SYMBOLS

<table>
<thead>
<tr>
<th>Notation</th>
<th>Dimensional Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = a length</td>
<td>L</td>
</tr>
<tr>
<td>c = the cohesion intercept</td>
<td>M L$^{-1}$ T$^{-2}$</td>
</tr>
<tr>
<td>$I_p$ = the plasticity index</td>
<td>-</td>
</tr>
<tr>
<td>P = a force</td>
<td>M L T$^{-2}$</td>
</tr>
<tr>
<td>u = the pore fluid pressure</td>
<td>ML$^{-1}$ T$^{-2}$</td>
</tr>
<tr>
<td>$\theta$ = an angle</td>
<td>Angle</td>
</tr>
<tr>
<td>$\mu_1$ = shear strength reduction factor related to time effects</td>
<td>-</td>
</tr>
<tr>
<td>$\mu_2$ = shear strength reduction factor related to fissuring</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma$ = the normal stress on a plane</td>
<td>M L$^{-1}$ T$^{-2}$</td>
</tr>
<tr>
<td>$\tau$ = the shearing stress on a plane</td>
<td>M L$^{-1}$ T$^{-2}$</td>
</tr>
<tr>
<td>$\phi$ = the angle of internal friction</td>
<td>Angle</td>
</tr>
<tr>
<td>$\psi$ = an angle related to cohesion</td>
<td>Angle</td>
</tr>
</tbody>
</table>

Subscripts etc. where not identified above
- parameter measured in terms of intergranular or effective quantities
- c = preconsolidation values
- cu = consolidated undrained parameters
- d = drained test parameters i.e. dissipated pore pressures
- e = so-called true parameters
- f = failure values
- max = maximum values
- n = normal to plane values
- nc = normally consolidated values
- o = overburden values
- u = unconsolidated undrained test parameters
- 1 = major principal values
- 2 = intermediate principal values
- 3 = minor principal values

1. INTRODUCTION

Stability analysis in geotechnical engineering includes all studies which attempt to determine whether or not the average shearing strength of soil over the assumed failure surface has a sufficient factor of safety against failure. Basically such studies consist of comparisons between all the forces which are or may act to cause failure and the resisting forces provided by the soil's shearing strength.

The shearing strength of a soil sample is generally defined as its maximum resistance to shearing forces. In special cases an ultimate, residual or post peak value is used. The peak and ultimate values are shown on the normal stress-strain plot of test results in Figure 1. The residual value (Skempton, 1964) may be taken as the value recorded on a presheared sample or on an intact sample which is subject to excessive shearing, such as in a ring shear apparatus (Bishop et al, 1971), where the soil particles can rearrange themselves into preferred orientations.

Normally stability problems are solved by approximating the stress-strain behaviour by an ideal rigid-plastic material as shown in Figure 2. Where such an assumption is made the stability can be expressed in terms of some definition of the shearing strength $\tau_f$ alone (this need not be a maximum value of $\tau$). In actual fact the failure strength varies with the normal stress on the failure plane. Coulomb is credited with being the first person to express this variable failure strength in terms of two engineering properties or parameters, namely:

1. cohesion (c), or the resistance due to the forces tending to bond or hold the soil particles together in a solid mass;
2. internal friction ($\phi$), or the rate of change of the resistance due to an increase of normal stress ($\sigma_n$) on the failure plane.

Coulomb's original shearing strength equation or Law is shown in Figure 3 and was expressed in terms of total stresses by

\[ \tau_f = c \%\sigma_n \tan \phi \]  

It should be noted that unless $\phi = 0$ the value of $\tau_f$ is not equal to the maximum shearing stress $\tau_{max}$. This relationship expresses both the variation of strength of a single sample to changing external stresses or the locus of test results using different samples of the same type soil. Another point worth noting is that the equation must have some limitation otherwise soil slopes with slope angles less than $\phi$ could theoretically be infinitely high. Particle crushing will obviously occur first.
2. EFFECTIVE STRESS CONCEPT
Unfortunately the values of $c$ and $\phi$ were not found to be constant values. Their usefulness was not universally accepted until Terzaghi's concept of effective intergranular stress ($\sigma'$) was postulated. This related the soil's behaviour to the difference between the total stresses ($\sigma$) and soil fluid pressure ($u$) by

$$\sigma' = \sigma - u$$  \hspace{1cm} (2)

In accordance with Terzaghi's concept the shearing strength of a soil may be expressed as

$$\tau_f = c' + \sigma_n \tan \phi'$$  \hspace{1cm} (3)

where the primes signify that the soil parameters are determined using effective stresses or equivalent intergranular stresses.

3. TRUE PARAMETER CONCEPT
Hvorslev (e.g. Gibson, 1953) working with multiple samples of remoulded soils, showed that the cohesion intercept was dependent on the moisture or water content of the failing soil sample. This led to what has become termed the true cohesion ($c_e$) and true angle of internal friction ($\phi_e$) which are determined as shown in Figure 4 and a restatement of equation (1) by

$$\tau_f = c_e + \sigma'_n \tan \phi_e$$  \hspace{1cm} (4)

$$c_e = \sigma'_{nc} \tan \psi_e$$  \hspace{1cm} (5)

where $\sigma'_{nc}$ is the effective intergranular pressure which would have to be applied normally to the failure plane of a normally consolidated sample of the same soil to give a failure water content of the same value as the failing sample, and $\psi_e$ is an angle which relates the true cohesion to $\sigma'_{nc}$. Hvorslev's proposals are useful as an academic model for soil strength but are normally not used in practice.

4. PRACTICAL CONSIDERATIONS
In nature soil has been affected by weathering and other phenomena so that fundamental relationships, such as suggested by Hvorslev, have not found general acceptance in practical engineering. In addition the shearing strength of soil is, like most other engineering material, dependent on factors such as creep and fatigue. For these and other more complex reasons the shearing strength parameters used in practice are generally based on simplifying conditions of determining the stability against immediate failure and (or) long term failure depending on the
nature of the problem.

In the analysis of immediate stability it is assumed that the soil has a very low permeability and the moisture content of the soil will remain unchanged during the course of the engineering works. For such conditions the soil is tested rapidly enough to ensure undrained conditions. Interpretation of the test results is then based on considering the soil as a single phase material much as most other engineering materials (i.e. steel, concrete and the like). Analysis is then performed by working in total stresses. The shearing strength equation for immediate, total stress or undrained stability analysis for a soil at a given moisture content

\[
\tau = c_n \% \sigma_n \tan \phi_n
\]  

(6)

where \( c_n \) = the undrained cohesion
\( \phi_n \) = the undrained angle of internal friction

\[ \phi_n = \frac{1}{2} \left( \tau - \sigma_n \right) \]  

(7)

This result is extensively used when dealing with immediate stability problems involving saturated or near saturated clays and relatively impermeable saturated silts for which, very simply (further corrections may be required prior to application) for the identical saturated samples

\[
\tau = c_u \% \sigma_u \tan \phi_u
\]  

(8)

For saturated samples with different moisture contents \( c_u \) would, of course, vary. Thus in a natural soil deposit \( c_u \) may vary with depth.

On the other hand where the permeability of the soil is high and rapid dissipation of pore fluid pressures occur, such as with clean sands and gravel, or in low permeability soils where a change in moisture content is likely during loading, the shearing strength is expressed in terms of effective intergranular stresses as given by equation (3). The parameters are sometimes referred to as drained (more logically dissipated pore fluid pressure)

\[
\tau = c_d \% \sigma_d \tan \phi_d
\]  

(9)

In a drained test the pore fluid pressures are zero (or used as the zero datum when not zero) so that

\[
c_d = c'
\]

(10)

\[
\sigma_d = \sigma_n (u = 0)
\]

(11)

\[ \phi_d = \phi' \]  

(12)

Thus equation (9) becomes identical to equation (3) or

\[
\tau = c' \% (\sigma_d \& u) \tan \phi'
\]

(13)

It should be clearly understood that the term "drained" refers to the dissipation of pore fluid pressures and not to the drainage under gravity of pore fluid from the soil. Basically what is being referred to is the open position of the drainage cock leading to the pore water of the soil. With the cock open pore water may (drain) enter or leave the sample to maintain zero pore pressure. A fully saturated soil specimen subject to a drained laboratory test remains fully saturated. In clean sands, gravels and normally consolidated clays \( c' \) is generally close to and assumed to be zero.

5. THEORY VERSUS APPLICATION

From a simple engineering mechanics point of view it is appropriate to develop theoretical engineering solutions in terms of Coulomb's expression for shearing strength (equation (1)) and then apply the result to practical problems in terms of short term undrained or total stress parameters (equation (6)) or long term drained or effective stress parameters (equation (13)).

6. RELATIONSHIP BETWEEN PRINCIPAL STRESSES AND FAILURE SHEARING STRESS

The relationship between the principal major and minor stress and failure shearing stress using the Coulomb failure criteria is illustrated by resolution of the forces on the element shown in Figure 5. (Note that the intermediate principal stress has no theoretical effect on the Coulomb failure criteria).

![Figure 5. Elemental stresses on random plane.](image)

Resolving forces (P) normal to any assumed plane making an angle \( \theta \) with the plane on which the major principal stress acts

\[
P_n = P_1 \cos \theta \% P_1 \sin \theta
\]

(14)

\[
\sigma_n = \frac{b}{\cos \theta} \% \sigma_1 \cos \theta \% \sigma_1 \sin \theta \tan \sin \theta
\]

(15)

\[ \tau = \sigma_2 \sin \theta \& \sigma_3 \cos \theta \]

(16)

where subscripts 1 and 3 refer to major and minor principal values, and b is a length over which P1 acts.

Resolving forces parallel to any assumed plane to find the shear force \( P_\parallel \) on the plane

\[
P_\parallel = P_1 \sin \theta \& P_3 \cos \theta
\]

(17)

\[ \tau = \frac{b}{\cos \theta} \cos \theta \% \sigma_1 \sin \theta \& \sigma_3 \cos \theta \]

(18)

\[ \tau = (\sigma_1 \& \sigma_3) \sin \theta \cos \theta \]

(19)

If the assumed plane is the failure plane and Coulomb's relationship is taken as being valid on the plane

\[
\tau = c_n \% \sigma_n \tan \phi
\]

(20)

Substituting equation (19) with suitable subscripts to indicate failure and equation (16) for \( \sigma_n \), in equation (20)

\[
(\sigma_1 \& \sigma_3) \sin \theta \cos \theta \% c \sin [\sigma_3 \% \sin \theta \tan \phi]
\]

(21)

\[
\sigma_1 \% c \% \sigma_3 \tan \phi
\]

(22)
The plane of least resistance would make this value of \( \sigma_i \), a minimum value to produce failure or

\[
\sin \theta \cos \theta \, \& \, \cos^2 \theta \, \tan \varphi \quad \text{Maximum} \tag{23}
\]

To find the location of \( \theta \) it is necessary to differentiate with respect to \( \theta \) and equate to zero.

\[
\frac{d}{d \theta} \left( \sin \theta \cos \theta \, \& \, \cos^2 \theta \, \tan \varphi \right) \quad 0 \tag{24}
\]

\[
\cos^2 \theta \, \sin^2 \theta \% 2 \cos \theta \, \sin \theta \, \tan \varphi \quad 0 \tag{25}
\]

\[
\& \, \cot 2 \theta \, \tan \varphi \tag{26}
\]

\[
90 \, \frac{\pi}{4} \% \, \tan \varphi \tag{27}
\]

Substituting back into equation (22) gives (at failure)

\[
\sigma_1 \cdot \sigma_2 \cdot N_v \% 2 \quad c \quad \sqrt{N_v} \quad \text{(28)}
\]

where

\[
N_v \% 1 \, \sin \varphi \quad \frac{1}{1 \, \& \, \sin \varphi} \quad \tan^2 \left( 45 \% \varphi \right) \quad \frac{1}{\tan^2 \left( 45 \% \varphi \right)} \tag{29}
\]

Also substituting equation (27) in equation (19)

\[
\tau' \left( \sigma_i \, \& \sigma_i \right) \sin \left( 45 \% \varphi \right) \cos \left( 45 \% \varphi \right) \tag{30}
\]

only when \( \varphi = 0 \) does the Coulomb failure shearing stress equal the maximum shearing stress. This may easily be seen on the Mohr circle shown in Figure 6. The Mohr circle is a useful method of verifying the above equations. It may be seen that the radius of the Mohr circle which must touch the failure locus shown in Figure 6 is (at failure)

\[
\sigma_1 \, \& \, \sigma_1 \quad \frac{\frac{c}{\tan \varphi}}{\% \, \sin \frac{\varphi}{2}} \sin \varphi \quad \text{(31)}
\]

which may be rearranged to give equation (28).

For the case of a saturated soil tested in an undrained condition \( \varphi = 0 \).

The total stress criterion using equation (31) then gives

\[
\tau' \quad c \quad \frac{\sigma_i \, \& \sigma_i}{\% 2} \tag{32}
\]

The other simple condition is for clean sands and gravels where \( c' = 0 \).

The effective stress criterion using equation (31) then gives

\[
\frac{\sigma_i \, \& \sigma_i}{\% 2} \, \quad N_v' \quad \text{(33)}
\]

7. MEASUREMENT OF SHEARING STRENGTH

The shearing strength of a soil can be determined in situ or in the laboratory. In situ tests are often preferred in the practice of engineering because great care and judgement are required in the sampling, transportation, storage and handling of laboratory samples prior to testing. Furthermore, cohesionless soils are badly disturbed during sampling and handling. Such disturbance makes correlation between laboratory testing and field performance questionable. Fortunately for granular soil \( c' \) may be taken as zero and field testing is then correlated with \( \varphi' \) only. The high permeability of most granular soil generally means that undrained failure is unlikely. For cohesive soils, however, the long term parameters cannot be satisfactorily determined in situ and these soils are often sampled and tested in the laboratory. Laboratory testing must also be relied on to determine the parameters of placed and compacted soils where testing of these soils is required.

Although \( c \) and \( \varphi \) are not true constants in practice they are generally regarded as constant at any given point (or depth) over the stress range likely to be encountered in the field problem being analyzed. Consequently, testing should be carried out at stress magnitudes appropriate to the solution and location being considered. For example in a \( \varphi = 0 \) analysis \( c_u \) is constant at a given depth but may vary, often linearly, with depth.

The values of \( c \) and \( \varphi \), if assumed constant may be determined by carrying out two or more (generally three is considered minimum) tests with different normal pressures acting on the plane of shear failure. If the shearing strength on the failure plane is measured directly, as in the shear box test shown in Figure 7, the shearing strength may be plotted directly against the normal stress on the induced failure plane to give \( c \) and \( \varphi \) as shown in Figure 3.

Alternatively where the external stresses are controlled, such as the principal stresses in the triaxial tests shown in Figure 8, the results may be plotted on a Mohr circle as shown in Figure 9. Once two or more failure circles are drawn a common tangent determines the values of \( c \) and \( \varphi \). Normally the triaxial test is done in compression with \( \sigma_1 = \sigma_2 \) however an extension test is sometimes done in research or expensive projects where warranted. In such cases \( \sigma_1 = \sigma_3 \). In either case the value of \( \sigma_3 \), has no theoretical effect on the Coulomb failure criteria, although in fact some differences have been noted (Bishop, 1966). Because of these differences a number of different laboratory testing equipment are available for research and special projects. The shear box and triaxial test equipment...
are the mainstay of a commercial laboratory particularly where both c and φ are required. A special form of triaxial test where \( \sigma_3 = 0 \) is known as the unconfined compression test.

Figure 8. Stresses acting on triaxial compression sample.

Figure 10. Experimental determination of c and φ.

Where either c or φ is assumed several tests are available both for in situ and laboratory testing. The most common in situ tests are the vane shear test shown in Figure 10 for use in soft clays to measure \( c_u (\phi_u = 0) \) and the standard penetration test shown in Figure 11 for use in sands where the number of blows of a standard sampling spoon is related to \( \phi' \) (\( c' = 0 \)). Other common field equipment includes the static or dynamic cone penetrometer for sands (Sanglerot, 1972) and the pressure meter (Baguelin et al, 1978) for stiff clays and soft rocks. Numerous more complex equipment are also available (e.g. the plate test and in situ shear box used by Marsland, 1971).

8. TYPES OF SHEAR TESTING

Three main types of tests are performed on soils dependent on the dissipation of pore pressures (termed drainage) from the specimens under test.

(A) Immediate or Undrained Test (also known as Quick Test): the samples are subject to an applied pressure (under conditions of no drainage) and as quickly thereafter sheared. Care is taken to prevent (in fact this is difficult) any dissipation of pore pressure since the results assume none has occurred. The test is most applicable to clays with low permeability where drainage is very slow and negligible if the test is performed quickly. If the soil samples being tested have the same stress history and are fully saturated then at failure \( \phi_u = 0 \) as shown in Figure 12. This is one way of establishing that a soil is fully saturated. Clays with fine sand lenses where cavitation of air from the pore fluid often occurs prior to failure and partially saturated soils with high degrees of saturation generally give low values of \( \phi_u \). The undrained immediate strength is also obtained in situ with the vane equipment. This test is most applicable to soft saturated soils. For such a result \( \phi = 0 \) is assumed.

Similarly in the laboratory when the unconfined compression test \( \sigma_3 = 0 \) is done \( \phi_u = 0 \) is assumed and \( (\sigma_1 - \sigma_3) = 2c_u \). For different sets of samples (i.e. from different depths) different values of \( c_u \) will obviously be obtained.

(B) Consolidated Undrained Tests: the samples are allowed to consolidate under an applied pressure. Once equilibrium is reached the drainage cock is closed and they are then sheared at constant moisture content under conditions of no drainage. The total stress parameters, obtained by using different consolidation pressures, \( c_{cu}, \phi_{cu} \) are of little value in practice so pore fluid pressure is generally measured during the test. Where the pore fluid pressures are measured the test must be performed slow enough to allow equalisation of pore pressure (normally 95%) since the sample may not be perfectly uniform in composition or external loading. This allows the effective or long term strength parameters \( c' \) and \( \phi' \) to be obtained as shown in Figure 13. Because of the slow rate of consolidation of soils having low permeability this test is generally preferred for obtaining the \( c' \) and \( \phi' \) values of clays. There is no equivalent in situ test. The value of \( c' \) and \( \phi' \) is affected by the consolidation pressure used in relationship to the soil's preconsolidation pressure. When the consolidation pressure greatly exceeds the preconsolidation pressure \( c' \) is normally observed to be close to zero which is a characteristic of normally consolidated remoulded clays.

(C) Drained Tests: the samples are allowed to consolidate as in the consolidated undrained test and then sheared slowly enough that any excess pore pressures dissipate completely (normally 95% dissipation is
9. SHEARING STRENGTH OF COHESIONLESS SOILS

The mobilisation of shearing resistance of cohesionless soils is illustrated by the stress-strain curves shown in Figure 1. For dense granular materials the resistance increases to a peak value and then decreases as the strain increases further to an ultimate value. During this post peak period the soil particles gradually loosen to a condition approximately the same as that of the granular material in the loose state. The value of \( \phi' \) for loose granular soil often being called the angle of repose. Several tests conducted at different confining pressures on loose material generally result in \( c' = 0 \). On dense material, as shown in Figure 14, the anomaly \( c' \neq 0 \) is often obtained due to the fact that \( \phi \) for dense cohesionless soils tends to decrease with increasing confining pressure. Indeed if the confining pressure is excessively high too severe particle crushing occurs. The decrease in \( \phi \) is important for high earth dams and the like. Under normal foundation engineering loads \( c' = 0 \) is generally assumed. Thus from one test result

\[
\phi' = \tan^{-1} \left( \frac{\tau_f}{\sigma_n'} \right)
\]

(34)

10. SHEARING STRENGTH OF COHESIVE SOILS

The selection of the shearing strength parameters appropriate to an engineering works built in cohesive soils is one of the most complex and difficult decisions facing the Geotechnical engineer. It is the intent herein
to deal only with the strength of cohesive soils in a simplistic manner. Cohesive soil properties need selection for three types of common analysis depending on the appropriateness of the problem

Table 1. Effect of Angularity and Grading on Peak Effective Friction Angle of Coarse Sand in Degrees (e.g. Sowers and Sowers, 1951).

<table>
<thead>
<tr>
<th>Shape and Grading</th>
<th>Symbol</th>
<th>Loose</th>
<th>Dense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounded, Uniform</td>
<td>SP</td>
<td>30</td>
<td>37</td>
</tr>
<tr>
<td>Rounded, Well Graded</td>
<td>SW</td>
<td>34</td>
<td>40</td>
</tr>
<tr>
<td>Angular, Uniform</td>
<td>SP</td>
<td>35</td>
<td>43</td>
</tr>
<tr>
<td>Angular, Well Graded</td>
<td>SW</td>
<td>39</td>
<td>45</td>
</tr>
</tbody>
</table>

Other factors such as anisotropy are also important but are generally not specifically determined in routine commercial investigations.

The undrained strength classification, often termed consistency, varies as shown in Table 4 from very soft to hard. These terms have no relationship to stress-strain properties since soft clays may, in some cases, be extremely brittle. It will be noted that the strength range covered by each higher consistency level is twice the range and values of the directly lower level.

Very soft and soft cohesive soils are generally intact and show little or no fissuring. Stiff to hard consistency clays, on the other hand, are very frequently fissured. Indeed as the strength increases some of the fissures may become classified as joints and these hard soils become indistinguishable from soft rocks. Clearly there is no clear cut division between these definitions and great care must be exercised in dealing with problems where fissures or joints are likely to be important in any stability analysis. Medium strength clays are likely to be either intact, showing no signs of a fissured pattern, or be clearly fissured. Their behaviour will be very much dependent on their physical nature.

The sensitivity of a cohesive soil is defined as the ratio of its undisturbed undrained strength to the remoulded undrained strength of the same soil. A sensitivity classification (Skempton and Northey, 1952) may be made and is shown in Table 5. It should be clearly understood that determination of a soil's sensitivity is not standardized. For the higher values of sensitivity there will normally be considerable differences in the values determined in the laboratory and in the field. The value determined by field vane is often less than that obtained by a laboratory triaxial or, more commonly, unconfined compression test. Further complications occur due to the fact that sensitive clays show different degrees of strength regain (known as thixotropic strength regain or thixotropy) after remoulding. Thus the time between remoulding and testing may have an appreciable effect on the measured value of sensitivity.

One of the best known type of clay deposits exhibiting high sensitivities are those deposited in a sea water environment and then leached by fresh water. Sensitivity in these deposits are sometimes related to residual salt content (Figure 17).
12. STRENGTH CHARACTERISTICS IN SOFT CLAYS

Very soft and soft clays are generally intact, rarely exhibiting any fissures. They have a liquidity index over 0.5 depending on their sensitivity and strength. Sensitive soils have a liquidity index close to or greater than 1.0 and insensitive soils a liquidity index less than 1.0. The liquidity index decreases as the strength (consistency) increases and for stiff and very stiff clays reduces to a value close to zero.

In a new, fresh or young deposit of uniform, fully consolidated soil the effective overburden pressure in most practical cases, increases relatively uniformly with depth (Skempton, 1948a) though, due to positive or negative artisan water pressure, this may be slower or faster than given by a static water table assumption. In such a normally consolidated soil deposit this uniform increase in effective overburden pressure \( \sigma'_o \) is associated with a decrease in moisture content of the soil and a uniform increase in undrained shearing strength \( \sigma'_u \). Note that if three samples of soil from the same depth are tested in an unconsolidated undrained test \( \phi'_u = 0 \), \( \sigma'_u = \) constant for the given depth. On the other hand if a series of such sets of tests are performed, each set from different depths, and the value of \( \sigma'_u \) for each depth is compared with the effective overburden \( \sigma'_o \) then (in practice \( \sigma'_u \) is obtained with an in situ vane test or unconfined compression test)

\[
\frac{\sigma'_u}{\sigma'_o} = \text{constant} \quad (35)
\]

This very important relationship is characteristic of young normally consolidated clay deposits. Where erosion of a young normally consolidated clay deposit has occurred the deposit becomes lightly overconsolidated. Because the expansion index of a soil is very much smaller than its compression index a small decrease in effective stress (due for example to erosion) has little effect on the soil's moisture content and on its undrained shearing strength \( \sigma'_u \) at a given depth. It does, however, have an effect on the effective overburden pressure. In lightly overconsolidated young deposits of clay equation (35) must be modified to

\[
\frac{\sigma'_u}{\sigma'_c} = \text{constant} \quad (36)
\]

where \( \sigma'_c \) = the preconsolidation pressure (in effective stresses)

As the amount of erosion increases the assumption of no change in \( \sigma'_u \) at a given depth becomes less valid and thus equation (36) decreases in correctness.

---

**Table 2. Typical Values of Frictional Angles for Granular Soils for \( \sigma'_n = 100 \text{ kPa} \)**

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Symbol</th>
<th>Loose</th>
<th>Medium</th>
<th>Dense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt (non-plastic)</td>
<td>ML, MH (PI&lt;6)</td>
<td>26 - 30</td>
<td>28 - 32</td>
<td>30 - 34</td>
</tr>
<tr>
<td>Uniform Sand and Silty Sand</td>
<td>SP, SM</td>
<td>26 - 30</td>
<td>30 - 34</td>
<td>32 - 36</td>
</tr>
<tr>
<td>Well Graded Sand</td>
<td>SW</td>
<td>30 - 34</td>
<td>34 - 40</td>
<td>38 - 46</td>
</tr>
<tr>
<td>Gravel</td>
<td>GW, GP, GM</td>
<td>32 - 36</td>
<td>36 - 42</td>
<td>40 - 48</td>
</tr>
</tbody>
</table>

---

**Table 3. Relationship for \( \phi'_u \) and In situ Tests in Clean Sands**

<table>
<thead>
<tr>
<th>Sand Density (SW,SP)</th>
<th>Relative Density</th>
<th>Standard Penetration Test N - blows/300 mm</th>
<th>Static Dutch-Cone Resistance ( q_u ) - MPa</th>
<th>Angle of Internal Friction ( \phi'_u ) Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Loose</td>
<td>&lt; 0.2</td>
<td>&lt; 4</td>
<td>2</td>
<td>&lt; 28</td>
</tr>
<tr>
<td>Loose</td>
<td>0.2 - 0.4</td>
<td>4 - 10</td>
<td>2 - 4</td>
<td>28 - 30</td>
</tr>
<tr>
<td>Medium</td>
<td>0.4 - 0.6</td>
<td>10 - 30</td>
<td>4 - 12</td>
<td>30 - 37</td>
</tr>
<tr>
<td>Dense</td>
<td>0.6 - 0.8</td>
<td>30 - 50</td>
<td>12 - 20</td>
<td>37 - 42</td>
</tr>
<tr>
<td>Very Dense</td>
<td>&gt; 0.8</td>
<td>&gt; 50</td>
<td>&gt; 20</td>
<td>&gt; 42</td>
</tr>
</tbody>
</table>

* Decreases 5\( \phi \) for non-plastic silts (ML,MH with PI < 6) and silty sands (SM)
* Increase 5\( \phi \) for gravel or gravel sand mixtures (GW,GP,GM)

---

**Table 4. Consistency of Saturated Clay Soils**

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Unconfined Compressive Strength (kPa)</th>
<th>Shearing Strength (kPa)</th>
<th>Standard Penetration Test N - blows/300 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Soft</td>
<td>&lt; 25</td>
<td>&lt; 12.5</td>
<td>&lt; 2</td>
</tr>
<tr>
<td>Soft</td>
<td>25 - 50</td>
<td>12.5 - 25</td>
<td>2 - 4</td>
</tr>
<tr>
<td>Medium/Firm</td>
<td>50 - 100</td>
<td>25 - 50</td>
<td>4 - 8</td>
</tr>
<tr>
<td>Stiff</td>
<td>100 - 200</td>
<td>50 - 100</td>
<td>8 - 15</td>
</tr>
<tr>
<td>Very Stiff</td>
<td>200 - 400</td>
<td>100 - 200</td>
<td>15 - 30</td>
</tr>
<tr>
<td>Hard</td>
<td>&gt; 400</td>
<td>&gt; 200</td>
<td>&gt; 30</td>
</tr>
</tbody>
</table>

**Table 5. Sensitivity of Clays**

(after Skempton and Northey, 1952).

<table>
<thead>
<tr>
<th>Classification</th>
<th>Strength Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insensitive</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Low Sensitivity</td>
<td>1 - 2</td>
</tr>
<tr>
<td>Medium Sensitivity</td>
<td>2 - 4</td>
</tr>
<tr>
<td>Sensitive</td>
<td>4 - 8</td>
</tr>
<tr>
<td>Extra Sensitive</td>
<td>8 - 16</td>
</tr>
<tr>
<td>Quick</td>
<td>&gt; 16</td>
</tr>
</tbody>
</table>
Normally consolidated clays may exhibit a continuing consolidation at approximately constant effective stress (known as secondary consolidation). This causes a slow decrease in moisture content and a slow increase in shearing strength. This rate of increase in strength has been shown by Bjerrum (1967) to be dependent on the effective overburden pressure such that in some aged normally consolidated clays both equations (35 and 36) are valid (the constants are different). For normally consolidated deposits aged and modified by secondary consolidation only, the induced preconsolidation pressure, known as quasi-preconsolidation, increases uniformly with depth so that

\[ \frac{\sigma'_c}{\sigma'_o} = \text{constant} \]  

(37)

Obviously both equations (35 and 36) will be valid although the constants will be different. An example of a deposit exhibiting this quasi-preconsolidation due to aging is shown in Figure 18. For glacial and post glacial clays the constants of equations (35 and 36) generally range with the limits shown in Figure 19.

Normally consolidated deposits may also be aged by other factors such as the bonding of particles by chemical action. These soils may also show a relationship expressed by equation (36) and it is common engineering practice in normally consolidated and lightly overconsolidated clay to check the validity of both equations (35) and (36).

Very soft and soft clay deposits of saturated or near saturated soils are generally intact (do not exhibit fissures). Their short term undrained strength is based on \( \phi_u = 0 \), \( c_u = 0 \) with a correction for the difference in rate of testing (Casagrande and Wilson, 1951), soil anisotropy and other factors. The correction to be made to the value of \( c_u \) has been the subject of an extensive study by Bjerrum (1973). The correction factor was correlated, with sufficient accuracy for practical purposes, to the soil's Plasticity Index (I_p). This is shown in Figure 20 and may be expressed by
where the engineering undrained stability strength is given by

$$\tau' \mu_1 c_u$$  \hspace{0.5cm} \text{(39)} \tag{39}

It is recommended that $\mu_1$ should not be taken as greater than 1. LaRochelle et al (1974) have suggested that the value of $\mu_1$ may be obtained by using the post peak strength (termed, by them, the undrained residual) recorded at relatively low strains since this strength could drop drastically on remoulding. Limited data is presently available on this procedure.

The long term or effective stress stability of very soft and soft normally consolidated and lightly overconsolidated clays are based on the peak values of $c'$ and $\phi'$. Great care must be taken to determine the parameters over the stress range applicable to the field since in aged clays the parameters change quite abruptly if the soils are subject to in situ shear stresses close to failure or close to or above the preconsolidation pressure.

According to Bjerrum (1973) the correction for rate of loading is similar in magnitude to that expressed by equation (39).

$$\tau' \mu_1 (c' \% \sigma' \tan \phi')$$  \hspace{0.5cm} \text{(40)} \tag{40}

Gibson (1953) has given a tentative relationship for $\phi'$ of normally consolidated remoulded clays, Figure 21. Further data confirming the guide has been presented by Kenney (1959).

Sensitive soils are particularly prone to major changes in their long term strength parameters if loaded close to their preconsolidation pressures along with major changes in their settlement parameters. These changes may result in large long term deformations due to creep and (or) consolidation. Sensitive soils may be treated, in terms of stability analysis, like insensitive soils except that higher factors of safety should be required particularly where failures are likely to cause large and possible catastrophic deformations.

13. STRENGTH CHARACTERISTICS OF STIFF TO HARD CLAYS

Intact, non-fissured very stiff to hard clays may be considered such a rarity that where they are reported it is recommended that a careful check be undertaken or they be regarded as fissured. Fissured clays exhibit weaknesses along the fissures which in random testing depends on the size of specimen. Where selected testing is done results similar to those shown in Figure 22 are obtained. For random testing the strength decreases as the specimen size increases since larger specimens are more likely to include more representative fissures of field scale. Considerable scatter must be expected in a testing program and this is illustrated in Figure 23 which shows the strength profile for a deep deposit of very stiff to hard London clay.

Provided that, in the field, water cannot enter the fissures and cause rapid softening a total stress or undrained analysis in fissured clay may be performed in the same way as any other undrained strength analysis except that a factor ($\mu_2$) for fissures needs to be included.

Thus

$$\tau' \mu_1 \mu_2 c_u$$  \hspace{0.5cm} \text{(41)} \tag{41}

Little information is available on the variation in values of $\mu_2$ for different soil types. Generally local or regional information exists where a record of case histories has been kept by local engineers. Quite often there are Governmental records since Governments are the major clients dealing with public works.
soil adjacent to the fissures decreases and also controls the 'global' strength of the deposit. Under such conditions the strength drops to values closely associated with the residual strength parameters. This may occur quite rapidly particularly where the prevention of fissure opening cannot be or is not engineered.

For long term stability involving increases in compressive lateral forces it is reasonable to assume that any fissures would remain closed and the long term stability may be based on peak effective stress parameters modified for time effects and fissure spacing.

\[
\tau = \mu_1 \mu_2 (c' - \%\sigma''_n \tan \phi')
\]

On the other hand when considering the long term stability involving decreases in compressive lateral forces the possibility of fissure opening is of major concern. In such circumstances the long term strength should be based on residual effective stress parameters. Since these should be obtained at very slow rates of loading and along presheared failure planes, as illustrated in Figure 24, no time or fissure coefficients are necessary and thus

\[
\tau = c' - \%\sigma''_n \tan \phi'
\]

A rough guide, shown in Figure 25, to the value of \(\phi'\), has been presented by Skempton (1964) who found the value to be very much affected by the clay content of a soil.

14. STRENGTH CHARACTERISTICS OF MEDIUM TO STIFF CLAYS

Medium to stiff intact clays should be treated in much the same way as soft and very soft clays. On the other hand medium to stiff fissured clays should be treated in much the same way as stiff to hard clays. The scatter in strength data may be expected to increase as the strength increases and as the overconsolidation ratio increases. In a strength depth profile a general curve in the data, as shown in Figure 26, may be expected due to the reduction in strength with loss of overburden or increase in overconsolidation ratio.

15. SOILS WITH STRUCTURALLY UNSTABLE SOIL FABRIC

Although not stated so far in the discussion of shearing strength it has been implicitly assumed that the soils under discussion have a soil fabric structure which is sufficiently stable that it may be simplistically modelled. These soils are characteristic of those found in post glacial regions and in alluvial deposits. They are composed in the main of relatively inert, natural or artificially compacted materials which are over 90% saturated. Of more complexity are the soils of diverse characteristics occurring in climatic regions which produce occasional or continuing aridity. Present fundamental knowledge of these soils is limited but has been summarized by Aitchison and Tokar (1973) under the term known as 'structurally unstable soils'. There is no clear definition of a 'structurally unstable soil' however such soils have stress-strain responses which cannot be quantified simply in terms of the applied stress level and an applied stress dependent pore fluid response. The definition includes high void ratio sands, silts and clays which are unsaturated and lightly cemented and which collapse or expand on wetting or leaching in the unloaded or lightly loaded condition. They also include those high void ratio soils which may be subject to dynamic loading and respond by liquefaction.

The solution to a problem on a 'structurally unstable soil' generally takes on one of the following

(a) to design for collapse (swelling) as quantified
(b) to design for avoidance of collapse (swelling) by precluding the operation of the triggering mechanism
(c) to induce collapse (swelling) prior to construction
(d) to apply soil stabilization processes to modify or remove the susceptibility of the soil to collapse (swelling).

At the present time a practical scientific approach to the problems involving 'structurally unstable soils' has not been developed which is generally accepted. This is mainly due to the fact that the hazard to life and injury is largely absent in these soils and thus research funding has been noticeably minor. Nevertheless it should be understood that property damage due to collapsing and swelling soils in terms of damage to houses, buildings, roads and pipelines is conservatively estimated for the U.S.A. by Jones and Holtz (1973) to be more than twice that due to damage from floods, hurricanes, tornadoes and earthquakes.

These soils will not be dealt with herein but the interested reader is referred to the State-of-the-Art statements in various Proceedings of the International Society of Soil Mechanics and Foundation Engineering Conferences as a suitable starting point.

16. REFERENCES
