Evaluation the Reliability and MTTF for Non – Maintained Standby Systems

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Abstract

In this paper the developing of the Reliability function *R(t)* and the expected time to system failure is presented for a non – maintained standby systems with considering a non – Markovain processes and indicate some cases where they can be reduced to simple Markov processes.

المستخلص

في هذا البحث تم دراسة وإيجاد دالة الوثوقية (المعوّلية) للأنظمة **البديلة** التي لا تخضع للصيانة بالإضافة إلى الوقت المتوقع لفشل النظام ويتم ذلك من خلال الاستفادة من بعض المفاهيم الأساسية لعمليات ماركوف.

1. Introduction

Many researchers works to evaluate the reliability for non - maintained (series and parallel) systems like Sandler(1963) , Bulgrasummy (1977) , Govil (1983) and Srinath (1985).

In this paper an application of the theory of Markov processes to derive the reliability of non – maintained standby systems is discussed. To describe the reliability of a given system it is necessary to specify the equipment failure process , the system diagram which describes how the equipments are connected and the rules of operation , finally the state in which the system is to be defined as failed . The evaluation of the system reliability is presented through illustrative examples and the conclusions is appended at the end .

2. Some definitions and general concepts

The terminology to follow is very important in creating and analyzing reliability block diagrams.

Definition (2-1) Non – Maintained Systems

These systems where maintenance action is not taken at the time of equipment failures .missiles and satellites , for example , would fall within this class of systems . see (Smith)

Definition (2-2) Reliability

The probability that an item will perform its intended function for a specified interval under stated conditions . see (Govil)

Definition (2-3) Mean Time to Failure (MTTF)

A basic measure of reliability for no repairable systems. Average failure free operating time , during a particular measurement period under stated conditions , see (Govil)

Definition (2-4) The transition probability matrix

The transition probabilities can be arranged as transition probability matrix $P = (p_{i,j})$

$$P = \begin{array}{c} \text{Final state} \longrightarrow \\ \begin{pmatrix} p_{0,0} & p_{0,1} & p_{0,2} & \cdots \\ p_{1,0} & p_{1,1} & p_{1,2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \end{array} \begin{array}{l} \\ \Big\downarrow \ \text{Initial state} \end{array}$$

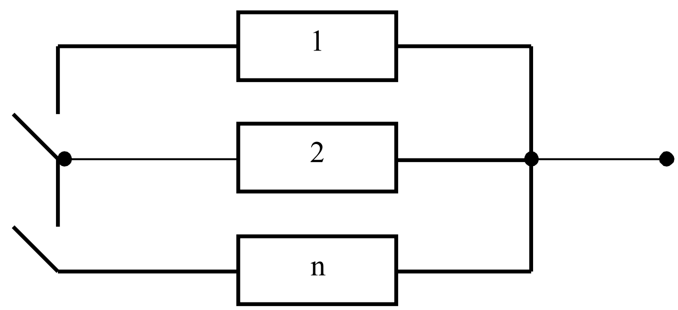
The row i contains the transition probabilities from state i to other states. Since the system always goes to some state, the sum of the row probabilities is 1. see (Smith)

Definition (2-5) A stochastic Matrix

Is A matrix with non-negative elements such that the sum of each row equals 1. see (Srinath)

3. Standby Systems Reliability

In this type of configuration only one equipment is on - line at a time . when it fails , a standby equipment is immediately switched on-line and the failed equipment taken off – line. This process continue until all n-1 stand by equipments have been exhausted. See Smith



We shall make the following statements and assumptions:

1. The system is failed when all n equipments are failed.
2. The failure probability of each equipment is independent of the other n- 1 equipments.
3. equipments can only fail in the on-line position with conditional probability λdt .
4. swathing is always perfectly reliable .

We defined the system as capable of performing its function if at least one of the n equipments is operable. Thus, the reliability in this case is simply the sum of the probabilities that the system is in a state other than failed .

$$R(t)=e^{-\lambda t}\sum_{i=0}^{n-1}(\lambda t)^{i}/i! \qquad \ldots(1)$$

Illustrative example:

Consider a two – equipment system operated in standby . for the system to be operated at a time t , there may have been none or one

Failure, but not two, therefore, from first two terms of the Poisson process we have: $R(t)=e^{-\lambda t}(1+\lambda t)$

In many cases it cannot be assumed that the off-line equipments are immune to failure until they are activated on – line status. To develop models for this case we can define λ_1 to be the failure rate of on-line equipments and λ_2 to be the failure rate off – line equipments. This leads us the following transition matrix for a two – equipment redundant system:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 1-(\lambda_1+\lambda_2) & (\lambda_1+\lambda_2) & 0 \\ 0 & (1-\lambda_1) & \lambda_1 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

Proceeding before, we obtain the following system of differential equations:

$P_0'(t) = -(\lambda_1+\lambda_2) P_0(t)$ …(2)

$P_1'(t) = (\lambda_1+\lambda_2) P_0(t) - \lambda_1 P_1(t)$ …(3)

$P_2'(t) = \lambda_1 P_1(t)$ …(4)

$P_0(0) = 1$, $P_1(0) = 0$, $P_2(0) = 0$

Transforming these equations results in :

$$p_0(s) = \frac{1}{s+\lambda_1+\lambda_2} \qquad \text{…(5)}$$

$$p_1(s) = \frac{\lambda_1+\lambda_2}{(s+\lambda_1)(s+\lambda_1+\lambda_2)} \qquad \text{…(6)}$$

$$p_2(s) = \frac{\lambda_1(\lambda_1+\lambda_2)}{s(s+\lambda_1)(s+\lambda_1+\lambda_2)} \qquad \text{…(7)}$$

Now , since the system will be in working order if we are in either state 0 or state 1 , $R(t) = P_0(t) + P_1(t)$. thus we need only find the inverse transforms of $P_0(t)$ and $P_1(t)$. it is possible to solve for $P_2(t)$ and subtract this result from unity since by definition $P_0(t) + P_1(t) + P_2(t) = 1$

The inverse transform of $P_0(s)$ is straightforward; it is simply:

$P_0(t) = L^{-1}\{P_0(s)\} = \exp - (\lambda_1 + \lambda_2)$ …(8)

To find the inverse of $P_1(s)$ we employ a partial – fraction expansion giving:

$$\frac{(\lambda_1+\lambda_2)}{(s+\lambda_1)(s+\lambda_1+\lambda_2)} = \frac{A}{(s+\lambda_1)} + \frac{B}{s+\lambda_1+\lambda_2}$$

A and B can be found rewriting the expression as two simultaneous equations.

$A(\lambda_1 + \lambda_2) + B\lambda_1 = \lambda_1 + \lambda_2$

$A s + \quad B s = 0$

Whose solution is

$$A = \frac{\lambda_1+\lambda_2}{\lambda_2} \quad \text{…(9)}$$

$$B = -\frac{(\lambda_1+\lambda_2)}{\lambda_2} \quad \text{…(10)}$$

Therefore

$$P_1(s) = \frac{\frac{\lambda_1+\lambda_2}{\lambda_2}}{(s+\lambda_1)} - \frac{\frac{\lambda_1+\lambda_2}{\lambda_2}}{s+\lambda_1+\lambda_2}$$

And the reliability function R(t) is

$$R(t) = [\exp. - (\lambda_1 + \lambda_2)t] + [\exp. - (\lambda_1 t)]\frac{\lambda_1+\lambda_2}{\lambda_2} - [\exp. -(\lambda_1 + \lambda_2)t]\frac{\lambda_1+\lambda_2}{\lambda_2}, \quad (\lambda_2 > 0) \quad \text{…(11)}$$

4. MTTF of Standby Systems

Let us define a Markov transition matrix as consisting of only one absorbing state , and all states communicate ; that is , we can go from any one state to another over a long enough period of time . we first define a matrix ***I* – *P*** , where ***I*** is an identity matrix and ***P*** is our transition matrix .

in this case where equipments are operated in standby and the off-line equipments cannot fail , ***I – P*** will be

$$I - P = \begin{array}{c|c} & \begin{array}{ccccc} 0 & 1 & 2 & \dots & n \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ . \\ . \\ . \\ n \end{array} & \begin{pmatrix} \lambda & \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \lambda & \cdots & 0 \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \end{array}$$

We now define a determinant D_n of the matrix I – P , which is the determinant obtained from the matrix after striking out the n^{th} row and n^{th} column . in a similar manner we define D_i to be the determinant obtained after striking out the i^{th} row and column of the matrix with the n^{th} row and column eliminated. The mean time to failure is defined as :

$$m_{0n} = \sum_{i=0}^{n-1} \frac{D_i}{D_n} \qquad \dots(12)$$

From inspection it can be seen that D_n , in this case , is simply λ^n . similarly all the D_i ' s are equal to λ^{n-1}; therefore

$$m_{0n} = \frac{n\lambda^{n-1}}{\lambda^n} = \frac{n}{\lambda} \qquad \dots(13)$$

This result would have been expected since the MTTF of each equipment is $1/\lambda$ hours on the average, all n will have failed by n/λ hours. If each equipment has a failure rate λ_1 when it is operating and λ_2 when it is standby, then for a two – equipment system it can be seen that :

$$m_{02} = \frac{2\lambda_1 + \lambda_2}{\lambda_1(\lambda_1 + \lambda_2)} \qquad \dots(14)$$

If $\lambda_1 = \lambda_2 = \lambda$, that is , if equipments fail at the same rate in both on – line and off – line status , $m_{02} = 3/2\lambda$.

5.Conclusions

1. To describe the reliability of a given system it is necessary to specify the equipment failure process and the system diagram which describes how the equipments are connected and the rules of operation .

2. The value of the system reliability for standby model is greater than that for the same model if its components connected in series which leads to get more efficiency at the case of standby systems.

3. In many cases it can't be assumed that the off – line equipments are immune to failure until they are activated on – line status.

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