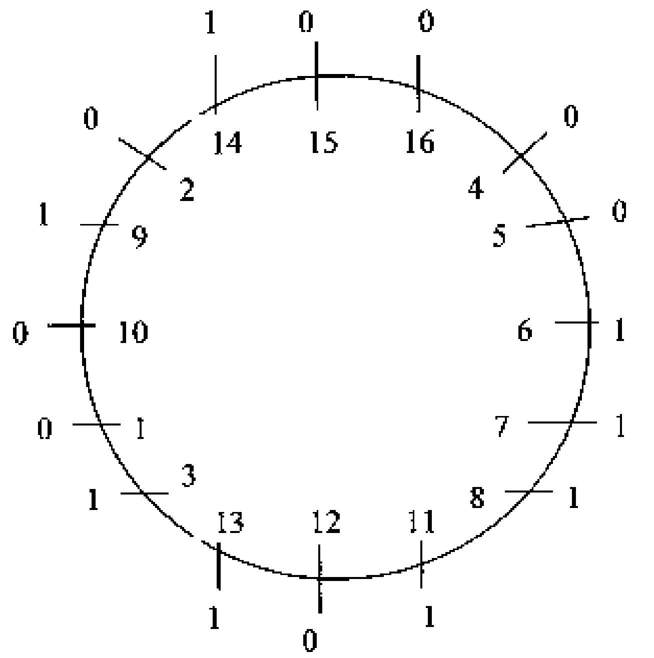
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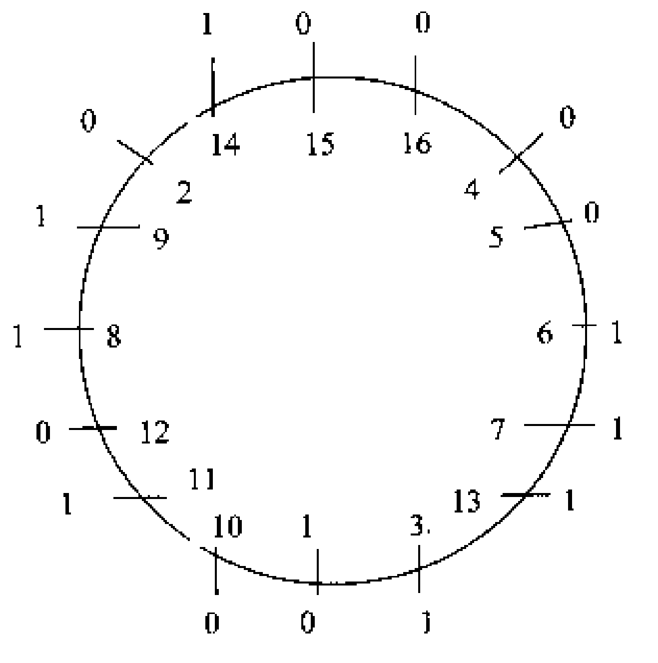
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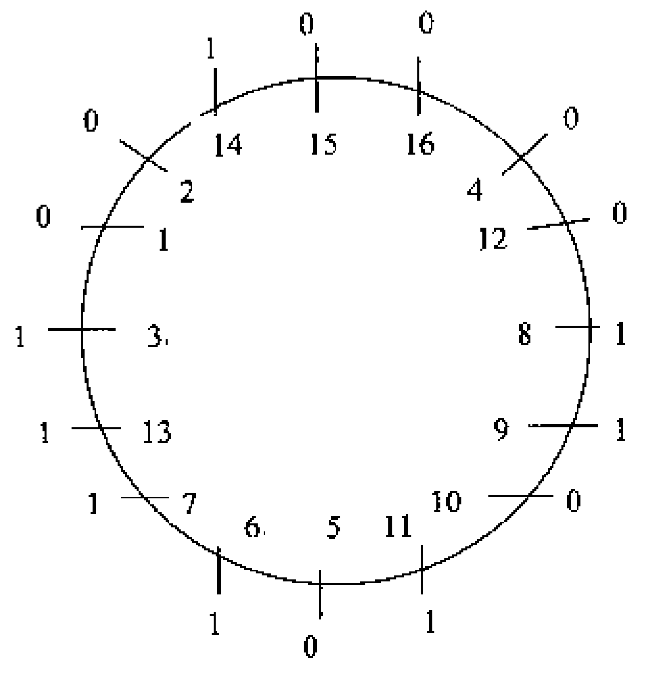
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8:Conclusion:

1-Now looking at the graph of the Königsberg bridges(fig.3-2), we find that not all its vertices are of even degree. Hence it is not an Euler graph. Thus it is not possible to walk over each of the seven bridges exactly once and return to the starting point. One often encounters Euler lines in various puzzles. The problem common to these puzzles is to find how a given picture can be drawn in one continuous line without retracing and without lifting the pencil from the paper.

2- and we can see that the graph Hilla bridges its Euler graph and its have Euler line.

3-and if we take r=4 in Tele printor's problem /De Bruijn cycle then there is 32 Euler line and if r=5 there is 64 Euler line so on.

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