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تهديكم هيئة تحرير مجلة بابل للعلوم الانسانية / جامعة بابل أطيب تحياتها ويسرها أن تعلمكم بآراء المقومين بشأن بحثكم الموسوم:

The solutions of differential equations with independent variable

علماً بأن الهيئة قد قررت قبول بحثكم أعلاه للنشر في العدد القادم من المجلة الذي يصدره قريباً.

مع التقدير..

الدكتور فاهم حسين الطريحي
سكرتير التحرير

THE SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH INDEPENDENT VARIABLE

Dr. Hayder Jabbar Abood
Department of Mathematics
College of Education
Babylon University

Abstract

Demonstrate to find a solution of the differential equation from 2[nd] degree which depend on premetary and initial condition .

Introduction

In this search we will studding and demonstrate to finding uniqueness solution to ordinary differential equation from 2[nd] degree and which depend on aprametery λ and initial condition .And for this purpose we will assume some condition which help us to arrived our target , So there are some searhers (Magnus W., Winkler S. 1986) which taked a differential equation studies for continous function as a 1[st] degree also taked (Sansone G., Contin R. 1995) a partial differential equation from 1[st] degree and others specified it (Arscott F.M. 1992) , (Cesaril., Hale J.K. 1998) , (Levy D.M., Keller J.B. 2002), and (Mathieu E. 1988) . So in this search we recognized from others to proving a singularity of solution for differential highet degree and we it possible to making for wide used for equations type .

1- We will study the following differential equation

$d^2x(t)/dt^2 = f[\,t , x(t) , x(t-\Delta) , \lambda\,]$ .... (1)

with the initial condition

$x(t,\lambda) = \phi(t,\lambda) \ , \ \forall t \in [-\Delta, 0]$ ....(2)

since $x = (x_1, x_2, \ldots, x_m) \in E^m$,
$f = (f_1, f_2, \ldots, f_m) \in E^m$ and
$\phi(t,\lambda) \in E^m$.

The solution looking at initial point $t_o = 0$

2- Suppose satisfying the following conditions :

a)The function $f(t,x,y,\lambda)$ know every point so that $(t,x,y,\lambda) \in [0,T] \times D \times D \times \Lambda$, since

D- open set in space E^m and Λ- set containing values parameter λ. for each $x \in D$, $y \in D$, $\lambda \in \Lambda$ the function $f(t,x,y,\lambda)$ measurable respect to the t and addition the kind of integration in the interval $[o,T]$ with function m(t) to which the

$\| f(t,x,y,\lambda) \| \le m(t)$ ....(3)

such that $f(t,x,y,\lambda)$ continous function for x,y .

b) Function $\phi(t,\lambda)$ know for each $t \in [-\Delta , 0]$, $\lambda \in \Lambda$ and continuous for t also $\| \phi(t,\lambda) \| \le K$, K – constant , $\phi(t,\lambda) \in D$ and

$\lim_{\lambda \to \lambda_o} \phi(t,\lambda) = \phi(t,\lambda_o)$

c) There exist function $\psi(\delta, \wp) = 0$ know for each $0 < \delta \le d$, $0 < \wp \le d$, $\lim\limits_{\delta,\wp\to 0} \psi(\delta, \wp) = 0$ and by using Lebik integration in [0,T] the function X(t) is

$$\int_0^t X(t)dt < \infty \;\ldots\ldots\ldots (4)$$

Since

$$\| f(t,x_1,y_1,\lambda) - f(t,x_2,y_2,\lambda) \| \le \psi(\| x_1-x_2 \|, \| y_1-y_2 \|)\, X(t) \ldots\ldots\ldots (5)$$

for $t \in [0,T]$, $x_1,x_2,y_1,y_2 \in D$, $\| x_1-x_2 \| \le d$, $\| y_1-y_2 \| \le d$, $\lambda \in \Lambda$.

d) The solutions equations (1) and (2) when $\lambda=\lambda_o$ definition in [0,T] uniquely in D.

Theorem 1 : Let satisfying conditions above and let

$$\lim_{\lambda\to\lambda_o} \int_0^t f(\tau,x,y,\lambda)\, dt = \int_0^t f(\tau,x,y,\lambda_o) \ldots\ldots\ldots (6)$$

Uniformly with respect to t,x,y . Then for all $\eta > 0$ exist $\delta > 0$ so that $/ \lambda-\lambda_o / < \delta$ the solution $x(t,\lambda)$ equations (1) , (2) meeting condition $\| x(0,\lambda)-x(0,\lambda_o) \| < \delta$ definite in [0,T] and inequality is correct

$$\| x(t,\lambda)-x(t,\lambda_o) \| < \eta, \text{ for all } t \in [0,T] \ldots\ldots\ldots (7)$$

$\delta,\wp\to 0$ proof of theory above requires the following results .

3-Lemma 1 : Let satisfying condition (6) and $\tilde{x}(t)$, $\tilde{y}(t)$ – constants functions defined in [o,T] , $\tilde{x}(t) = c_i \in D$, $\tilde{y}(t) = \ell_i \in D$ for all $\tau_{i-1} \le t < \tau_i$ (i=1,2,…) ,$O=\tau_0 < \tau_1 < \cdots < \tau_k = T$,

then

$$\lim_{\lambda\to\lambda_o} \int_0^t f[\tau, \tilde{x}(\tau), \tilde{y}(\tau), \lambda]\, dt = \int_0^t f[\tau, \tilde{x}(\tau), \tilde{y}(\tau), \lambda_o]\, d\tau$$

uniformly with respect t .

proof : Lemma (1) easily it should be from (6) .

Lemma 2 : Let $x^*(t,\lambda)$ – continuous function of the variable t,$x^*(t,\lambda) \in D$, meeting the following condition

$$\lim_{\lambda\to\lambda_o} \sup_{t\in[-\Delta,T]} \| x^*(t,\lambda) - x^*(t,\lambda_o) \| = 0$$

Then

$$\lim_{\lambda\to\lambda_o} \int_0^t f[\tau, x^*(\tau,\lambda), x^*(\tau-\Delta,\lambda), \lambda]\, d\tau = \int_0^t f[\tau, x^*(\tau,\lambda_o), x^*(\tau-\Delta,\lambda_o), \lambda_o]\, d\tau,$$

uniformly with respect $t \in [0,T]$.

Proof : from supposition , function X(t) integrable by lebik and satisfying condition (d) the function $\psi(\delta, \wp)$ so that $\lim\limits_{\delta,\wp\to 0} \psi(\delta, \wp) = 0$, then for each ε can be found $\delta=\delta(\varepsilon)$, $\wp=\wp(\varepsilon)$ such that

$$\psi(\delta,\wp)\int_0^T X(t)\,dt < \frac{\varepsilon}{6}.$$

Let $\tilde{x}(t)$ – constant function so that $\sup\limits_{t\in[-\Delta,T-\Delta]} \|\tilde{x}(t)-x^*(t,\lambda_o)\| < \wp$ **and**

$$\sup_{t\in[0,T]} \|\tilde{x}(t)-x^*(t,\lambda_o)\| < \delta.$$

We denote by $\cup(\lambda_o)$ - neighborhood point λ_o, since for all $\lambda\in\cup(\lambda_o)$

$$\sup_{t\in[-\Delta,T-\Delta]} \|x^*(t,\lambda)-x^*(t,\lambda_o)\| < \wp,$$

$$\sup_{t\in[0,T]} \|x^*(t,\lambda)-x^*(t,\lambda_o)\| < \delta \quad \text{..............(8)}$$

$$\left\|\int_0^t \{f[\tau,\tilde{x}(\tau),\tilde{x}(\tau-\Delta),\lambda]-f[\tau,\tilde{x}(\tau),\tilde{x}(\tau-\Delta),\lambda_o]\}d\tau\right\| < \frac{\varepsilon}{2}.$$

This choice $\cup(\lambda_o)$ it is possible by lemma 1 and the conditions lemma 2, then for $\lambda\in\cup(\lambda_o)$ inequality is correct

$$\int_0^t \| f[\tau,x^*(\tau,\lambda),x^*(\tau-\Delta,\lambda),\lambda] - f[\tau,x^*(\tau,\lambda_o),x^*(\tau-\Delta,\lambda_o),\lambda] \|d\tau \le \psi(\delta,\wp)\int_0^T X(t)dt < \frac{\varepsilon}{6},$$

$$\int_0^t \| f[\tau,x^*(\tau,\lambda_o),x^*(\tau-\Delta,\lambda_o),\lambda]-f[\tau,\tilde{x}(\tau),\tilde{x}(\tau-\Delta),\lambda] \| d\tau \le \psi(\delta,\wp)\int_0^T X(t)\,dt < \frac{\varepsilon}{6},$$

$$\int_0^t \| f[\tau,\tilde{x}(\tau),\tilde{x}(\tau-\Delta),\lambda_o]-f[\tau,x^*(\tau,\lambda_o),x^*(\tau-\Delta,\lambda_o),\lambda_o] \|d\tau \le \psi(\delta,\wp)\int_0^T X(t)dt < \frac{\varepsilon}{6}.$$

Hence and from (c) can be found :

$$\left\|\int_0^t \{f[\tau,x^*(\tau,\lambda),x^*(\tau-\Delta,\lambda),\lambda]-f[\tau,x^*(\tau,\lambda_o),x^*(\tau-\Delta,\lambda_o),\lambda_o]\}d\tau\right\| \le \frac{\varepsilon}{6}\ \frac{\varepsilon}{6}+\frac{\varepsilon}{2}+\frac{\varepsilon}{6}=\varepsilon.$$

lemma 2 . proved .

Lemma 3 : Let $\lambda_n \to \lambda_o$ with $n \to \infty$, $\lambda_n \in \Lambda$ and let functions $x(t,\lambda_n)$ are solutions equations (1) ,(2) defined in $[0,T_n]$, $0< T_n \le T$. Then $x(t,\lambda_n)$ is equicontinuous sequence and uniformly bounded function .

Proof : The solution equations (1) and (2) satisfy equation

$$X(t,\lambda n) = \phi(0,\lambda_n) + \int_0^t f[\tau,x(\tau,\lambda_n),x(\tau-\Delta,\lambda_n),\lambda_n]\, d\tau ,$$

$x(t,\lambda_n) = \phi(t, \lambda_n)$, for $t\in[-\Delta ,0]$............(9)

Then for all $t_1, t_2 \in [0,T_n]$, $t_1 < t_2$ we can be found

$$X(t_2 , \lambda_n) - X(t_1 , \lambda_n) = \int_{t1}^{t2} f[\tau,x(\tau,\lambda_n),x(\tau-\Delta,\lambda_n),\lambda_n]\, d\tau \ .$$

From here

$$\| x(t_2,\lambda_n) -x(t_1,\lambda_n) \| \le \| \int_{t1}^{t2} f[\tau,x(t_1,\lambda_n), x(t_1-\Delta,\lambda_n),\lambda_n]\, d\tau \| + \int_{t1}^{t2} \| f[\tau,x(\tau,\lambda_n),$$

$$x(\tau-\Delta,\lambda_n),\lambda_n] - f[\tau,x(t_1,\lambda_n), (t_1-\Delta,\lambda_n),\lambda_n] \|\, d\tau \ ....(10)$$

By virtue (6) exist function $A(\delta)$ definite for all $\delta>0$, and $\lim_{\delta\to 0} A(\delta)=0$, so that

$$\int_{t1}^{t2} f(\tau,u,v,\lambda)d\tau = \int_{t1}^{t2} f (\tau,u,v,\lambda_o)d\tau+R(t_1,t_2,u,v,\lambda), \quad u,v\in D,\|R\|\le A(|\lambda-\lambda_o|).........(11)$$

Taking into account (3) we obtain

$$\int_{t1}^{t2} f(\tau,u,v,\lambda_o)\, d\tau \le c (t_2-t_2),.........(12)$$

$c(\delta)$– decreasing function , with $\delta>0$ and $\lim c(\delta) = 0$

To prove that the function $x(t,\lambda_n)$ - equicontinuous it is necessary show , for all $\varepsilon, 0 < \varepsilon \le d$ there exist $N(\varepsilon)$ and $\delta(\varepsilon)$ that from

$t_1, t_2 \in [0,T_n]$, $0< t_2-t_1< \delta(\varepsilon)$, $n >N(\varepsilon)$ ....(13)

it should be $\| x(t_2,\lambda_n)-x(t_1,\lambda_n) \| < \varepsilon$.

We shall take N , that for $n > N$, $A(|\lambda_n - \lambda_o|) < \frac{\varepsilon}{3}$ . ...(14)

And later δ, such that : $C(\delta) < \frac{\varepsilon}{3}$, $\nu(\delta)\, \psi(\varepsilon,d) < \frac{\varepsilon}{3}$, ...(15)

where $\nu(\delta)$ – decreasing function , $\lim_{\delta\to 0} \nu(\delta) = 0$,

$$\nu(\bar{t}_2 - \bar{t}_1) \ge \int_{\bar{t}_1}^{\bar{t}_2} X(t)\, dt , \text{ for } 0 \le \bar{t}_1 \le t_2 \le T .$$

We assume that (13) is holds and $\| x(t_2,\lambda_n) - x(t_1,\lambda_n) \| \ge \varepsilon$ ....(16)

since $x(t,\lambda_n)$ – continuous function then exist t_3 , $t_1< t_3 \le t_2$, that

$\|x(t_3,\lambda_n)-x(t_1,\lambda_n) \| =\varepsilon$

but $\| x(\tau,\lambda_n)- x(t_1,\lambda_n) \| < \varepsilon$ for $t_1 \le \tau < t_3$. Then

$$\int_{t_1}^{t_3} \| f[\tau,x(\tau,\lambda_n),x(\tau-\Delta,\lambda_n),\lambda_n]-f[\tau,x(t_1,\lambda_n),x(t_1-\Delta,\lambda_n),\lambda_n] \, d\tau \le \int_{t_1}^{t_3} X(\tau)\psi(\| x(\tau,\lambda_n)-$$

$$x(t_1,\lambda n) \| , \| x(\tau-\Delta,\lambda_n)-x(t_1-\Delta,\lambda_n) \| \times d\tau \le \nu(\delta)\, \psi(\varepsilon,d) \cdots\cdots\cdots (17)$$

Inequality (10) , (11) and (12) , in place of t_2 it should be write t_3 with inequality (14) , (15) and (17) we will get contradiction

$\varepsilon < A(|\lambda_n-\lambda_o|) + c(\delta) + \nu(\delta)\, \psi(\varepsilon,d) < \varepsilon$

that mean the assumption (17) is incerrect . The function $x(t,\lambda_n)$ is uniformly bounded it should be from (3) , $\|\phi(t,\lambda)\| \le K$, that obviously from (9) .

Lemma 3 . proved .

4- proof theory 1 : By virtue condition **a)** for each constant λ the solution equations (1) , (2) exist in finite interval $[0,T_\lambda]$. From Lemma (3) any solution $x(t,\lambda_n)$ equations (1) , (2) is uniformly bounded function and equicontinuous sequence . Then from this sequence we can define sequence $x(t,\lambda_{nk})$, uniformly convergent in $[0,T_o]$ to continuous function $y(t)$, moveorer $y(t) \in D$, for $t \in [0,T_o]$. We will prove , that

$y(t) = x(t,\lambda_o)$. In equation (9) we can substituten to n_k with $k \to \infty$ and by virtue Lemma 2 we get

$$y(t) = \phi(0,\lambda_o) + \int_0^t f[\tau,y(\tau,\lambda_o), y(\tau-\Delta,\lambda_o),\lambda_o]\, d\tau$$

$y(t) = \phi(t,\lambda_o)$ for $t \in [-\Delta,0]$.

That mean , the function $y(t)$ is solution equations (1) , (2) with $\lambda=\lambda_o$ from uniqueness solution $x(t,\lambda_o)$ it should be that $y(t) = x(t,\lambda_o)$ for $t \in [0,T_o]$ .Thus , all uniformly convergent sequence $\{ x(t,\lambda_{nk}) \}$ in $[0,T_o]$ converge to $x(t,\lambda_o)$. Therefore it is possible choice neighborhood $U(\lambda_o)$ so that for all $\lambda \in U(\lambda_o)$ in interval $[0,T_o]$ be fulfilled inequality

$\| x(t,\lambda) - x(t,\lambda_o) \| < \eta$.

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الخلاصة : البرهنة على وجود حل لمعادلة تفاضلية من الدرجة الثانية والتي تعتمد على معَلمة وشرط ابتدائي .