D.C GENERATORS

1.1 Generator principle

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power). Induced e.m.f is produced in it according to Faraday's law of electromagnetic induction. This e.m.f cause a current to flow if the conductor circuit is closed.

Hence, two basic essential parts of an electrical generator are:

a) Magnetic field.

b) Conductor or conductors which can move as to cut the flux.

Generators are driven by a source of mechanical power, which is usually called the prime mover of the generator (steam turbine, diesel engine, or even an electric motor).

1.2 Simple loop generator

In fig.(1.1) is shown a single turn rectangular copper coil (AA'BB') rotating about its own axis in a magnetic field provided by either permanent magnets or electromagnets. The two end of the coil are joined to two slip-rings which are insulated from each other and from the central shaft. Two collecting brushes (carbon or copper) press against the slip-rings. The rotating coil may be called (armature) and the magnets as (field magnets).

One way to generate an AC voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field, fig. (1.1). (slip rings and brushes connect the coil to the load). The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut (faraday's law), and its polarity is dependent on the direction the coil sides move through the field.
The direction of an induced e.m.f can be predetermined by using 
**Flemings Right-hand** rule (often called the gene**R**ator rule) fig.(1.2).

**First finger - Field**
**Thu**Mb – **M**otion
**s**Econd finger – **E.m.f**

Since the rate of cutting flux varies with time, the resulting voltage will also vary with time. For example in (a), since the coil sides are moving parallel to the field, no flux lines are being cut and the induced voltage at this instant (and hence the current) is zero. (this is defined as the 0° position of the coil). As the coil rotates from the 0° position, coil sides AA’ and BB’ cut across flux lines, hence, voltage builds, reaching a peak when flux is cut at the maximum rate in the 90° position as in (b). Note the polarity of the voltage and the direction of current. As the coil rotates further, voltage decrease, reaching zero at the 180° position when the coil sides again move parallel to the field as in (c). At this point, the coil has gone through a half-revolution. During the second half-revolution, coil sides cut flux in directions opposite to that which they did in the first half revolution, hence, the polarity of the induced voltage reverses. As indicated in (d), voltage reaches a peak at the 270° point, and, since the polarity of the voltage has changed, so has the direction of current. When the coil reaches the 380° position, voltage is again zero and the cycle starts over. Fig. (1.1) shows one cycle of the resulting waveform. Since the coil rotates continuously, the voltage produced will be a repetitive, periodic waveform as you saw in fig. (1.1). E.m.f. generated in one side of loop= $Blv \cdot \sin \phi$, and total e.m.f. generated in loop= $2 \times Blv \cdot \sin \phi$ (volts), where

(B): flux density in (teslas), (l): length in (meters), (v): the conductor velocity, is measured in meters per second.
Fig.(1.1) Generating an AC voltage. The 0° position of the coil is defined as in (a) where the coil sides move parallel to the flux lines.
1.3 Construction of DC Generators

The parts of a simple DC generator are shown in fig.(1.3). The principle of operation of a DC generator is similar to that of the AC generator, which was discussed previously. A rotating armature coil passes through a magnetic field that develops between the north and south polarities of permanent magnets or electromagnets. As the coil rotates, electromagnetic induction causes current to be induced into the coil. The current produced is an alternating current. However, it is possible to convert the alternating current that is induced into the armature into a form of direct current. This conversion of AC into DC is accomplished through the use of a commutator. The conductors of the armature of a DC generator are connected to commutator segments.

The commutator shown in fig. (1.3) has two segments, which are insulated from one another and from the shaft of the machine on which it rotates. An end of each armature conductor is connected to each commutator segment. The purpose of the commutator is to reverse the armature coil connection to the external load circuit at the same time that the current induced in the armature coil reverses. This causes DC at the correct polarity to be applied to the load at all times.

![Fleming's Right-hand rule](image)

Figure (1.2) Fleming's Right-hand rule.
Fig. (1.3) Simple drawing of the basic parts of DC generator

Fig. (1.4) Output waveforms of a DC generator.

(A) Pulsating DC developed by a simple single-coil generator.

(B) Pure DC developed by a more complex generator using many turns of wire and many commutator segments.
1.4 Armature Windings

Armature windings can be divided into two groups, depending on how the wires are joined to the commutator. These are called (lap windings) and (wave windings). These windings will be examined individually below, and their advantage and disadvantage will be discussed.

1.4.1 The Lap Winding

The simplest type of winding construction used in modern DC machines is the simplex lap winding. A simplex lap winding is a rotor (armature) winding consisting of coils containing one or more turns of wire with the two end of each coil coming out at adjacent commutator segments fig. (1.5). The number of current paths in a machine is:

\[ a = mp \]

Where:
- \( a \): number of current path in the rotor.
- \( m \): plex of the windings (1,2,3,etc….)
- \( p \): number of poles on the machines.

Lap wound generators produce high current, low voltage output.

Fig. (1.5) Lap-wound DC machine.
1.4.2 The Wave Winding (الف تموجي)

The wave winding is an alternative way to connect the rotor (armature) coils to the commutator segments. Fig. (1.6) shows a simple wave winding. In this simplex wave winding, every other rotor coil connects back to a commutator segment adjacent to the beginning of the first coil. Therefore, there are two coils in series between the adjacent commutator segments. Furthermore, since each pair of coils between adjacent segments has a side under each pole face, all output voltage are the sum of the effects of every pole, and there can be no voltage imbalances. wave windings, generators produce higher-voltage, low current outputs, since the number of coils in series between commutator segments permits a high voltage to be built up more easy than with lap windings.

\[ a = 2m \]

multiplex wave

Fig.(1.6) Wave wound DC machine.
1.5 Electromotive Force (e.m.f) Equation

The induced voltage in any given machine depends on three factors:

1. **The flux \( \phi \) in the machine**
2. **The speed \( \omega \) of the machine's rotor.**
3. **A constant depending on the construction of the machine.**

The voltage out of the armature of a real machine is equal to the number of conductors per current path time the voltage on each conductor. The voltage in any single conductor under the pole faces was previously shown to be:

\( e_{in} = Blv \)

Where \( B \), the flux density, is measured in teslas, \( l \), the length of conductor in the magnetic field, is measured in meters, and \( v \), the conductor velocity, is measured in meters per second.

The voltage out of the armature of a real machine is thus:

\[ E_s = \frac{ZBlv}{a} \]

Where \( z \) is the total number of conductors and \( a \) is the number of current paths. The velocity of each conductor in rotor can be expressed \( v = r\omega \), where \( r \) is the radius of the rotor, \( \omega \), angular velocity in radians per second, so

\[ E_s = \frac{ZBlr\omega}{a} \]

This voltage can be re-expressed in a more convenient form by noting that the flux of a pole is equal to the flux density under the pole times the pole's area:

\[ \phi = BA_p \]

The rotor of the machine is shaped like a cylinder, so its area is:

\[ A = 2\pi rl \]
If there are \( P \) poles on the machine, then the portion of the area associated with each pole is the total area \( A \) divided by the number of poles \( P \):
\[
A_p = \frac{A}{P} = \frac{2\pi rl}{P}
\]

The total flux per pole in the machine is thus
\[
\phi = BA_p = \frac{B(2\pi rl)}{P} = \frac{2\pi rlB}{P}
\]

Therefore, the internal generated voltage in the machine can be expressed as:
\[
E_A = \frac{Zrl\omega \phi}{a} = \left( \frac{ZP}{2 \pi a} \right) \left( \frac{2\pi rlB}{P} \right) \omega
\]

Finally,
\[
E_A = K \phi \omega
\]

Where
\[
K = \frac{ZP}{2 \pi a}
\]

In modern industrial practice, it is common to express the speed of a machine in revolutions per minute instead of radians per second. The conversion from revolutions per minute to radians per second is:
\[
\omega = \frac{2\pi}{60} n
\]

So the voltage equation with speed expressed in terms of revolutions per minute is
\[
E_A = \bar{K} \phi n
\]

Where
\[
\bar{K} = \frac{ZP}{60a}
\]
1.6 Types of D.C Generators:

D.C Generators are classified according to the way in which a magnetic field is developed in the stator of the machine. Thus, there are three basic classification DC generators (1) permanent-magnet field (2) separately-excited field and (3) self-excited field.

1) permanent-magnet field

permanent-magnet DC machines are widely found in a wide variety of low-power applications. The field winding is replaced by a permanent magnet, resulting in simpler construction. Chief among these is that they do not require external excitation and its associated power dissipation to create magnetic fields in the machine the space required for the permanent magnets may be less than that required for the field winding, and thus machine may be smaller, and in some cases cheaper, than their externally excited counter parts. Notice that the rotor of this machines consists of a conventional DC armature with commutator segments and brushes.

Fig. (1.7) Cross section of a typical permanent-magnet machines.
2) Separately-excited field

Separately-excited generators are those whose field magnets are energized from an *independent external source* of DC current. It is shown diagrammatically in fig (1.8).

![Figure 1.8](image_url)

*Fig. (1.8) Simplified illustration of a separately DC generator.*
3) Self-excited field

Self-excited generators are those whose field magnets are energized by the current produced by the generators themselves. Due to residual magnetism, there is always present some flux in poles. When the armature is rotated, some e.m.f and hence some induced current is produced which is partly or fully passed through the field coils thereby strengthening the residual pole flux.

There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to armature.

(a) Shunt -Wound

The field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them fig. (1.9).

![Fig. (1.9) Simplified illustration of a self-excited, shunt wound DC generator.](image-url)
(b) Series-Wound

In this case, the field windings are joined in series with the armature conductors fig. (1.10). As they carry full load current, they consist of relatively few turn of thick wire or strips. Such generators are rarely used except for special purposes.

Fig. (1.10) Simplified illustration of a self-excited, series-wound DC generator.
(c) Compound –Wound

The compound-wound D.C generator has two sets of field windings. One set is made of low-resistance windings and is connected in series with the armature circuit. The other set is made of high-resistance wire and is connected in parallel with the armature circuit. A compound-wound D.C generator is illustrated in figure (1.11), can be either short-shunt or long-shunt. In a compound generator, the shunt field is stronger than the series field. When series field aids the shunt field, generator is said to be cumulatively-compounded. On the other hand if series field opposes the shunt field, the generator is said to be differentially compounded. Various types of DC generators have been shown separately in fig. (1.12).

![Diagram of Compound-Wound DC Generator](image)

Fig. (1.11) Simplified illustration of a compound- wound DC generator.
Example (1.1): A four-pole generator, having lap-wound armature winding has 51 slot, each slot containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 r.p.m assuming the flux per pole to be 7 mWb.?

Solution

\[ E_s = \frac{Zn\phi}{60} \left( \frac{P}{a} \right) \]

\[ \phi = 7 \times 10^{-3} \text{ Wb}, \ Z = 51 \times 20 = 1020 \]

\[ a = P = 4 \text{ (lap-wound)} \]

\[ E_s = \frac{7 \times 10^{-3} \times 1020 \times 1500}{60} \left( \frac{4}{4} \right) = 178.5 \text{ V} \]
Example (1.2): A shunt generator delivers 450A at 230 V and the resistance of the shunt field and armature are 50 Ω and 0.03 Ω respectively. Calculate the generated e.m.f.?

Solution:

Current through shunt field winding is

\[ I_f = \frac{230}{50} = 4.6 \text{ A} \]

∴ Armature current

\[ I_a = I_L + I_f \]

\[ = 450 + 4.6 = 454.6 \text{ A} \]

Armature voltage drop

\[ I_a R_a = 454.6 \times 0.03 = 13.6 \text{ V} \]

Now,

\[ E_g = \text{terminal voltage} + \text{armature drop} \]

\[ E_g = V + I_a R_a \]

∴ e.m.f generated in the armature

\[ E_g = 230 + 13.6 = 243.6 \text{ V} \]
Example (1.3): An 8-pole D.C shunt generator with 778 wave-connected armature conductors and running at 500 r.p.m. supplies a load of 12.5Ω resistance at terminal voltage of 250 V. The armature resistance is 0.24Ω and the field resistance is 250Ω. Find the armature current, the induced e.m.f and the flux per pole.?

Solution

Load current

\[ I_L = \frac{V}{R} = \frac{250}{12.5} = 20 \text{ A} \]

Shunt current

\[ I_f = \frac{250}{250} = 1 \text{ A} \]

Armature current

\[ I_a = 20 + 1 = 21 \text{ A} \]

Induce e.m.f = 250 + (21 × 0.24) = 255.04 V

Now

\[ E_g = \frac{Zn\phi}{60} \left( \frac{P}{a} \right) \]

\[ a = 2 \quad \text{(wave-wound)} \]

\[ 255.04 = \frac{778 \times 500 \times \phi}{60} \left( \frac{8}{2} \right) \]

\[ \phi = 9.83 \text{ mWb.} \]

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Example (1.4): A 4-pole, long-shunt lap-wound compound generator delivers a load current of (50 A) at (500 V). The armature resistance is (0.03 Ω), series field resistance is (0.04 Ω) and shunt field resistance is (200 Ω). The brush drop may be taken as (1V). Determine the e.m.f. generated. Calculate also the no. of conductors if the speed is (1200 r.p.m) and flux per poles (0.02 Wb). Neglect armature reaction.

Solution:

\[ I_{sh} = \frac{500}{200} = 2.5 \text{ A} \]

\[ I_a = I_{sh} + I = 50 + 2.5 = 52.5 \text{ A} \]

Series field drop = \(52.5 \times 0.04 = 2.1\) V

Armature drop = \(52.5 \times 0.03 = 1.575\) V

Brush drop = \(2 \times 1 = 2\) V

\(e.m.f \ E_g = 500 + 2.1 + 1.575 + 2 = 505.67\) V

now,

\[ E_g = \frac{Z \cdot \phi \cdot n \cdot \left(\frac{p}{a}\right)}{60} \]

\[ 505.67 = \frac{Z \times 0.02 \times 1200 \left(\frac{4}{4}\right)}{60} \]

\[ Z = 1264 \]
1.7 Armature Reaction

An armature reaction is meant the effect of magnetic field set up by armature current on the distribution of flux under main poles of a generator. The armature magnetic field has two effects:

(a) it demagnetizes or weakens the main flux.
(b) it cross-magnetizes or distorts it.

The first effect leads to reduced generator voltage and the second to the sparking at the brushes. These effects are well illustrated in fig(1.13) which shows the flux distribution of a bipolar generator when there is no current in the armature conductors.

Magnetic neutral axis (M.N.A) may be defined as the axis along which no (e.m.f) is produced in the armature conductors because they then move parallel to the lines of flux or (M.N.A) is the axis which is perpendicular to the flux passing through the armature, brushes are always placed along (M.N.A).

In general, the magnetic neutral axis shifts in the direction of motion for a generator and opposite to the direction of motion for motor. Furthermore, the amount of the shift depends on the amount of rotor current and hence on the load of machine.

Fig.(1.13)
1.8 Compensating Windings

These are used for large direct current machines. Their function is to neutralize the cross-magnetizing effect of armature reaction. The compensating windings are embedded in slots in poles shoes and are connected in series with armature in such away that the current in them flows in opposite direction to that flowing in armature conductors directly below with pole shoes. An elementary scheme of compensating winding is shown in fig.(1.14)

Ampere-turns of compensating winding are equal and opposite to those due to armature conductors that are opposite the pole face.

![Compensating Winding Diagram](image-url)
1.9 Characteristics of a D.C Generators

Following are the three most important characteristics of curves of a DC generator:

1. **No-load saturation characteristic** \( (E_o/I_f) \)

   It is also known as magnetic (c/s) or open-circuit characteristic (O.C.C). It shows the relation between the no-load generated e.m.f in armature \( E_o \), and the field or exciting current \( I_f \) at a given fixed speed. It is just the magnetization curve for the material of the electromagnets. Its shape is practically the same for all generators whether separately-excited or self-excited.

2. **Internal Characteristic** \( (E/I_a) \)

   It gives the relation between the e.m.f, \( E \) actually induced in the armature and the armature current \( I_a \). This (c/s) is of interest mainly to the designer.

3. **External Characteristic** \( (V_L/I_L) \)

   It is also referred to as performance (c/s) or sometime voltage-regulation curve.

   It gives relation between the terminal voltage \( V_L \) and the load current \( I_L \). This curve lies below the internal (c/s) between it takes into account the voltage drop over the armature circuit resistance. The values of \( V_L \) are obtain by subtracting \( I_a R_a \) from corresponding values of \( E \). This (c/s) is of great importance in judging the suitability of a generator for a particular purpose.
1.10 Separately-Excited Generator

a) Open-circuit characteristics ($E_o/I_f$)

The arrangement for obtaining the necessary data to plot this curve is shown in fig. (1.15). The exciting or field current ($I_f$) is obtained from an external independent D.C source.

![Diagram](image)

Fig.(1.15)

It can be varied ($I_f$) from zero upwards by a rheostat and its value read by an ammeter ($A$) connected in the field circuit as shown. Now, the voltage equation of a D.C generator is:

$$E_v = \phi Zn \left(\frac{P}{a}\right) \text{ volt}$$

Hence, if the speed is constant, the above relation becomes:

$E = K\phi$

It is obvious that when ($I_f$) is increased from its initial small value, the flux ($\phi$) and hence generated e.m.f, $E_o$ increase directly as current while the poles are unsaturated. This is represented by straight portion (o d) in fig.(1.15). But as the flux density increases, the poles become saturated, so a greater increase in ($I_f$) is required to produce a given increase in voltage than on the lower part of curve. That is why the upper portion (d b) of curve (o d b) bends over as shown.
(b) Internal and External Characteristic

Let us consider a separately-excited generator giving its rated no-load voltage of \(E_o\) for a certain constant field current. If there were no armature reaction and armature voltage drop, then this voltage would have remained constant as shown in fig.(1.16) by the dotted horizontal line (I). But when the generator is loaded, the voltage falls due to these two causes, thereby giving slightly drooping (c/s). If we subtract from \(E_o\) the values of voltage drops due to armature reaction for different loads. Then we get the value of \(E\) the e.m.f actually induced in the armature under load conditions. Curve (II) is plotted in this way and is know as the internal characteristic. The straight line (o a) represents the \(I_aR_a\) drops corresponding to different armature currents. If we subtract from \(E\) the armature drop \(I_aR_a\) we get terminal voltage \(V_L\). Curve (III) represents the external (c/s) and is obtained by subtracting ordinates of line (o a) from those of curve (II).

\[V = E - I_aR_a\]
1.11 Self-Excited Generator

a) Open-circuit characteristic or magnetization curve for self-excited generator

The (O.C.C) or no-load saturated curves for self-excited generators whether shunt or series-connected, are obtained in a similar way.

The field winding of the generator whether (shunt or series wound) is disconnected from the machine and connected to an external source of direct current as shown in fig.(1.17). The field or exciting current (I_r) is varied rheostatically and its value read on the ammeter (A). The machine is driven at constant speed by the prime mover and the generator e.m.f on No-load is measured by voltmeter connected across the armature. (I_r) is increased by suitable steps (starting from zero) and the corresponding values of (E_o) are measured. On plotting the relation between (I_r) and (E_o) a curve of the form shown in fig.(1.17) is obtained. Due to residual magnetism in the poles, some e.m.f is generated even when (I_r=0). Hence, the curve starts a little way up. The straight curvature at the lower end is due to magnetic inertia. It is seen that the first part of the curve is practically straight.

![Diagram](image-url)
Now, connect the field windings back to the armature and run the machine as **shunt generator**. A shunt generator will excite only if the poles have some **residual magnetism** and the **resistance of the shunt circuit is less than some critical value**, the actual value depending upon the machine and upon the speed at which the armature is driven.

Suppose curve in fig. (1.18) to represent the open-circuit characteristic of a shunt generator. With increasing excitation. Then, for a shunt current ($I_f$) OA, the e.m.f. is AB and.

\[
E_o = \frac{AB}{OA} = \tan(BOA) = \text{slope of OB}
\]

The resistance line OB represents smaller resistance to which the machine will build up and represent the maximum voltage AB. If field resistance is increased, then slope of the resistance line increase, and hence the maximum voltage to which the generator will build up at a given speed, decreases.
If field resistance increased so much that the resistance line does not cut the O.C.C at all (like OE) then obviously the machine will fail to excite, there will be no "build up" of the voltage. The value of the resistance represented by the tangent to the curve, is known as critical resistance \( R_C \) for a given speed.

**How to draw O.C.C at Different Speed**

Suppose we are given the data for (O.C.C) of a generator run at a fixed speed, say, \( n_1 \). It will be shown that (O.C.C) at any other constant speed \( n_2 \) can be deduced from the (O.C.C) for \( n_1 \). In fig.(1.19) is shown the (O.C.C) for speed \( n_1 \).

![Diagram](image)

**Fig.(1.19)**

Since \((E\alpha n)\) for any fixed excitation, hence

\[
\frac{E_1}{E_2} = \frac{n_1}{n_2} \quad \text{Or} \quad E_2 = E_1 \times \frac{n_1}{n_2}
\]

As seen, for \( I_f = OH \), \( E_1 = HC \)

The value of new voltage for the same \( I_f \) but at \( n_2 \)

\[
E_2 = HC \times \frac{n_2}{n_1} = HD
\]

In a similar way, other such points can be found and the new O.C.C at \( n_2 \) draw.
(b) External Characteristic of a shunt generator

We will now proceed to find its external characteristic \((V_L/I_L)\) when loaded. It is found that if after building up, a shunt generator is loaded, then its terminal voltage \((V_L)\) drops with increase in load current. Such a drop in voltage is undesirable especially when the generator is supplying current for load and power for which purposes it is desirable that \((V_L)\) should remain practically constant and independent of the load.

There are three main reasons for the drop in terminal voltage of a shunt generator when under load.

I) Armature resistance drop.

II) Armature reaction.

III) The drop in terminal voltage due to armature resistance and armature reaction results in a decreased field current \((I_f)\) which further reduces the induced (e.m.f).

The terminal voltage

\[ V = E - I_a R_a \]

\[ E = K\phi \]

The shunt generator is first excited on no-load so that it gives its full open circuit voltage \((o a)\). Then, the load is gradually applied and, at suitable intervals, the terminal voltage \((V_L)\) (as read by the voltmeter) and the load current \((I_L)\) (as read by the ammeter \(A_2\)) are noted. The field current as recorded by ammeter \((A_1)\), is kept constant by a rheostat. By plotting these readings, the external (c/s) of fig.(1.20) is obtained. The portion \((a b)\) is working part of this curve. Over this part, if the load resistance is decreased, load current is increased as usual, although this results in a comparatively small additional drop in voltage. These condition hold good till point \((b)\) is reached. This point is known as break-down point.
It is found that beyond this point (where load is maximum =O B) any effort to increase load current by further decreasing load resistance results in decreased load current like(O A) due to a very rapid decrease in terminal voltage.

\[
\text{Fig.(1.20)}
\]

(c) Internal characteristic of a shunt generator

As defined before, internal (c/s) gives the relation between \(E\) and \(I_a\). Now in a shunt generator,

\[
I_a = I_f + I_L \quad \text{and} \quad E = V + I_a R_a , \quad I_f = \frac{V}{R_f}
\]

Hence, \((E/I_a)\) curve can be obtained from \((V_L I_L)\) curve as shown in fig.(1.21). In this figure, (a b) represents the external (c/s) as discussed above. The field resistance line (O B) is drawn as usual. The horizontal distances from (O Y) line to the line (O B) give the values of field currents for different terminal voltages. If we add these distances horizontally to the external characteristic (a b), then we get the curve for the total armature current i.e. dotted curve (a c). For example, point (d) on (a c) is obtained by making (g d=e f). The armature resistance drop line (O M) is the plotted as usual.
If brush contact resistance is assumed constant, then armature voltage drop is proportional to the armature current. For any armature current (O K), armature voltage drop \((I_a R_a = M K)\). If we add these drops to the ordinates of curve (a c), we get the internal characteristic.

Fig.(1.21)

(c) Series Generator

The magnetization curve of a series DC generator looks very much like the magnetization curve of any other generator. At no load, however, there is no field current, so \((V_L)\) is reduced to a small level given by the residual flux in the machine. As the load increases, the field current rises, so \((E)\) rises rapidly. The \(I_a(R_a+R_f)\) drop goes up too, but at first the increase in \((E)\) goes up more rapidly than \(I_a(R_a+R_f)\) drop rises, so \((V_L)\) increase.
\[ V_L = E - I_a R_a - I_f R_f \]

\[ I_a = I_f = I_L \]

\[ V_L = E - I_a (R_a + R_f) \]

After a while, the machine approaches saturation, and (E) becomes almost constant. At that point, the resistive drop is predominant effect, and \( V_L \) starts to fall.

This type of characteristic is shown in fig.(1.22). It is obvious that this machine would make a bad constant-voltage source. In fact, its voltage regulation is a large negative number.

Series generators are used only in a few specialized applications, where the steep voltage (c/s) of the device can be exploited. On such application is arc welding. Series generators used in arc welding are deliberately designed to have a large armature reaction, which gives them a terminal (c/s) like the one shown in fig.(1.23). Notice that when the welding electrodes make contact with each other before welding commences, a very large current flows. As the operator separates the welding electrodes, there is a very steep rise in generator's voltage, while the current remains high. This voltage ensures that a welding arc is maintained through the air between electrodes.

Fig. (1.22)  
Fig. (1.23)
(d) Compound-Wound Generator

If the full-load voltage is thereby made the same as the no-load voltage, this is known as a **level-compound** characteristic, though the curve is not actually flat because armature reaction demagnetizing effects are not exactly linear with current.

If the series field amp-turns are such that the rated-load voltage is greater than the no-load voltage, then generator is **over-compounded**.

If rated-load voltage is less than the no-load voltage, then the generator is **under-compounded** but such generators are seldom used.

![Diagram](image_url)

Fig.(1.24)

### 1.12 Condition for Build up of a self-excited

We may summarize the conditions necessary for the build-up of a (self-excited) generator as follows:

1. There must be some **residual** magnetism in the generator poles.
2. For the given **direction of rotation**, the (shunt or series) field coils should be **correctly connected** to the armature i.e. they should be so connected that the induced current **reinforces** the e.m.f produced initially due to residual magnetism.
Example (1.5): The magnetization curve of a D.C generator has the following points, all taken at (1000)r.p.m:

<table>
<thead>
<tr>
<th>$I_f$ (Amperes)</th>
<th>1.5</th>
<th>1.25</th>
<th>1</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_o$ (Volts)</td>
<td>250</td>
<td>230</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

(a) If the field current is adjusted at (1.25 A), what must be speed to generate (250 V)?

(b) What is the field current to generate (200 V) at speed (1000 r.p.m) on no-load?

Solution:

(a) From the given data, for $I_f=1.25$ A, $E_o=230$ V at 1000 r.p.m. If $n$ is the speed for generating $E_o=250$ V, then

$$E_g = K \cdot \phi \cdot n$$

$$\phi_1 = \phi_2$$

$$\frac{n_2}{1000} = \frac{250}{230}$$

$$n_2 = 1087 \text{ r.p.m}$$

(b) From the given data, Value of ($I_f$) for $E_o=200$ V, is (1 A).
Example (1.6): A shunt generator gave the following results in the O.C.C. test at a speed of (800 r.p.m).

<table>
<thead>
<tr>
<th>I_f (A)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_a (V)</td>
<td>90</td>
<td>185</td>
<td>251</td>
<td>290</td>
<td>324</td>
<td>345</td>
<td>360</td>
</tr>
</tbody>
</table>

The field resistance is adjusted to (50 Ω) and the terminal voltage is (300 V) on load. Armature resistance is (0.1 Ω) and assuming that the flux is reduced by (5%) due to armature reaction, find the load supplied by the generator.

Solution:

When the terminal voltage is (300 V) and (R_{sh}=50 Ω), then field current is

$$I_f = \frac{300}{50} = 6 \text{ A}$$

With this shunt current, the induced e.m.f. as seen from the given table (we need not draw the O.C.C) is (=324 V).

Due to armature reaction, the flux and hence the induced e.m.f is reduced to (0.95) of its no-load value.

Hence, induced e.m.f when generator is on load

$$= 324 \times 0.95 = 307.8 \text{ V}$$

Armature drop at given load

$$= 307.8 - 300 = 7.8 \text{ V}$$

$$I_a \cdot R_a = 7.8,$$

$$I_a = \frac{7.8}{0.1} = 78 \text{ A}$$

Load current =78-6=72 A

***********************************************************************************************************************************************
Example(1.7): The O.C.C. of a D.C shunt generator running at 300 r.p.m. is as follows.

<table>
<thead>
<tr>
<th>$I_f$ (A)</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{o1}$ (V)</td>
<td>7.5</td>
<td>92</td>
<td>132</td>
<td>160</td>
<td>183</td>
<td>190</td>
<td>212</td>
</tr>
</tbody>
</table>

(i) Plot the O.C.C. for 375 r.p.m. and determine the voltage to which the machine will excite if the field resistance is 40 $\Omega$.

(ii) Determine the load current supplied by the generator, when its terminal voltage is 200 V. Take armature resistance 0.3 $\Omega$. Assume speed to constant and armature reaction may be ignored.

(iii) What additional resistance would have to be inserted in the field circuit to reduce the voltage to 200 V at 375 r.p.m. (no-load).

Solution:

(i) plot the O.C.C. at 375 r.p.m., increase the e.m.f. induced in ratio (375/300).
\[
\frac{E_{s1}}{E_{s2}} = \frac{n_1}{n_2}
\]

\[
E_{s2} = \frac{375}{300} \times E_{s1}
\]

<table>
<thead>
<tr>
<th>I_f (A)</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_{o2} (V)</td>
<td>9.4</td>
<td>115</td>
<td>165</td>
<td>200</td>
<td>228.8</td>
<td>248.8</td>
<td>265</td>
</tr>
</tbody>
</table>

The new O.C.C. at 375 r.p.m. is shown. Line OL represent 40 Ω line. The voltage (corresponding to point L) to which the machine will excite if the field circuit resistance is (40 Ω = 255 V).

(ii) \( R_f = 40 \) Ω

The terminal voltage \( V = 200 \) V

\[ I_f = \frac{V}{R_f} = \frac{200}{40} = 5 \text{ A} \]

Generated e.m.f. for shunt field current of (5 A = 228.8 V)

\[
E_{g2} = V + I_a R_a
\]

\[ 228.8 = 200 + I_a \times 0.3 \]

\[ I_a = \frac{228.8 - 200}{0.3} = 96 \text{ A} \]

∴ Load current

\[ I_L = I_a - I_f \]

\[ I_L = 96 - 5 = 91 \text{ A} \]

(iii) It is clear that for exciting the generator to 200 V, field current should be 4 A.

Corresponding resistance of shunt circuit \( R_f = \frac{200}{4} = 50 \text{Ω} \)

∴ Additional resistance required = 50-40 = 10 Ω
Example (1.8): The following is the magnetic (c/s) of a D.C. generator driven at 1000 r.p.m.

\[
\begin{array}{c|cccccc}
I_f (A) & 1 & 2 & 4 & 6 & 8 & 10 \\
E_o (V) & 160 & 260 & 390 & 472 & 522 & 550 \\
\end{array}
\]

Determine:

(i) The voltage to which it will excite on open circuit.

(ii) The approximate value of the critical resistance of shunt circuit.

(iii) The terminal potential difference and load current for a load resistance of 4Ω the armature and field resistance are 0.4 Ω and 60 Ω respectively.

Solution:

(i) Draw the O.C.C. as shown in figure, draw the shunt resistance line (60 Ω) as usual. The intersection of shunt resistance line and O.C.C. gives the open circuit voltage of 540 V.

(ii) To find the critical resistance, draw the line (OL) which tangential to the initial straight part of the O.C.C. The slope of (OL) gives the critical resistance. Take any point on line (OL), it is seen

\[
\text{Critical resistance } = \frac{400}{2.5} = 160\Omega
\]
(iii) To obtain the value of terminals voltage and load current for a given load resistance, we have to draw the external (c/s) \( (V_L/I_L) \) and the load resistance line. The intersection of these two curves will give the required values.
The external (c/s) can be calculated from the table given below.

<table>
<thead>
<tr>
<th>$I_f$ (A)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_o$ (V)</td>
<td>160</td>
<td>260</td>
<td>390</td>
<td>472</td>
<td>522</td>
<td>550</td>
</tr>
</tbody>
</table>

$(V_L = 60 \times I_f)$

| $V_L$ (V) | 60  | 120 | 240 | 360 | 480 | 600 |

$I_a = \frac{E_o - V}{R_a}$

| $I_a$     | 250 | 350 | 375 | 280 | 105 | ..... |

$I_L = I_a - I_f$

| $I_L$     | 249 | 348 | 371 | 274 | 97  | ..... |

Draw the $(V_L/I_L)$ (c/s) as shown in figure. Draw the 4Ω resistance line.

Take $I_L=100$ A, terminal voltage $=4 \times 100 = 400$ V, hence point N lies on the load resistance line. The 4 Ω load resistance line cuts the external $(V_L/I_L)$ (c/s) at point L.

Terminal potential difference $=470$ V

Load current $=117.5$ A
1.13 D.C Machine Losses

A generator is a machine for converting mechanical energy into electrical energy and a motor is a machine for converting electrical energy into mechanical energy. When such conversions take place, certain losses occur which are dissipated in the form of heat. The principle losses of machines are:

(i) **Copper loss**, due to $I^2R$ heat losses in the armature and field windings.

(ii) **Iron (or core) loss**, due to hysteresis and eddy-current losses in the armature. This loss can be reduced by constructing the armature of silicon steel laminations having a high resistivity and low hysteresis loss. At constant speed, the iron loss is assumed constant.

(iii) **Friction and windage losses**, due to bearing and brush contact friction and losses due to air resistance against moving parts (called windage). At constant speed, these losses are assumed to be constant.

(iv) **Brush contact loss** between the brushes and commutator. This loss is approximately proportional to the load current.
1.14 Efficiency of a D.C generator

The efficiency of an electrical machine is the ratio of the output power to the input power and is usually expressed as a percentage, and since the units are power/power, then efficiency has no units, thus

\[
\eta = \left( \frac{\text{output \cdot power}}{\text{input \cdot power}} \right) \times 100\%
\]

If the total resistance of the armature circuit (including brush contact resistance) is \( R_a \), then the total loss in the armature circuit is \( (I_a^2 \times R_a) \). If the terminal voltage is \( V \) and the current in the shunt circuit is \( I_f \), then the loss in the shunt circuit is \( (I_f \times V) \). If the sum of the iron, friction and windage losses is \( C \) the total losses is given by:

\[
(I_a^2 \times R_a) + (I_f \times V) + C
\]

\( (I_a^2 R_a + I_f V) \) is, in fact, the "copper loss" if the output current is \( I \), then the output power is \( V \times I \).

Total input power = \( VI + I_a^2 R_a + I_f V + C \)

Hence,

\[
\eta = \left( \frac{VI}{VI + I_a^2 R_a + I_f V + C} \right) \times 100\%
\]
Example (1.9): A 10 KW shunt generator having an armature circuit resistance of 0.75 Ω and a field resistance of 125 Ω, generates a terminal voltage of 250 V at full load. Determine the efficiency of the generator at full load, assuming the iron, friction and windage losses amount to 600 W.

Solution:

Output power = \( 10000 = VI \)

Load current \( I = \frac{10000}{250} = 40 \, \text{A} \)

Field current, \( I_f = \frac{V}{R_f} = \frac{250}{125} = 2 \, \text{A} \)

Armature current, \( I_a = I_f + I = 2 + 40 = 42 \, \text{A} \)

\[
\eta = \left( \frac{VI}{VI + I_a^2 R_a + I_f V + C} \right) \times 100 \%
\]

\[
= \left( \frac{10000}{10000 + (42)^2 (0.75) + (2)(250) + 600} \right) \times 100
\]

\[
= 80.5 \%
\]
D.C MOTORS

2.1 Introduction

An electric motor is a machine which converts electric energy to mechanical energy.

Why the D.C motors so common, when D.C power systems themselves were fairly rare?

1. The D.C power systems are still common in trucks, aircraft and cars (starting motor on all automobiles, outsider rear-view mirrors, windshield wipers, fuel pump, water injection pump, cooling fan).

2. D.C Motor was a situation in which wide variations in speed are needed.

If a current-carrying conductor is placed in a magnetic field produced by permanent magnets, then the field due to the current-carrying conductor and the permanent magnets interact and cause a force to be exerted on the conductor as shown in fig.(2.1). The force on the current-carrying conductor in a magnetic field depends upon:

(a) The flux density of the field \( B \) (teslas).

(b) The strength of the current, \( I \) (Amperes).

(c) The length of the conductor perpendicular to the magnetic field, \( \ell \) (meters).

(d) The direction of the field and current (angle).

When the magnetic field, the current and the conductor are mutually at right angles then

\[
\text{Force } F = B\ell I \quad \text{Newton's}
\]

When the conductor and the field are at an angle \( \theta \) to each other then

\[
\text{Force } F = B\ell I \sin(\theta) \quad \text{Newton's}
\]
As shown in figure (2.1), the field is strengthened above the conductor and weakened below, thus tending to move the conductor downward. This is the basic principle of operation of the electric motor.

![Diagram](image)

**Fig.(2.1)**

The direction of the force exerted on a conductor can be predetermined by using **Fleming's left-hand rule** (often called the **motor rule**).
2.2 Principle of Operation of a Simple D.C Motor

A rectangular coil which is free to rotate about a fixed axis shown placed inside a magnetic field produced by permanent magnets in fig.(2.3). A direct current is fed into the coil via carbon brushes bearing on a commutator, which consists of a metal ring split into two halves separated by insulation.

When current flows in the coil a magnetic field is set up around the coil which interacts with the magnetic field produced by the magnets. This causes a force (F) to be exerted on the current-carrying conductor which by Fleming's left-hand rule, is **down wards** between point (A) and (B), **up ward** between (C) and (D) for the current direction shown. This causes a torque and the coil rotates **anticlockwise**.

![Diagram of D.C Motor Principle](image)

Fig.(2.3)
When the coil has turned through \((90^\circ)\) from the position shown in figure, the brushes connected to the positive and negative terminals of supply make contact with different halves of the commutator ring, thus reversing the direction of the current flow in the conductor. If the current is not reversed and the coil rotates past this position the forces acting on it change direction and it rotates in the opposite direction thus never making more than half a revolution.

The current direction is reversed every time the coil swing through the vertical position and thus the coil rotates anti-clockwise for as long as the current flows. This is the principle of operation of a D.C motor which is thus a device that takes in electrical energy and converts it into mechanical energy.

### 2.3 Significance of the Back e.m.f.

When the motor armature rotates, the conductors also rotate and hence cut the flux. In accordance with the laws of electromagnetic induction, e.m.f. is induced in them whose direction, as found by Fleming's **Right-hand Rule**, is in opposition to supplied voltage. Because of its opposing direction, it is referred to as counter e.m.f. or back e.m.f. \((E_b)\). It will be seen that

\[
V = E_b + I_a R_a
\]

\[
I_a = \frac{V - E_b}{R_a}
\]

Where \((R_a)\) is the resistance of the armature circuit. As pointed out above

\[
E_b = \frac{Z \cdot \phi \cdot n}{60} \left( \frac{P}{a} \right) \text{ Volts}
\]
Back e.m.f. depends, among other factors, upon the armature speed. If speed is high, \( E_b \) is large, hence armature current \( (I_a) \), as seen from the above equation, is small.

If the speed is less, then \( (E_b) \) is less, hence more current flows which develops more torque. So, we find that \( (E_b) \) acts like a governor i.e. it makes a motor self-regulating so that it draws as much current as is just necessary.

2.4 Induce Torque equation of a D.C Machine

The torque in any D.C machine depends on three factor:

1. The flux \( (\phi) \) in the machine.
2. The armature (or rotor) current \( (I_a) \) in the machine.
3. A constant depending on the construction of the machine.

The torque on the armature of a real machine is equal to the number of conductors \( (Z) \) times the torque on each conductor. The torque in any single conductor under the pole faces is.

\[
T_{\text{Cond.}} = r.F \\
F = B.\ell .I_{\text{Cond.}} \\
T = r.B.\ell .I_{\text{Cond.}}
\]

If there are \( (a) \) current paths in the machine, then the total armature current \( (I_a) \) is split among the \( (a) \) current path, so the current in a single conductor is given by

\[
I_{\text{Cond.}} = \frac{I_a}{a}
\]

and the torque in a single conductor on the motor may be expressed as

\[
T_{\text{Cond.}} = \frac{r.B.\ell .I_a}{a}
\]
Since there are \((Z)\) conductors, the **total induce torque** in a D.C machine rotor is:

\[
T_{\text{ind.}} = \frac{ZrB\ell I_a}{a}
\]

The flux per pole in this machine can be expressed as

\[
\phi = B.A_p = \frac{B(2\pi r\ell)}{p}
\]

So the total induce torque can be re-expressed as

\[
T_{\text{ind.}} = \frac{Z\phi I_a}{2\pi} \left(\frac{P}{a}\right)
\]

Finally,

\[
T_{\text{ind.}} = K\phi I_a
\]

Where

\[
K = \frac{ZP}{2\pi a}
\]

The induce torque equations given above are only **approximations**, because not all the conductors in the machine are under the pole faces at any given time.
2.5 Types of D.C Motors

(a) Permanent-Magnet D.C Motor

The permanent-magnet D.C motor shown in fig.(2.4), is construction in the same manner as its D.C generator counterpart.

When this type of motor is used, the D.C power supply is connected directly to the armature conductors through the brush to commutator assembly. The magnetic field is produced by permanent-magnets mounted on the stator. The permanent-magnet motor has several advantages over conventional types of D.C motors. The advantage is a reduced operational cost, and the direction of rotation of a permanent-magnet motor can be reversed by reversing the two power lines. The speed (c/s) of the permanent-magnet motor are similar to those of the shunt wound D.C motor.
(b) Shunt- Wound D.C Motor

Shunt-wound D.C motor are more commonly used than any other type of D.C motor. As shown in figure (2.6), the shunt-wound D.C motor has field coils connected in parallel with its armature. This type of D.C motor has field coils that are wound of many turns of small-diameter wire and have a relatively high resistance. Since the field is a high-resistant parallel path of the circuit of the shunt motor, a small amount of current flows through the field. A strong electromagnetic field is produced because of the many turns of wire that form the field windings. Since the field current has little effect on the strength of the field, motor speed is not affected appreciably by variation in load current.

\[ V = E_b + I_a . R_a \]

\[ I = I_a + I_f \]

Because of its good speed regulation, and its ease of speed control, the D.C shunt motor is commonly used for industrial applications.
(c) Series-Wound D.C Motor

In the series-wound motor the field winding is in series with the armature across the supply as shown in fig.(2.7). There is only one path for current to flow from the D.C voltage source. Therefore, the field is wound of relatively few turns of large-diameter wire, giving the field a low resistance. Changes in load applied to the motor shaft cause change in current through the field. If the mechanical load increase, the current also increase. The increased current creates a stronger magnetic field. The speed of a series motor varies from very fast at no load, to very slow at heavy loads. Since large currents may flow through the low resistance field, the series motor produces a high torque output. Series motors are used when heavy loads must be moved, and speed regulation is not important. A typical application is automobile starter motors.

\[ V = E_b + I(R_a + R_f) \]

\[ I = I_a \]
(d) Compound-Wound D.C Motor

The compound-wound D.C motor, has two sets of field windings, one in series with the armature and one in parallel. This motor combines the desirable characteristics of the series- and shunt-wound motors. It has high torque similar to that of a series-wound motor, along with good speed regulation similar to that of a shunt motor. Therefore, when good torque and good speed regulation are needed, the compound-wound D.C motor can be used. There are two common types of compound motor connection, the long-shunt connection and short-shunt connection, as shown in fig.(2.8). And there are two different types of compound motors in common use, they are the cumulative compound motor and the differential compound motor.

Fig.(2.8)
2.6 Motors Characteristics

The characteristic curves of a motor are those curves which shown relationship between the following quantities:

1. Torque and armature current i.e. \((T/I_a)\) characteristic.
2. Speed and armature current \((n/I_a)\) characteristic.
3. Speed and torque \((n/T)\) characteristic.

2.6.1 Characteristic of shunt-wound motor

1. \((T/I_a)\) Characteristic

The theoretical torque/ armature current \((c/s)\) can be derived from the expression \(T \propto \phi I_a\) for a shunt-wound motor, the field winding is connected in parallel with the armature circuit and thus the applied voltage gives a constant field current, i.e. a shunt-wound motor is a constant flux machine. Since \((\phi)\) is constant, it follows that \(T \propto I_a\), and the \((c/s)\) is as shown in fig.(2.9).

![Fig.(2.9)](image-url)
2. \((n / I_a)\) Characteristic

For a shunt motor, \(V\), \(\phi\) and \(R_a\) are **constants**, hence as armature current \((I_a)\) increases, \(I_a\ R_a\) increase and \((V-I_a\ R_a)\) **decrease**, and the speed is proportional to a quantity, which is decreasing and is shown in fig.(2.10).

As the load on the shaft of the motor increases, \((I_a)\) increases and the speed drops slightly. In practice, the speed falls by about \((10\%)\) between no-load and full-load on many D.C shunt-wound motors. Due to this relatively **small drop** in speed, the D.C shunt-wound motor is taken as basically being a **constant-speed** machine.

From equation :

\[ V = E_b + I_a R_a \]
\[ E_b = V - I_a R_a \]
\[ E_b \propto \phi n \quad \text{hence } \phi \text{ is constant then} \]

\[ n = \frac{V - (I_a \times R_a)}{\phi} \]

3. \((n / T)\) Characteristic

The theoretical speed/ torque Characteristic can be deduced from (1) and (2) above and is shown in fig.(2.11).

\[ E_b = K\phi \omega \]
\[ T = K\phi I_a \quad I_a = \frac{T}{K\phi} \]
\[ E_b = V - I_a R_a \]
\[ K\phi \omega = V - \frac{T}{K\phi} R_a \]

\[ \omega = \frac{V}{K\phi} - \frac{T}{(K\phi)^2} R_a \]

This equation is just a straight line with a negative slope.
2.6.2 Characteristic of series-wound motor

1. \((T/I_a)\) Characteristic

In a series motor, the armature current flows in the field winding and is equal to the supply current \((I)\). The torque \(T \propto \phi I_a\) over a limited range, before magnetic saturation of the magnetic circuit of the motor is reached. Thus \((\phi \propto I)\) and \((T \propto I^2)\). Hence, \((T/I_a)\) curve is a parabola as shown in fig.(2.12). After magnetic saturation, \(\phi\) almost becomes a constant and \((T \propto I)\), so the characteristic becomes a straight line.
2. \( (n / I_a) \) Characteristic

In a series motor, \( I_a = I \) and below the magnetic saturation level, \( \phi \propto I \). Thus \( n \propto \frac{(V - IR)}{I} \) when \( R \) is the combined resistance of the series field and armature circuit.

Since \( (I.R) \) is small compared with \( (V) \), then an approximate relationship for the speed is \( n \propto \frac{V}{I} \) since \( (V) \) is constant. Hence, \[
    n = \frac{1}{I}
\]

Speed varies inversely as armature current as shown in fig.(2.13). The high speed at small values of current indicate that this type of motor must not be run on very light loads and invariably. Such motors are permanently coupled to their loads.
3. \( \frac{n}{T} \) Characteristic

The theoretical speed/torque \((c/s)\) may be derived from (1) and (2) above by obtaining the torque and speed for various values of current and plotting the co-ordinates on the speed/torque \((c/s)\). The series-wound motor has a large torque when the current is large on starting. A typical speed/torque \((c/s)\) is shown in fig.(2.14).

\[
E_b = V - I \times R
\]

\[
T = K \times \phi \times I , \quad \phi \propto I , \quad I = \frac{T}{\sqrt{K}} , \quad E_b = K \phi \omega
\]

\[
K \phi \omega = V - IR
\]

\[
KI \omega = V - IR
\]

\[
K \sqrt{\frac{T}{\sqrt{K}}} \omega = V - \sqrt{\frac{T}{\sqrt{K}}} R
\]

\[
\omega \sqrt{\frac{T}{\sqrt{K}}} = V - \sqrt{\frac{T}{\sqrt{K}}} R
\]

\[
\omega = \frac{1}{\sqrt{T}} \cdot \frac{V}{\sqrt{K}} - \frac{R}{K}
\]

unsaturated  the speed varies of square root of torque.

![Graph](image)

Fig.(2.14)
2.6.3 Characteristic of Compound-wound motor

A compound-wound motor has both a series and a shunt field winding, (i.e. one winding in series and one in parallel with the armature circuit), by varying the number of turns on the series and shunt windings and the directions of the magnetic fields produced by these windings (assisting or opposing), families of (c/s) may be obtained to suit almost all applications. There are two common types of compound motor connection, the long-shunt connection and short-shunt connection. And there are two different types of compound motors in common use, they are the cumulative compound motor and the differential compound motor. In the cumulative compound motor, the field produced by the series winding aids the field produced by the shunt winding. The speed of this motor falls more rapidly with increasing current than does that of the shunt motor because the field increases. In the differential compound motor, the flux from the series winding opposes the flux from the shunt winding. The field flux, therefore, decreases with increasing load current. Because the flux decreases, the speed may increases with increasing load. Depending on the ratio of the series-to-shunt field ampere-turns, the motor speed may increases very rapidly.

Fig.(2.15)
The torque-speed (c/s) of a cumulatively compound D.C motor

In the cumulative compounded D.C. motor, there is a component of flux which is constant and another component which is proportional to its armature current (and thus to its load). Therefore, the cumulatively compounded motor has a higher starting torque than a shunt motor (whose flux is constant) but a lower starting torque than a series motor (whose entire flux is proportional to armature current). At light loads, the series field has a very small effect, so the motor behaves approximately as a shunt D.C. motor. As the load gets very large, the series flux becomes quite important and the torque-speed curve begins to look like a series motor's (c/s). A comparison of the torque-speed (c/s) of each of these type of machines is shown in figure (2.16).

The torque-speed (c/s) of a differentially compound D.C motor

In a differentially compound D.C. motor, the shunt magneto motive force and series magneto motive force subtract from each other. This means that as the load on the motor increases, $I_a$ increases and the flux in the motor decreases. But as the flux decreases, the speed of the motor increases. This speed increases causes another increase in load, which further increases $I_a$, further decreasing the flux, and increasing the speed again. The result is that a differentially compounded motor is unstable and tends to run away. It is so bad that a differentially compounded motor is unsuitable for any application.

![Figure (2.16)](image-url)
2.7 D.C Motor Starter

If a D.C motor whose armature is stationary is switched directly to its supply voltage, it is likely that the fuses protecting the motor will burn out. Because the armature resistance is small, frequently being less than one ohm. Thus, additional resistance must be added to the armature circuit at the instant of closing the switch to start the motor.

As the speed of the motor increases. The armature conductors are cutting flux and a generated voltage, acting in opposition to the applied voltage, is produced, which limits the flow of armature current. Thus the value of the additional armature resistance can then be reduced.

When at normal running speed, the generated e.m.f. is such that no additional resistance is required in the armature circuit. To achieve this varying resistance in the armature circuit on starting a D.C motor starter is used, as shown in fig.(2.17). The starting handle is moved slowly in a clockwise direction to start the motor. For a shunt-wound motor, the field winding is connected to stud (1) or (M) via a sliding contact on the starting handle. To give maximum field current hence maximum flux, hence maximum torque on starting, since $T \propto \phi I_a$.

![Fig.(2.17)](image)
2.8 Speed Control of D.C Motor

2.8.1 Shunt-Wound Motor

The speed of a shunt-wound D.C motor, \( n \), is proportional to \( \frac{(V - I_a R_a)}{\phi} \). The speed is varied either by varying the value of flux, \( (\phi) \), or by varying the value of \( (R_a) \). The former is achieved by using a variable resistor in series with the field winding, as shown in fig.(2.18) and such a resistor is called the shunt field regulator. As the value of resistance of the shunt field regulator is increased, the value of the field current, \( (I_f) \), is decreased. This results in a decrease in the value of flux, \( (\phi) \), and hence an increase in the speed, since \( n \propto \frac{1}{\phi} \). Thus only speeds above that given without a shunt field regulator can be obtained by this method.

Speeds below those given by \( \frac{(V - I_a R_a)}{\phi} \) are obtained by increasing the resistance in the armature circuit, as shown in fig.(2.18), where

\[
n \propto \frac{V - I_a (R_a + R)}{\phi}
\]

Since resistor \( (R) \) is in series with the armature, it carries the full armature current and results in a large power loss in large motors where a considerable speed reduction is required for long periods.

Fig.(2.18)
The speed control of series-wound motors is achieved using either (a) **field resistance**, or (b) **armature resistance** techniques.

(a) The speed of a D.C series-wound motor is given by:

\[ n = K \left( \frac{V - IR}{\phi} \right) \]

Where \( K \) is a constant, \( V \) is the terminal voltage, \( R \) is the combined resistance of the armature and series field and \( \phi \) is the flux.

Thus, a reduction in flux results in an increase in speed. This is achieved by putting a variable resistance in **parallel** with the **field winding** and reducing the field current, and hence flux, for a given value of supply current. A circuit diagram of this arrangement is shown in fig. (2.19). A variable resistor connected in **parallel** with the series-wound field to control speed is called a diverter speeds above those given with no diverter are obtained by this method.

![Circuit Diagram](image)
(b) speed below normal are obtained by connecting a variable resistor in series with the field winding and armature circuit, as shown in fig.(2.20). This effectively increases the value of (R) in the equation.

\[
n = K \left( \frac{V - I.R}{\phi} \right)
\]

And thus reduces the speed. Since the additional resistor carries the full supply current, a large power loss is associated with large motors in which a considerable speed reduction is required for long periods.

![Diagram](image)

**Example (2.1)** A D.C motor has a speed of (900 r.p.m) when connected to a (460 V) supply. Find the approximate value of the speed of the motor when connected to a (200 V) supply, assuming the flux decreases by (30%) and neglecting the armature volt drop?

**Solution:**

\[
E_{b1} = K\phi_1 n_1 \quad \quad E_{b2} = K\phi_2 n_2
\]

\[
\phi_2 = \phi_1 - \phi_1 \times 0.3 \quad \quad \phi_2 = 0.7\phi_1
\]

Now

\[
\frac{E_{b1}}{E_{b2}} = \frac{\phi_1 \times 900}{0.7\phi_1 \times n_2}
\]

\[n_2 = 559\text{ r.p.m}\]
Example (2.2): A series motor has an armature resistance of (0.2 \ \Omega) and a series field resistance of (0.3 \ \Omega). It is connected to a (240 V) supply and at a particular load runs at (1440 \ \text{r.p.m}) when drawing (15 \ \text{A}) from the supply.

(a) Determine the back e.m.f at this load.

(b) Calculate the speed of motor when the load is changed such that the current is increased to (30 \ \text{A}). Assume that this cases a doubling of flux.

Solution:

(a) at initial load, is given by

$$E_{b1} = V - I_a(R_a + R_f)$$

$$E_{b1} = 240 - 15(0.2+0.3)$$

$$= 232.5 \ \text{Volt}.$$  

(b) When the current is increased to (30 \ \text{A}), the back e.m.f. is given by.

$$E_{b2} = V - I_a(R_a + R_f)$$

$$=240 - 30(0.2+0.3)$$

$$=225 \ \text{volt}$$

Now back e.m.f  \( E_b \propto \phi \cdot n \)

Thus

$$\frac{E_{b1}}{E_{b2}} = \frac{\phi_1 \cdot n_1}{2 \cdot \phi_1 \cdot n_2}$$

i.e.

$$\frac{232.5}{225} = \frac{\phi_1 \times 1440}{2 \times \phi_1 \times n_2}$$

$$n_2 = \frac{1440 \times 225}{232.5 \times 2} = 696.77 \ \text{r.p.m}$$

****************************************************
Example (2.3): A series motor runs at (800 r.p.m) when the voltage is (400 V) and the current is (25 A). The armature resistance is (0.4 Ω) and the series field resistance is (0.2 Ω). Determine the resistance to be connected in series to reduce the speed to (600 r.p.m) with same current.

Solution:

at (800 r.p.m)

\[ E_{b1} = V - I(R_a + R_f) \]

\[ = 400 - 25(0.4+0.2) \]

\[ = 385 \text{ volt} \]

at (600 r.p.m), since the current is unchanged, the flux is unchanged.

Thus \( E_b \propto \phi \times n \), or \( E_b \propto n \), and \( \frac{E_{b1}}{E_{b2}} = \frac{n_1}{n_2} \)

\[ E_{b2} = \frac{(385)(600)}{(800)} = 288.75 \text{ volt} \]

And

\[ E_{b2} = V - I(R_a + R_f + R) \]

\[ 288.75 = 400 - 25(0.4+0.2+R) \]

Rearranging gives

\[ 0.6 + R = \frac{400 - 288.75}{25} = 4.45 \]

From which, extra series resistance,

\[ R = 4.45 - 0.6 \]

i.e., \( R = 3.85 \Omega \)

thus the addition of a series resistance of (3.85 Ω) has reduced the speed from (800 r.p.m) to (600 r.p.m).
Example (2.4): On full load a (300 V) series motor takes (90 A) and runs at (900 r.p.m) the armature resistance is (0.1 Ω) and the series winding resistance is (50 mΩ). Determine the speed when developing full load torque but with a (0.2 Ω) diverter in parallel with the field winding. (assume that the flux is proportional to the field current).

Solution:

at (300 V)

\[ E_{b1} = V - I(R_a + R_f) \]

\[ = 300 - 90(0.1 + 0.05) \]

\[ = 286.5 \text{ Volts} \]

With the (0.2 Ω) diverter in parallel with (R_f)

The equivalent resistant

\[ R = \frac{0.2 \times 0.05}{0.2 + 0.05} = 0.04\Omega \]

By current division, current \[ I_x = I \left( \frac{0.2}{0.2 + 0.05} \right) \]

\[ I_x = 0.8I \quad , \quad I_x = 0.8I_{a2} \]

Torque, \[ T \propto I_a\phi \] and for full load torque \[ I_a\phi_1 = I_{a2}\phi_2 \]

Since flux is proportional to field current \[ \phi_1 \propto I_{a1} \] and \[ \phi_2 \propto 0.8I_{a2} \]

Then \[ (90)(90) = (I_{a2})(0.8I_{a2}) \]

\[ I_{a2}^2 = \frac{(90)^2}{0.8} \quad \text{and} \quad I_{a2} = 100.62A \]

Hence \[ E_{b2} = V - I_{a2}(R_a + R) \]

\[ = 300 - 100.62(0.1 + 0.04) = 285.9 \text{ Volts} \]

Back e.m.f. , \[ E_b \propto \phi_n \] from which \[ \frac{E_{b1}}{E_{b2}} = \frac{\phi_1 \times n_1}{\phi_2 \times n_2} = \frac{I_{a1} \times n_1}{0.8 \times I_{a2} \times n_2} \]

new speed \[ n_2 = \frac{285.9 \times 90 \times 900}{286.5 \times 0.8 \times 100.62} = 1004.4 \text{ r.p.m} \]
2.9 The efficiency of a D.C. motor

It was stated in section (1.14), that the efficiency of a D.C. machine is given by.

Efficiency, \( \eta = \frac{\text{output power}}{\text{input power}} \times 100\% \)

Also, the total losses = \( I_a^2 R_a + I_f V + C \) (for a shunt motor) and,

total losses = \( I^2 R + C \) (for a series motor), where C is the sum of the iron, friction and windage losses, \( R \) is the total resistance for series motor \( R = (R_a + R_f) \)

for a motor, the input power = \( VI \)
and the output power = \( VI - \text{losses} \)

hence,

\[
\eta = \left( \frac{VI - I_a^2 R - I_f V - C}{VI} \right) \times 100\% \quad \text{(for shunt motor)}
\]

\[
\eta = \left( \frac{VI - IR - C}{VI} \right) \times 100\% \quad \text{(for series motor)}
\]

The efficiency of a motor is a maximum when the load is such that

\( I_a^2 R_a = I_f V + C \) (for shunt motor), \( I^2 R = C \) (for series motor)

Example (2.5): A 250 V series motor draws a current of 40 A. The armature resistance is 0.15 \( \Omega \) and the field resistance is 0.05 \( \Omega \).

Determine the maximum efficiency of the motor.

Solution:

For series motor \( \eta = \left( \frac{VI - I^2 (R_a + R_f) - C}{VI} \right) \times 100\% \)

For maximum efficiency, \( \eta = \left( \frac{VI - 2I^2 (R_a + R_f)}{VI} \right) \times 100\% \)

\[
= \left( \frac{(250)(40) - 2(40)^2 (0.15 + 0.05)}{(250)(40)} \right) \times 100 = 93.6\%
\]
2.10 D.C Stepping Motors

D.C stepping motors are unique D.C motors that are used to control automatic industrial processing equipment. D.C motors of this type are found in numerically controlled machines and robotic systems used by industry. They are very efficient and develop a high torque. The stepping motor is used primarily to change electrical pulses into a rotary motion that can be used to produce mechanical movements.

The shaft of a D.C stepping motor rotates a specific number of mechanical degrees with each incoming pulse of electrical energy. The amount of rotary movement or angular displacement produced by each pulse can be repeated precisely with each succeeding pulse from the drive source. The resulting output of this device is used to accurately locate or position automatic process machinery.

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</table>

Fig.(2.20)
2.11 Electromechanical power control equipment

There are so many types of electromechanical power control equipment used today that it is almost impossible to discuss each type. However, some of the very important types will be discussed in the following paragraphs.

2.11.1 Relays

Relays represent one of the most widely used control devices available today. The electromagnet of a relay contains a **stationary core**. Mounted close to one end of the core is a **movable piece** of magnetic material called the **armature**. When the coil is activated electrically, it produces a magnetic field in the metal core. The armature is then attracted to the core, which in turn produces a mechanical motion. When the coil is de-energized, the armature is returned to its original position by spring action. Figure (2.21) shows a simplified diagram of the construction of a relay that is used to control.

![Relay Diagram](image-url)

*Fig.(2.21)*
Relays use a small amount of current to create an electromagnetic field that is strong enough to attract the armature. When the armature is attracted, it either opens or closes the contacts. The contacts then either turn (on) or (off) circuits that are using large amounts of current.

There are two types of contacts used in conjunction with most relays. Normally open (N.O) and normally close (N.C). The (N.O) contacts remain open when the relay coil is de-energized, and are closed only when the relay is energized. The (N.C) contacts remain closed when the relay is de-energized, and are open only when the coil is energized.

**Application**

**D.C Motor Reversing**

The direction of rotation of a permanent-magnet D.C motor can be reversed by reversing the two power line as shown in fig.(2.22).

![Diagram of D.C Motor Reversing](image)
2.11.2 Solenoids

A solenoid, shown in fig.(2.23) is an electromagnetic coil with a **movable core** that is constructed of a magnetic. The **core**, or **plunger**, is sometimes attached to an external spring. This spring causes the plunger to remain in a fixed position until moved by the electromagnetic field that is created by current through the coil. This external spring also causes the core or plunger to return to its original position when the coil is de-energized.

Solenoids are used for a variety of control applications. Many gas and fuel oil furnaces use solenoid valves to automatically turn the fuel supply (on) or (off) upon demand. Dishwashers used one or more solenoids to control the flow of water.

![Solenoid Diagram](image)

**Fig.(2.23)**
Petrol Engine Idle Speed Control

Idle speed control is an important element of the control strategy for any engine management system. The control strategy engine idling must take account of factors such as engine coolant temperature, engine load, power assisted steering alternator load, etc. many systems are fitted with an idle speed control valve that provides a supply of air that by-passes the throttle valve, whilst other systems may make use of the electronic throttle control. Two types of valve are used to provide a computer controlled idle air supply. One makes use of a stepper motor, as shown in fig.(A.1), and the other uses a solenoid operated valve as shown in fig.(A.2).
(a) Stepping motor-operated valve

Figure (A.1) shows a simplified arrangement of the extra air (air-by-pass) valve that is built into the throttle body of some petrol injection system, the ECU (electronic control unit) pulses the transistor bases, in the correct sequence, so that the stepper motor moves the air valve to provide the correct air supply, for any given condition. In addition, other sensor signals will enable the ECU to provide the correct amount of fuel to ensure that the engine continues to run smoothly.

Figure (A.3) shows the stepper motor with the air valve attached. The multiple pin connection is typical of the type of connection that is used to electrically connect the stepper motor to the ECU.

(b) Solenoid-operated valve
This type of valve regulates the amount of air that bypasses the throttle valve through the medium of a solenoid-operated valve of the type shown in fig.(A.2).

In the rest position shown, the valve (4) is closed by the spring (5) and the armature of the solenoid (2) is pushed back inside the solenoid coil (3). When operating, the energized solenoid opens the valve (4) and admits air to the induction system. The quantity of air admitted is controlled by duty cycle pulses that are sent from the ECM.

**Computer Controlled Petrol Fuelling Systems**

Computer controlled petrol injection is now the normal method of supplying fuel-in a combustible mixture form to the engine's combustion chambers. There are two basic petrol injection system in common use. There are single-point (or throttle body) injection and multi-point injection. The single point injection is a single injector, this injector is placed at the throttle body while multi-point injector, in these systems there is an injector for each cylinder.

![Fig.(A.4)](image)
The actual central unit is shown in fig.(A.5). The injector valve is operated by the solenoid (3) which receives electric current in accordance with signals from the engine control computer. When the engine is operating at full or part load the injector sprays fuel during each induction stroke. When the engine is idling the injector operates once per revolution of the crankshaft. Because the fuel pressure regulator maintains a constant fuel pressure at the injector valve, the amount of fuel injected is determined by the length of time for which the solenoid holds the valve in the open position.

Fig.(A.5)