The Best Hydraulic Section:

It is known that the conveyance of a channel section increases with increase in the hydraulic radius or with decrease in the wetted perimeter.

Section \( \propto R = \frac{A}{p} \)

*For Rectangular Section:

\[ P = B + 2y \]

\[ \frac{dp}{dy} = \frac{-A}{y^2} + 2 \]

\[ -\frac{A}{y^2} + 2 = 0 \]

\[ -\frac{A}{y^2} = -2 \]

\[ A = 2y^2 \quad \text{By} = 2y^2 \quad \text{B} = 2y \]

\[ R = \frac{A}{p} = \frac{By}{B+2y} = \frac{2y^2}{2y+2y} = \frac{2y^2}{4y} = \frac{y}{2} \]

\[ R = \frac{1}{2}y \quad \text{For rectangular section} \]

*For Trapezoidal Section:

\[ P = b + 2y\sqrt{1 + z^2} \]

\[ \frac{dp}{dy} = -\frac{A}{y^2} - z + 2\sqrt{1 + z^2} \]

\[ [-\frac{A}{y^2} - z + 2\sqrt{1 + z^2} = 0] \times y^2 \]

\[ -A - zy^2 + 2y^2\sqrt{1 + z^2} = 0 \]

\[ A = by + zy^2 \quad \text{Or} = (b + zy) y \]

\[ P = b + 2y\sqrt{1 + z^2} \]

\[ b = \frac{A}{y} - zy \]
\[-by - zy^2 - zy^2 + 2y^2\sqrt{1 + z^2} = 0\]

\[-by - 2zy^2 + 2y^2\sqrt{1 + z^2} = 0 \quad \rightarrow \quad -2zy^2 + 2y^2\sqrt{1 + z^2} = by\]

\[b = 2y\sqrt{1 + z^2} - 2zy\]

\[P = b + 2y\sqrt{1 + z^2}\]

\[\frac{dp}{dz} = -y + y(1 + z^2)^{-1/2} \times 2z\]

\[-y + y(1 + z^2)^{-1/2} \times 2z = 0\]

\[-y + \frac{2zy}{\sqrt{1 + z^2}} = 0\]

\[2y\sqrt{1 + z^2} + 2zy = 0\]

\[2z = \sqrt{1 + z^2}\]

\[4z^2 = 1 + z^2 \quad \rightarrow \quad 4 = \frac{1}{z^2} + 1 \quad \rightarrow \quad z^2 = \frac{1}{3}\]

\[Z = \frac{1}{\sqrt{3}}\]

\[\tan \theta = \sqrt{3} \quad \rightarrow \quad \theta = 60^\circ\]

\[b = 2y\sqrt{1 + \left(\frac{1}{\sqrt{3}}\right)^2} - \frac{2y}{\sqrt{3}} = 0\]

\[b = \frac{4y}{\sqrt{3}} - \frac{2y}{\sqrt{3}} = \frac{2y}{\sqrt{3}} \quad \rightarrow \quad b = \frac{2y}{\sqrt{3}}\]

\[R = \frac{A}{P} = \frac{by + zy^2}{b + 2y\sqrt{1 + z^2}}\]

\[R = \frac{\frac{2y^2}{\sqrt{3}} + \frac{y^2}{\sqrt{3}}}{\frac{2y}{\sqrt{3}} + \frac{y}{\sqrt{3}}} = \frac{3y^2}{6y} = \frac{1}{2}y\]

\[R = \frac{1}{2}y\]

Notes:
When the circular section used:
\[P = 2\pi r\]
\[A = \pi r^2 \quad \rightarrow \quad R = \frac{A}{P}\]
Example:
A trapezoidal channel carrying 11.34 m$^3$/sec is built with non erodible bed having aslope of 0.0016 and $n = 0.025$ proportion the section dimension by using best hydraulic section.

Solution:
$$Q = \frac{1}{n} R^{2/3} S^{1/2} A$$

$$R = \frac{1}{2} y$$

$$A = by + zy^2$$

$$b = \frac{2y}{\sqrt{3}}$$

$$b = \frac{2y^2}{\sqrt{3}} + \frac{1}{\sqrt{3}} y^2 = \frac{3y^2}{\sqrt{3}} = \sqrt{3} y^2$$

$$11.34 = \frac{1}{0.025} \left(\frac{1}{2} y\right)^{2/3} (0.0016)^{1/2} (\sqrt{3} y^2)$$

$Y = 2m$

$B = 2.31m$

$Z = \frac{1}{\sqrt{3}} = 0.577m$

*Control of Flow:*

The control of flow in an open channel is defined as the establishment of a definite flow condition in the channel or a definitive relationship between the stage and discharge of the flow. When the control of flow is achieved at a certain section of the channel this section is called a control section.

Solution:
always stage above sea surface = 220m or 230m

$$y_c = \frac{3q^2}{g}$$
At the critical state of flow a definitive stage – discharge relationship as shown in figure:

1) Subcritical flow ($y_1 > y_c$)

```
\begin{align*}
\text{Se} &< \text{Sc} \\
\text{mild} &
\end{align*}
```

```
\begin{align*}
\text{Super critical flow} \\
\text{yc} > y_2
\end{align*}
```

2) Mild flow

```
\begin{align*}
\text{Se} &> \text{Sc} \\
\text{Steep flow}
\end{align*}
```

3) Free Flow

```
\begin{align*}
\text{yn} > \text{yc} \\
\text{mild} \\
\text{yn} &< \text{yc} \\
\text{S}_0 &< \text{Sc}
\end{align*}
```

```
\begin{align*}
\text{yn} &> \text{yc} \\
\text{0.7yc} &
\end{align*}
```

```
\begin{align*}
\text{S}_0 &> \text{Sc} \quad \text{“Steep flow”}
\end{align*}
```

```
\begin{align*}
\text{yn} &< \text{yc} \\
\text{S}_0 &> \text{Sc} \quad \text{“Steep flow”}
\end{align*}
```
4) 

yc = control flow

5) 

Example:
A rectangular channel $B=1.5m$, $Q=900L/s$, the depth of flow before the hump is 1m and $\Delta z=200mm$, compute the depth of flow above the hump and calculate the required $\Delta z$ cause the critical flow above it.

Solution:

\[
y_1 + \frac{v_1^2}{2g} = y + \frac{v^2}{2g} + \Delta z
\]

$V_1 = \frac{Q}{A_1} = 0.9 \text{ m}^3/\text{s} / (1.5 \times 1) = 0.6 \text{ m/s}$

$V = \frac{Q}{(1.5 \times y)} = 0.9 / 1.5 y$
\[
y = 0.765m
\]
\[
y_c = \sqrt[3]{\frac{q^2}{3g}} = \sqrt[3]{\frac{(0.9/1.5)^2}{9.81}} = 0.3323m
\]
\[
V_c = \frac{Q}{Ac} = \frac{0.9}{(1.5 \times 0.332)} = 1.807 m/s
\]
\[
1 = \left(\frac{0.6}{2g}\right)^2 + 0.332 + \left(\frac{1.807}{2g}\right)^2 + \Delta z
\]
\[
\Delta z = 0.52 m
\]

*Non uniform flow: (varied flow)*

For design of open channel and analysis of their performance the engineer must be able to predict forms and positions of water surface profiles of varied flow and acquire some facility in their calculation.

\[
\begin{align*}
S_0 \, dx + y + \frac{v^2}{2g} & = y + dy + \frac{v^2}{2g} + d\left(\frac{v^2}{2g}\right) + S_e \, dx \\
\frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) & = S_0 - S_e \\
\frac{dy}{dx} & = \frac{S_0 - S_e}{1 + \frac{d}{dy}\left(\frac{v^2}{2g}\right)} \quad \text{surface profile for varied flow} \\
\frac{d}{dy}\left(\frac{v^2}{2g}\right) & = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) \\
& = \frac{Q^2}{gA^3} \\
\frac{dy}{dx} & = \frac{S_0 - S_e}{1 + \frac{Q^2f^2}{gA^2}} = \frac{S_0 - S_e}{1 - f^2}
\end{align*}
\]
\[
\frac{dy}{dx} = \frac{s_0 - se}{1 - Fr^2}
\]
in non uniform

\[y = \frac{A}{T} = \text{hydraulic depth}\]

T = Top surface width

\[Fr = \sqrt{\frac{v}{yg}} = \frac{v}{\sqrt{gh}}\]

Depth decrease with distance

Aproaching

To channel bottom \(\frac{dy}{dx} < 0\)

Depth increase with distance

Direction to up

Driving from channel bottom

\[\frac{dy}{dx} > 0\]

“general form”

\[Z_1 + y_1 + \frac{v_1^2}{2g} = Z_2 + y_2 + \frac{v_2^2}{2g} + h_f\]

\[\frac{Z_1 - Z_2}{\Delta x} = S_0 ,\quad S_e = \frac{h_f}{\Delta x}\]

\[(y_1 + \frac{v_1^2}{2g}) - (y_2 + \frac{v_2^2}{2g}) = S_e \Delta x - S_0 \Delta x\]

\[\Delta x = \frac{E_1 - E_2}{S_e - S_0}\]

\[V_{ave} = \frac{1}{n} R_{ave}^{2/3} S_{ave}^{1/2}\]

\[S_e = \left(\frac{v_{ave} \times n}{R_{ave}^{7/3}}\right)^2\]
\[ S_0 = \text{constant} \]
\[ V_{ave} = \frac{v_1 + v_2}{2} \]
\[ R_{ave} = \frac{R_1 + R_2}{2} \]

Example:
A flow rate 10 m\(^3\)/s occurs in a rectangular channel \(n = 0.013\), 6m in wide, slope =0.0001, at point in this channel the depth is 1.5m. how far from this point will the depth be 1.65m.
Solution:

<table>
<thead>
<tr>
<th>(y)</th>
<th>(A = B \times y)</th>
<th>(P)</th>
<th>(R_{ave})</th>
<th>(V_{ave})</th>
<th>(V^2/2g)</th>
<th>(\Delta E) (E=(y + v^2/2g))</th>
<th>(\Delta X (m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>9</td>
<td>9</td>
<td>1</td>
<td>1.111</td>
<td>0.0629</td>
<td>1.5629</td>
<td></td>
</tr>
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<td>1.55</td>
<td>9.3</td>
<td>9.1</td>
<td>1.022</td>
<td>1.075</td>
<td>0.0589</td>
<td>1.6089</td>
<td>465</td>
</tr>
<tr>
<td>1.60</td>
<td>9.6</td>
<td>9.2</td>
<td>1.043</td>
<td>1.042</td>
<td>0.0553</td>
<td>1.6553</td>
<td>570</td>
</tr>
<tr>
<td>1.65</td>
<td>9.9</td>
<td>9.3</td>
<td>1.0645</td>
<td>1.01</td>
<td>0.05199</td>
<td>1.7099</td>
<td>709</td>
</tr>
</tbody>
</table>

\(\Delta x = 1744m\)

Also we find
\[ S_e = \left( \frac{V_{ave} \times n}{R_{ave}^{2/3}} \right)^2 \]

Example:
When the depth of water just upstream from a frictionless broad-crested weir of 0.6m height is 0.9m, what is the flow rate per meter of crest length that can be expected.

\[ q = vy \]
\[ \frac{v^2}{2g} + y_1 = \frac{v_1^2}{2g} + y_2 \]
\[ \frac{(\frac{q}{y_1})^2}{2g} + y_1 = \frac{(\frac{q}{y_2})^2}{2g} + y_2 \]
\[ y_c = \sqrt[3]{\frac{q^2}{g}} \]
\[ y_2 = y_c \]
\[ y_1 = 0.9m \]
\[ \frac{v_1^2}{2g} \]
\[ \frac{v_2^2}{2g} \]

0.6m
\[ \frac{y_1^3}{2gy_1^2} + y_1 = \frac{y_c^3}{2gy_c^2} + y_c + 0.6 \]

\[ \frac{y_1^3}{2y_1^2} + y_1 = \frac{y_c}{2} + y_c + 0.6 \]

\[ \frac{y_c^3}{2(0.9)^2} + 0.9 = \frac{y_c}{2} + y_c + 0.6 \]

\[ \frac{y_1^3}{2 \times 0.81} + 0.3 = 1.5 y_c \]

\[ y_c = 0.2 m \]

\[ Q = 0.28 \text{ m}^3/s.m \]