مجلة جامعة بابل

سلسلة (جـ-٣) العلوم الصرفة والتطبيقية

المجلد /١٣ ، العدد /٣ / تشرين ثاني / ٢٠٠٦

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SOLUTION OF CONVERGENT SEQUENCE BY USING THE HEAT DISTRIBUTION IN A THIN ROD

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Abstract

In this paper a studying for the heat equation in three dimensions is presented under boundary conditions and initial condition and through these conditions above, the solution for a convergence, in three dimension is performed.

Introduction

Many researchers studied the heat equation like [2],[3] and [4] because of the wide application of it in different fields. In this paper a study of heat equation in three dimension is considered with respect to the operators which effect on the accuracy of the solutions. We emphasize on the operators through the studying of the regular equations, corner equation and the boundary equations for a thin rod.

1. In this research we study the following problem

$$\varepsilon^2 \frac{\partial u}{\partial t} - a(\varepsilon^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) = \varepsilon^2 \sum_{|k|\le m} f_k(u,x,t,\varepsilon)e^{ikwt}$$

$(x,y,t)\in\Omega=(0\le x\le 1) \times (0\le y\le 1)\times (0\le t\le T)$ ... (1)

$$u\big|_{t=0} = \varphi(x,y,\varepsilon),\ u\big|_{x=0} = \sum_{|k|\le m} \bar{\bar{\Psi}}_{1k}(y,t,\varepsilon)e^{ikwt}$$

$$u\big|_{x=1} = \sum_{|k|\le m} \bar{\bar{\Psi}}_{2k}(y,t,\varepsilon)e^{ikwt}, \frac{\partial u}{\partial y}\Big|_{y=0} = 0,\ \frac{\partial u}{\partial y}\Big|_{y=1} = 0 \quad \ldots(2)$$

∍ u is an arbitrary vector, a = const. >0 w,ε >0 a very small parameter. Later we shall discuss the problem in the region Ω, and we shall prove that the analytic convergent series

$$u(x,y,t,\varepsilon) = \sum_{i=1}^{\infty} \varepsilon^i u_i(x,y,t)$$

is a solution for the problem (1) with boundary conditions and initial condition in (2). In order to prove the existence of a solution for (1) and (2) we find an estimation for the convergent series, in other meaning for construct like this convergence and more accurate it contains two functions.

The first is the function the boundary of Ω when t=0 , x=o, x=1, and the corner function at the points (x,t)=(0,0) and (x,t)=(1,0) in the region Ω.

2- The convergent series which is analytic with respect to, ε to solve (1), and (2) it must given as follows:-

$$u(x,y,t,\varepsilon) = \sum_{k=0}^{\infty} \varepsilon^k \begin{bmatrix} u_k(x,y,t) + \upsilon_k(x,y,t,\tau) + B_k(x,y,\tau) + p_k(\zeta,y,t) + q_k(\zeta,y,t,\tau) \\ + r_k(\eta,y,t) + Z_k(\eta,y,t,\tau) + P_k(\zeta,y,\tau) + Q_k(\eta,y,\tau) \end{bmatrix} \quad \ldots(3)$$

where u_k and ν_k are the regular functions, B_k,P_k,q_k and r_k are boundary functions, and the corner functions P_k , Q_k such that

$\tau\frac{t}{\varepsilon^2}, \xi = \frac{x}{\varepsilon}, \eta = (1-x)|\varepsilon$. moreover coefficient series (3) ν_k, q_k and Z_k are with period 2π-periodic τ with zero average i.e.

$<\nu_k(x,y,t,.)>=(q_k(\xi,y,t,.)=(Z_k(\eta,y,t,.)>=0$

i.e $<\nu_k(x,y,t,.)>= \frac{1}{2\pi}\int_0^{2\pi} \upsilon_k(x,y,t,\tau)d\tau = 0$

To find its coefficients (3) must be put in (1) and (2), we get the following equation

$$\frac{\partial u}{\partial \tau} - a(\varepsilon^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) = \varepsilon^2\left(\sum_{i=0}^{\infty}\varepsilon^i \sum_{|k|\le m} f_{ki}(u,x,t)e^{ik\tau} - \frac{\partial u}{\partial t}\right),$$

$$u\big|_{t,\tau=0} = \sum_{i=0}^{\infty}\varepsilon^i \varphi_i(x,y), u\big|_{x=0} = \sum_{i=0}^{\infty}\varepsilon^i \sum_{|k|\le m}\Psi_{1ki}(y,t)e^{ik\tau}$$

$$u\big|_{x=1} = \sum_{i=0}^{\infty}\varepsilon^i \sum_{|k|\le m}\Psi_{2ki}.(y,t)e^{ik\tau}, \frac{\partial u}{\partial y}\Big|_{y=0,1} = 0 \qquad \dots (4)$$

From the standard procedures for the analysis of series with respect to ε powers and by putting the coefficients for the equal powers with respect to ε, we get an equation for any convergent series

(a) The functions u_o, and ν_o when ε^0 posses the following equations

$$a\frac{\partial^2 u_o}{\partial y^2} = 0, \frac{\partial u_o}{\partial y}\Big|_{y=0,1} = 0 \qquad \dots(5)$$

$$\frac{\partial \upsilon_o}{\partial \tau} - a\frac{\partial^2 \upsilon_o}{\partial y^2} = 0, \frac{\partial \upsilon_o}{\partial y}\Big|_{y=0,1} = 0, <\nu_o(x,y,t,.)>=0 \qquad \dots(6)$$

Which can be found $u_o=\alpha_o(x,t)$, where α_o-arbitrary function, and for the equation (6) we get $\nu_o=0$. By the same way we can fined $u_1=\alpha_1(x,t)$, where α_1-arbitrary function, $\nu_1=0$.

For the coefficients u_2, ν_2, we get the following equations:

$$a\frac{\partial^2 u_2}{\partial y^2} = a\frac{\partial^2 \alpha_o}{\partial x^2} - \frac{\partial \alpha_o}{\partial t} + f_{oo}(\alpha_o, x, t), \frac{\partial \upsilon_2}{\partial y}\Big|_{y=0,1} = 0 \qquad \dots (7)$$

$$\frac{\partial \upsilon_2}{\partial \tau} - a\frac{\partial^2 \upsilon_2}{\partial y^2} = \sum_{0<|k|\le m} f_{ko}(\alpha_o, x, t)e^{ikt} \qquad \frac{\partial \upsilon_2}{\partial y}\Big|_{y=0,1} = 0, (\upsilon_2(x,y,t,.)>=0$$

Decidability condition (7) we have the equation $\dots(8)$

$$\frac{\partial \alpha_o}{\partial t} - a\frac{\partial^2 \alpha_o}{\partial x^2} = f_{oo}(\alpha_o, x, t)$$

The solution of (8) evidently keep in mind $\dots(9)$

$$\nu_2 = \sum_{0<k\le m} \frac{1}{ik} f_{ko}(\alpha_o, x, t)e^{ikt}.$$

Now coefficients u_i, ν_i, $i\ge 3$ can be obtain problems:

$$a\frac{\partial^2 u_i}{\partial y^2} = a\frac{\partial^2 \alpha_{i-2}}{\partial x^2} - \frac{\partial \alpha_{i-2}}{\partial t} + \frac{\partial f_{oo}(\alpha_o, x, t)}{\partial u}\alpha_{i-2}$$

$$\Phi_i(x,t)\frac{\partial u_i}{\partial y}\Big|_{y=0,1} = 0 \qquad \ldots(10)$$

where Φ_i can be expressed through the functions νs, $s\le i-3$ and ν_s, $s\le i-2$,

$$\frac{\partial^2 \nu_i}{\partial y^2} - a\left(\frac{\partial^2 \nu_{i-2}}{\partial x^2} + \frac{\partial^2 \nu_i}{\partial y^2}\right) = \sum_{0<|k|\le m_i} \Psi_{ki}(x,t)e^{ikt} \equiv \chi_i(x,t,\tau),$$

$$\frac{\partial \nu i}{\partial \tau}\Big|_{y=0,1} = 0, < \nu_i(x, y, t,.) >= 0 \qquad \ldots(11)$$

where Ψk_i can be expressed through the functions u_s and ν_s , $s\le i-2$. For example $\Phi_3(x,t)=f_{01}(\alpha_o,x,t)$,

$$\chi_3(x,t,\tau) = \sum_{0<|k|\le m}\left[\frac{\partial f_{ko}(\alpha_o, x, t)}{\partial u}\alpha_1(x,t) + f_{k1}(\alpha_o, x, t)\right]e^{ikt}.$$

By decidability condition (10) we have the equation

$$\frac{\partial \alpha_{i-2}}{\partial t} - a\frac{\partial^2 \alpha_{i-2}}{\partial x^2} - \frac{\partial f_{oo}(\alpha_o, x, t)}{\partial u}\alpha_{i-2} = \Phi_i(x,t). \qquad \ldots(12)$$

The solution of equation (11) becomes

$$\nu_i = \sum_{0<|k|\le m_i} \frac{1}{ik}\Psi_{ki}(x,t)e^{ikt}.$$

(b) We find coefficients $P_k(\xi,y,t)$ and $q_k(\xi,y,t,\tau)$.

In order that consideration Eq. (1) in neighborhood left boundary at the region Ω, for that change of variables $\xi=x\varepsilon^{-1}$, lead to the problem

$$\frac{\partial u}{\partial \tau} - a\left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial y^2}\right) = \varepsilon^2\left(\sum_{i=0}^{\infty}\varepsilon^i \sum_{|k|\le m} f_{ki}(u, \varepsilon\xi, t)e^{ikt} - \frac{\partial u}{\partial t}\right)$$

$$u\big|_{\xi=0} = \sum_{i=0}^{\infty}\varepsilon^i \sum_{|k|\le m}\Psi_{1ki}(y,t)e^{ikt}, \frac{\partial u}{\partial y}\Big|_{y=0,1} = 0. \qquad \ldots(13)$$

By substituting series (3) in eq. (13) we get coefficients q_o and p_o as follows:

$$\frac{\partial^2 P_o}{\partial \xi^2} + \frac{\partial^2 P_o}{\partial y^2} = 0, \frac{\partial P_o}{\partial y}\Big|_{y=0,1} = 0,$$

$$(u_o + P_o)\big|_{x=0} = \Psi_{100}(y,t) \equiv \sum_{0\le s\le moo} a_s(t)\cos(s\pi y), P_o\big|_{\xi=\infty} = 0. \qquad \ldots(14)$$

$$\frac{\partial q_o}{\partial \tau} - a\left(\frac{\partial^2 q_o}{\partial \xi^2} + \frac{\partial^2 q_o}{\partial y^2}\right) = 0, \frac{\partial q_o}{\partial y}\Big|_{y=0,1} = 0,$$

$$q_o\Big|_{\xi=0} = \sum_{0\le|k|\le m} \Psi_{1ko}(y,t)e^{ik\tau} \equiv \sum_{0\le|k|\le m}\ \sum_{0\le s\le m_{ko}} b_{ks}(t)\cos(s\pi y)e^{ik\tau}, \quad \ldots (15)$$

$$< q_o(\xi, y, t, .) >= 0, q_o\Big|_{\xi=\infty} = 0$$

The solution of eq. (14) seek in the form:

$$P_o = \sum_{0\le|k|\le m} Cs(\xi st)\cos s\pi y,$$

in this connection we get unknown coefficients Cs from the following eq.:

$$\frac{d^2Cs}{d\xi^2} - s^2\pi^2 cs = 0, \quad Co(0,t) = q_o(t) - \alpha_o(0,t),$$

$$Cs(0,t) = a_s(t) \text{ at } s \ne 0, Cs\Big|_{\xi=\infty} = 0$$

Then we find

$$P_o = \sum_{0\le|k|\le m_{oo}} a_s(t)e^{-s\pi s}\cos s\pi y,$$

$$\alpha_o(0,t) = a_o(t) \equiv < \Psi_{100}(.,t) >.$$

The solution eq. (15) seek in the form:

$$q_o\Big|_{\xi=0} = \sum_{0\le|k|\le m}\ \sum_{0\le|k|\le m} C_{ks}(\xi,t)\cos\pi)c^{ik\tau}, \quad \ldots(16)$$

Then

$$ikC_{ks} - a\left(\frac{\partial^2 C_{ks}}{\partial \xi^2} - \pi^2 S^2 C_{ks}\right) = 0, C_{ks}(0,t) = b_{ks}(t).$$

λ_{ks}- can be expressed characteristic root of an equation
$a\lambda^2 = ik + a\pi^2 S^2$, at $Re\lambda > 0$. Thus we get
$C_{ks} = b_{ks}(t)e^{-\lambda ks\xi}$,

By virtue (16):

$$q_o\Big|_{\xi=0} = \sum_{0\le|k|\le m}\ \sum_{0\le s\le m_{ko}} b_{ks}(t)e^{-\lambda ks\xi}\cos\pi os^{ik\tau}$$

Similar, we find coefficient Pi, qi, i≤1: i≥1

$$\frac{\partial P_{i-2}}{\partial t} - a\left(\frac{\partial^2 P_i}{\partial \xi^2} + \frac{\partial^2 P_i}{\partial y^2}\right) = \sum_{0\le s\le m_i} M_{is}(t)P_{is}(\xi\xi)^{-Mis\xi}\cos\pi o$$

$$\frac{\partial P_i}{\partial y}\Big|_{y=0,1} = 0, \quad P_i\Big|_{\xi=0} = \Psi_{10i}(y,t) - \alpha_i(0,t), P_i\Big|_{\xi=\infty} = 0 \quad \ldots(17)$$

$$\frac{\partial q_i}{\partial t} - a\left(\frac{\partial^2 q_i}{\partial \xi^2} + \frac{\partial^2 q_i}{\partial y^2}\right) = \sum_{0\le|k|\le ti_i} \sum_{0\le s\le ri_i} N_{iks}(t) Q_{iks}(\xi) e^{-Niks} \cos s\pi os^{ik\tau},$$

$$\frac{\partial q_i}{\partial y}\Big|_{y=0,1} = 0, \quad q_i\Big|_{\xi=0} = \sum_{0\le|k|\le m_i} \Psi_{1kii}(y,t) eik\tau - v_i\Big|_{x=\infty} = 0 \qquad \ldots(18)$$

$$< q_i(x,y,t,.) >= 0, \quad q_i\Big|_{\xi=\infty} = 0$$

where

constant-

$\sqrt{ni,\ ti, vi>0}$.

The solution eq. (17) seek in the form:

$$P_i\Big|_{\xi=0} = \sum_{0\le s\le ni} C_{si}(\xi,t)\cos s\pi)$$

The functions C_{si} are solution the following eq.

$$\frac{d^2 Cis}{d\xi^2} - s^2\pi^2 C_{si} = M_{si}(t) P_{is}(\xi) e^{-Mis\xi},$$

$$C_{si}(0,t) = a_{si}(t) \text{ at } s \neq 0, C_{si}\Big|_{\xi=\infty} = 0 \qquad \ldots(19)$$

which uniquely solution [3] and from equation $\alpha_i(0,t)=a_{oi}(t)-C_{oi}(0,t)$ we get formula

$$\alpha_i(0,t)\Big|_{t=0} = < \Psi_{10i}(.,0) >. \qquad \ldots(20)$$

Now, we find the solution eq. (18). Let

$$\sum_{0\le|k|\le m_i} \Psi_{1kii}(y,t) e^{ik\tau} - v_i\Big|_{x=\infty} = \sum_{0\le|k|\le p_i} \sum_{0\le s\le d_i} a_{ksi}(t).\cos s\pi y e^{ik\tau}$$

and

supposed

$\sqrt{t_i=P_i}$, $v_i=d_i$. The solution qi will be seek in the form:

$$q_i = \sum_{0\le|k|\le t_i} \sum_{0\le s\le r_i} C_{ks}(\xi,t).\cos s\pi y e^{ik\tau}.$$

In this connection we get unknown coefficients C_{ks} form the following eq.:

$$ik - a\left(\frac{d^2 C_{ks}}{d\xi^2} - s^2\pi^2 C_{ks}\right) = N_{iks}(t) Q_{ikss}(\xi) e^{-viks},$$

$$C_{ks}(0,t) = a_{ksi}(t), C_{ks}\Big|_{\xi=\infty} = 0$$

which it is obvious uniquely solution [2]. Coefficients r_k and Z_k are defined similar way. Moreover, in specifically we get:

$$\alpha_i(1,t)\Big|_{t=0} = < \Psi_{20i}(.,0) >. \qquad \ldots(21)$$

(c) we fined boundary function $B_k(x,y,\tau)$ in neighborhood lower boundary (t=0) at the region Ω and use change of variables $t=\varepsilon^2\tau$ in the problem (1), we get

$$\frac{\partial u}{\partial \tau} - a\frac{\partial^2 u}{\partial y^2} = \varepsilon^2\left(a\frac{\partial^2 u}{\partial x^2} + \sum_{|k|\le m} f_k(u, x, \varepsilon^2\tau, \varepsilon)e^{ikk\tau}\right)$$

$$\frac{\partial u}{\partial y}\Big|_{y=0,1} = 0,\ u\big|_{\tau=0} = \vartheta(x, y, \varepsilon) = \sum_{|k|\le m} \varepsilon^i \vartheta_i(x, y). \quad \ldots (22)$$

From (3) and (22) function $b_o(x,y,\tau)$ have eq.

$B_o(x,y,o)= \varphi(x,y,o)-u_o(x,y,o)-\nu_o(x,y,o,o)$. ...(23)

Taking into account $u_o(x,y,t)=\alpha_o(x,t)$ and $\nu_o(x,y,t,\tau)=0$ the eq. (23) can be write

$$B_o(x, y, o) = \vartheta_o(x, y) - \alpha_o(x, o) \equiv \sum_{0<s\le n_o} b_{so}(x)\cos s\pi y.$$

where no-number and b_s-functions. The solution (23) seek in the form:

$$B_o(x, y, \tau) = \sum_{0\le s\le n_o} C_s(x, \tau)\cos s\pi y.$$

and be found

$C_o(x,\tau)=0$, $C_s(x,\tau)=b_s(x)e\text{-}as^2\pi 2\tau$

$B_o(x,y,\tau)=0$, $B_o\big|_{\tau=\infty} = 0.$

That mean $\alpha_o(x,o)=<\varphi o(x,.)>$. ...(24)

Similar, we have coefficient $B_1(x,y,o)$, from that we can be define $\alpha_1(x,o)=<\varphi_1(x,.)$. Coefficients B_i, $i\ge 2$ series (3) are the following solutions equations:

$$\frac{\partial Bi}{\partial \tau} - \left(\frac{\partial^2 B_{i-2}}{\partial x^2} + \frac{\partial^2 B_i}{\partial y^2}\right)\sum_{0\le k\le ni} d_{ksi}(x)P_{ksi}(\tau)e^{ik\tau k}e^{-\gamma ik\tau}\cos s\pi y \quad \ldots (25)$$

$$B_i(x, y, o) = \vartheta_i(x, y) - \alpha_i(x, o) - \nu_i(x, y, o, o),\ B_i\big|_{\tau=\infty} = 0$$

where numbers n_i, m_i, $\gamma_{ksi} > 0$, functions d_{ksi} and P_{ksi} can be expressed through the functions B_s, ν_s and ν_s, $s\le i\text{-}2$, $\nu_i(x,yo,o)=0$.

Let

$$\vartheta_i(x, y) = \sum_{0\le s\le ni} d_{si}(x)P_{ksi}\cos s\pi y$$

Then we shall seek the solution equations (25) in the form:

$$B_i(x, y, \tau) = \sum_{0\le s\le ni} C_{si}(x, \tau)\cos s\pi y \quad \ldots (26)$$

Thus we get for coefficients C_{si} the following equation:

$$\frac{\partial C_{si}}{\partial \tau} + as^2\pi^2 C_{si} = \sum_{0\le l\le ni} d_{ksi}(x)P_{ksi}(\tau)e^{ik\tau}e^{-\gamma_{ksi}\tau}$$

$$C_{si}\big|_{\tau=\infty} = 0,\ C_{si}(x, o) = C_{si}(x),\ 0 \le s \le mi.$$

From that, we obtain uniquely functions $C_{si}(x,\tau)$ and taking into account (26), we define that

$$C_{oi}(x,o)\Big|_{x=0,1}=0\,,\ \alpha_i(x,o)=a_{oi}(x)-C_{oi}(x,o)\,,$$

$$\alpha_i(x,o)\Big|_{x=0,1}=<\vartheta_i(x,.)\Big|_{x=0,1} \qquad \ldots(27)$$

(d) We observe that, by virtue equation (20), (21) and (27) are make consistent up to the continuity.

(e) We find coefficient P_k and Q_k. For P_o, as follow by using the mathematic induction.

$$\frac{\partial Po}{\partial \tau}-a\left(\frac{\partial^2 P_o}{\partial \xi^2}+\frac{\partial^2 P_o}{\partial \xi^2}\right)=0\ \text{, at k=0} \qquad \ldots(28)$$

$$P_o(\xi\xi y,o)=-\rho_o(\xi\xi y,o)-q_o(\xi\xi y,o,o),P_o(o,y,\tau)-B_o(o,y,\tau)$$

$$\frac{\partial P_o}{\partial y}\Big|_{y=0,1}=0$$

According to (b) and (C) it should are representation

$$P_o(\xi,y,o)=\sum_{0\le s\le ni}a_s(o)e^{-\lambda s\xi}\cos s\pi y,$$

$$B_o(o,y,\tau)=\sum_{0\le s\le no}b_s(o)e^{-\lambda s\xi}\cos s\pi y$$

where $R_e\lambda s>o$ and in this case, evidently, $P_0(o,y,o)=B_0(o,y,o)$. That means $m_{oo}=n_o$, $a_s(o)=b_s(o)$. The solution equation (28) we shall seek in the form:

$$P_o=\sum_{0\le s\le o}N_s(\xi,\tau)-b_s(o)e^{-(\lambda s\xi+as^2\pi^2\tau)}\cos s\pi y., \qquad \ldots(29)$$

Then for v_s we obtain the equation:

$$\frac{\partial v_s}{\partial \tau}-a\frac{\partial^2 v_s}{\partial \xi^2}+a\pi^2 s^2 v_s=-b_s(o)a\lambda_s^2 e^{-(\lambda\lambda s+as^2\pi^2 t)}\equiv h_s(\xi,\tau),$$

$$v_s(\xi,o)=0,v_s(o,\tau)=0. \qquad \ldots(30)$$

Introducing Greena function [2] we define

$$G_s=\frac{1}{2\sqrt{a\pi(\lambda-u)}}e^{-a\pi^2 s^2(\tau-v)}\left[e^{-\frac{(\xi-S)^2}{4a(\tau-u)}}-e^{-\frac{(\xi-S)^2}{4a(\tau-u)}}\right] \ldots(31)$$

We produce solution equation (30) in the form [2]:

$$v(\xi,t)=\int_0^\infty du\int_0^\infty G_s(\xi,S,\tau-u)h_s(s,u)ds$$

for finding estimation function v we examine integral [3]:

$$J=\int_0^\infty du\int_0^\infty\frac{1}{2\sqrt{a(\tau-u)}}e^{-\gamma_1(\tau-u)}e^{-\frac{(\xi-S)^2}{4a(\tau-u)}}e^{-\gamma_2(u+s)}ds$$

where $\gamma_1,\gamma_2>0$. If we divide by substitution

$\beta = \dfrac{\xi - S}{2\sqrt{a(\tau - \nu)}}$, we get

$$J = \int_0^{\tau} du \int_{-\frac{\xi}{2\sqrt{a(\tau-\nu)}}}^{\infty} e^{-\gamma_1(\tau-u)} e^{-\beta^2} e^{-\gamma_2(u+\xi+2\sqrt{a(\tau-\nu)})} d\beta \le$$

$$e^{-\gamma^2\xi} \int_{-\infty}^{\infty} e^{-(\beta+\gamma_2\sqrt{a(\tau-\nu)})^2} d\beta \int_0^{\tau} e^{-(\gamma_1+\gamma 2a)(\tau-\nu)} . e^{-\gamma_2 \nu} d\nu \le c e^{\gamma(\xi+\tau)}$$

where c-const., $\gamma = \min \frac{1}{2}(\gamma_2, \gamma_1 + \gamma^2{}_2 a)$. From relations (29), (31), (33) it should be existence positives numbers C_o and γ_o have the following power estimation[4]:

$|P_o(\xi, y, \tau)| \le C_o e^{-\gamma_0(\xi+\tau)}$, $\xi, \tau \ge 0$.

For functions P_k, $k \ge 1$, similar we find power estimation of boundary layer variables ξ and τ.

In the same way are construct equations for the corner boundary functions Q_k and have deduce power estimation.

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