Theory of Machines

References:

4- Kinematics and Dynamics of Machines. By: G.H. Martin.

Topics to be considered:

2. Balancing of Machines.
   a. For Rotating masses.
   b. For Reciprocating masses.
3. Cams and Followers.
4. Flywheel.
5. Gears and Gears trains.
6. Friction Clutches.
7. Belt drives and Band Brakes.
8. Power Screw.
11. Speed Governors.
12. Steering mechanism.
Chapter One

Kinematics and Kinetics of Mechanisms

Introduction:

Theory of Machines: may be defined as that branch of engineering science, which deals with the study of relative motion between the various parts of machine, and forces which act on them. The knowledge of this subject is very essential for an engineer in designing the various parts of a machine.

Sub-divisions of theory of Machines:

They Theory of Machines may be sub-divided into the following four branches:

1- **Kinematics:** is that branch of theory of machines which is responsible to study the motion of bodies without reference to the forces which are cause this motion, i.e it’s relate the motion variables (displacement, velocity, acceleration) with the time.

2- **Kinetics:** is that branch of theory of machines which is responsible to relate the action of forces on bodies to their resulting motion.

3- **Dynamics:** is that branch of theory of machines which deals with the forces and their effects, while acting upon the machine parts in motion.

4- **Statics:** is that branch of theory of machines which deals with the forces and their effects, while the machine parts are rest.
There are some definitions which are concerned with this subject, must be known:

**Mechanism:** is a combination of rigid bodies which are formed and connected together by some means, so that they are moved to perform some functions, such as the crank- connecting rod mechanism of the I.C. engines, steering mechanisms of automobiles…… etc.

The analysis of mechanisms is a part of machine design which is concerned with the kinematics and kinetics of mechanisms (or the dynamics of mechanisms).

**Rigid Body:** is that body whose changes in shape are negligible compared with its overall dimensions or with the changes in position of the body as a whole, such as rigid link, rigid disc…..etc.

**Links:** are rigid bodies each having hinged holes or slot to be connected together by some means to constitute a mechanism which able to transmit motion or forces to some another locations.

**Absolute motion:** the motion of body in relative to another body which is at rest or to a fixed point located on this body.

**Relative motion:** the motion of body in relative to another moved body.

**Scalar quantities:** are those quantities which have magnitude only e.g. mass, time, volume, density etc.

**Vector quantities:** are those quantities which have magnitude as well as direction e.g. velocity, acceleration, force etc.
Disc in motion (rigid body)

- Slot, used for the purpose of connection with another link by slider.
- Hinged hole used for the purpose of the connection with another link by hinge pin.
- Hinged hole

Rigid link

- Connecting rod
- Crank
- Fixed point

Crank-Connecting rod mechanisms

Piston moved on horizontal path
**Part one: Kinematics of Mechanisms:**

1- **The connection of mechanism parts:**

The mechanism is a combination of rigid bodies which are connected together using different methods:

1-1: **Hinged part:**

The hinge connection may be used to connect the links together or connect a link to a fixed point, piston, disc ….. etc, the connection is achieved using pin, which is pass through the hinge holes.

1-2: **Sliding Parts:**

The sliding connection may be used to connect two links rotate about fixed points by means of slot, pin and hinge.
1-3: **Rolling without slipping parts:**

![Diagram](image)

2- **Translated bodies:**

There are some bodies in the mechanism which are constrained to move in translation manner, such as the piston of crank-connecting rod mechanism, the body is used to be in translation motion, if any line remain in some configuration during motion; then all the points have the same absolute velocity and acceleration.

**Velocity diagram:**

- ∴ the motion is absolute, then select any fixed point such as o be as a reference point (i.e point of zero velocity).
- Draw the path of translation.
- If \( v_B \) is known, select a scale factor to draw the velocity diagram (denoted by SFv)

\[
SFv = \frac{\text{draw value in mm}}{\text{actual value of velocity in (m/s)}} = \frac{ob}{v_B}
\]

The draw a line \( ob = (v_B)(SFv) \) in direction of \( v_B \) parallel to the path of translation.
Then all points on the piston have the same velocity, such as point D, i.e on the velocity diagram, the point d coincide on the point b.

**Acceleration diagram:**

- ∵ the motion is absolute, then select any fixed point such as o be as a reference point (i.e point of zero acceleration).
- Draw the path of translation.
- If $a_B$ is known, select a scale factor to draw the acceleration diagram (denoted by SFa)

$$SFa=\frac{\text{draw value in mm}}{\text{actual value of acceleration in } (m/s^2)} = \frac{ob}{a_B}$$

In which $ob=(a_B)(SFa)$.

Then all points on the piston have the same acceleration value.

Note: the base (ref.) point o of $v_o=0$, $a_o=0$. 
Dynamic review:

Translation motion can by treatment by the dynamics of particles i.e body B can be treatment as a particle moved on straight or curved path.

Then: \[ v = \frac{ds}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \Rightarrow vdv = ads. \]

Where:

- \( s \): displacement
- \( v \): velocity
- \( a \): acceleration

\[ v_2 - v_1 = \int_{t_1}^{t_2} a \, dt \], if \( a \) uniform

\[ \frac{1}{2}(v_2^2 - v_1^2) = \int_{s_1}^{s_2} a \, ds \], if \( a \) uniform

\[ ∴ s_2 - s_1 = v_1(t_2 - t_1) + (t_2^2 - t_1^2). \]

3-Bodies rotate about fixed point:

Consider the link shown which is rotate about the fixed point o, the motion of this link can be analyzed using the principle of absolute motion as follow:

If \( θ \): angular displacement about fixed rotation centre.

\( ω \): angular velocity about fixed rotation centre.

\( α \): angular acceleration about fixed rotation centre.
Then:

\[ \omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}, \quad \Rightarrow \omega d\omega = \alpha d\theta \]

\[ \therefore \omega_2 - \omega_1 = \int_{t_2}^{t_1} \alpha(t)dt \quad \text{but if } \alpha \text{ is uniform } \Rightarrow \]

\[ \omega_2 - \omega_1 = \alpha(t_2 - t_1) \]

and \[ \omega_2^2 - \omega_1^2 = \int_{\theta_1}^{\theta_2} \alpha(\theta) d\theta \quad \text{but if } \alpha \text{ is uniform } \Rightarrow \]

\[ \omega_2^2 - \omega_1^2 = 2\alpha(\theta_2 - \theta_1) \]

\[ \therefore \theta_2 - \theta_1 = 2v_1(t_2 - t_1) + \frac{1}{2} \alpha(t_2^2 - t_1^2). \]

**Velocity diagram:**

In order to analyze the velocity of any point we follow with one of the following methods:

1- **If \( \omega \) is given:-**

- Draw the link by SFp (scale factor for position),
  \[ \text{SFp} = \frac{\text{drawn length of link}}{\text{actual length of link}} \]
- \[ v_A = (\omega)(oA). \]
- Select \( \text{SFv} = \frac{oA}{v_A} \), then select a reference point of zero velocity, such as \( o \).
- Draw from \( o \), a line of length \( (oA) = v_A \cdot \text{SFv} \perp oA \) in direction of \( \omega \).
- To find the velocity of any point located on the link, such as \( D \), specify point \( d \) on \( oA \) such that \[ \frac{od}{oa} = \frac{oD}{oA} \quad \Rightarrow od = \left( \frac{oD}{oA} \right) oa. \]

Then:
\[ v_D = \frac{od}{\text{SFv}}. \]
2- If $v_A$ is given:-
- Select SFv, specify reference point of zero velocity.
- Draw $oa$ of length $(v_A)(SFv)$ in the same direction given.
- To find value and direction of $\omega$:

\[
\text{Value of } \omega = \frac{v_A}{oa}.
\]

**Acceleration diagram:-**

Also we have two method:-

1- If $\alpha$ is given:-

\[
a_{An} = (oA)\omega^2 \quad \text{normal component of acceleration of A relative to rotation centre.}
\]

\[
a_{At} = (oA)\alpha \quad \text{normal component of acceleration of A relative to rotation centre.}
\]

- Select a reference point of zero acceleration (point o)
- Select $SFa = \frac{\text{drawn value of } a_{An} \text{ or } a_{At}}{\text{actual value of } a_{An} \text{ or } a_{At}}$, depend on which is greater $a_{An}$ or $a_{At}$.
- Start from o to draw $oa$ // $OA$ directed into the rotation centre, by value of $oa = a_{An}$. $SFa$.
- From point $\bar{a}$ draw $\bar{a}a \perp OA$ in direction of $\alpha$ by value $\bar{a}a = a_{At}.SFa$.
- Finally connect $oa$ to represent the absolute value of acceleration of point A. $\Rightarrow a_A = a_{Ao} = \frac{oa}{SFa}$.

To find the acceleration of any point located on the link, such as point D. specify $d$ on $oa$ such that $\frac{od}{oa} = \frac{OD}{OA}$

\[
\Rightarrow od = \left(\frac{OD}{OA}\right)oa.
\]
2- If $a_A$ is given as a value and direction. (absolute acceleration of point A).

- Find $a_{An} = (OA)(\omega^2) = \frac{v_A^2}{OA}$.

- Select $SFa = \frac{\text{draw value of } a_A}{\text{actual value of } a_A}$, select refence point of zero acceleration. (point O).

- Start from O, draw two lines.
  - First line $o \vec{a} = a_{An} \cdot SFa // OA$ directed in to point O.
  - Second line $oa = a_A \cdot SFa$ in direction of $a_A$ (given).

Then connect $\vec{a}o$ to represent the drawn tangential component of acceleration of A.

\[ \Rightarrow a_{At} = \frac{a \vec{a}}{SFa} \]

\[ \alpha = \frac{a_{At}}{OA} \]

4-Bodies under general plane motion:-

If a body under general plane motion, then it’s motion can be analyzed using the principle of relative motion.

The motion of any point can be discretized into translation and rotation, if consider the link shown under general plane motion, the ends, B of absolute velocities $v_A$, $v_B$, and absolute accelerations $a_A$, $a_B$ then:

\[ \vec{V}_A = \vec{V}_B + \vec{V}_{AB} \quad \text{or} \quad \vec{V}_B = \vec{V}_A + \vec{V}_{BA} \]

\[ \vec{a}_A = \vec{a}_B + \vec{a}_{AB} \quad \text{or} \quad \vec{a}_B = \vec{a}_A + \vec{a}_{BA} \]
Where:

\[ \vec{V}_{BA} \] is the relative velocity of B w.r.t A.

\[ \vec{V}_{AB} \] is the relative velocity of A w.r.t B.

\[ \vec{a}_{BA} \] is the relative acceleration of B w.r.t A.

\[ \vec{a}_{AB} \] is the relative acceleration of A w.r.t B.

i.e the state of velocity can be replaced by one of the following:-

\[
\vec{V}_A = \vec{V}_B + (\vec{w} \times (\vec{AB})) \quad \text{vector notation.}
\]

\[
\vec{V}_B = \vec{V}_A + (\vec{w} \times (\vec{BA}))
\]

\[ \therefore V_{AB} \text{ and } V_{BA} \perp AB. \]

\[ \therefore \omega_{AB} = \frac{V_{AB}}{AB} = \frac{V_{BA}}{AB} \]

To specify direction of \( \omega \):  \[ \Rightarrow \]
\( \mathbf{V}_{AB} \): mean that B is a fixed rotation a center, and A moved a round A.

\( \mathbf{V}_{BA} \): mean that A is a fixed rotation a center, and A moved a round B.

And the state of acceleration can be represented by one of the following:-

\[
\begin{align*}
\vec{a}_A &= \vec{a}_B + \vec{\omega} \times \vec{\omega} \times \overrightarrow{AB} + \vec{\alpha} \times \overrightarrow{AB} \\
\vec{a}_B &= \vec{a}_A + \vec{\omega} \times \vec{\omega} \times \overrightarrow{BA} + \vec{\alpha} \times \overrightarrow{BA}
\end{align*}
\]

\( \therefore \) \( \mathbf{a}_{AB} \) contain two

\[
\begin{align*}
\text{comps.} & \begin{cases} 
\text{normal component} = \omega^2 \overrightarrow{AB} \parallel \overrightarrow{AB} \text{ into } B. \\
\text{tangential component} = \alpha \overrightarrow{AB}. \overrightarrow{AB} \text{ in direction of } \alpha.
\end{cases}
\end{align*}
\]

\( \mathbf{a}_{BA} \) contain two

\[
\begin{align*}
\text{comps.} & \begin{cases} 
\text{normal component} = \omega^2 \overrightarrow{BA} \parallel \overrightarrow{BA} \text{ into } A. \\
\text{tangential component} = \alpha \overrightarrow{AB}. \overrightarrow{AB} \text{ in direction of } \alpha.
\end{cases}
\end{align*}
\]
**Velocity diagram:**

Consider the shown link under general plane motion, to specify the velocity of any point, it’s required one of following:-

1- * Absolute velocity of any point (value **and** direction).
   * Absolute velocity of other point (value **or** direction).

2- * Absolute velocity of any point (value **and** direction).
   * Angular velocity of the link which is the same for all points.

**Steps:**

- Draw the link position by scale (SFp).
- If $V_B$ is known (value and direction), then select the scale factor of velocity diagram (SFv).
  - Specify the point of zero velocity. (point O).
  - $SFv = \frac{ob}{V_B} \implies ob$ known in mm.
  - Draw $ob$ in direction of $V_B$. 
To continue we require other direction of absolute velocity of other point or \( \omega \).

If the direction of absolute velocity of point \( c \) is known then:

- Start from \( o \) to draw line // direction of \( V_c \).
- Start from \( b \) to draw line \( \perp \) link to be \( V_{bc} \) or \( V_{cb} \), which is intersected with the line // direction of \( V_c \) at \( c \).

If \( \omega \) is known:

- \( V_{BC} = \omega \cdot (BC) \), then \( bc = SFv \cdot V_{BC} \).
- Draw \( bc \) from \( b \) \( \perp \) link.
- Draw line between \( o \) and \( c \) produce \( oc \).

To find \( V_A \), \( V_D \):

- Specify \( ba \) such that \( \frac{cd}{bc} = \frac{CD}{BC} \).
- Measure \( od \Rightarrow V_D = ad / SFv \).

To find \( \omega \) value and direction, if unknown measure \( bc \Rightarrow V_{BC} = \frac{bc}{SFv} \).

\[ \Rightarrow \omega = \frac{V_{BC}}{BC}. \]

**Acceleration diagram:**

To draw the acceleration diagram it’s required one of following:

1- \( \omega \) or \( V_{BC} \).

- Absolute acceleration of any point (value **and** direction).
- Absolute acceleration of other point (value **or** direction).

2- \( \omega \) or \( V_{BC} \).

- Absolute acceleration of any point (value **and** direction).
- Angular acceleration of the link.
Steps:

- Find $a_{BCn} = \omega^2 BC = \frac{v_{BC}^2}{BC}$
- If $a_c$ is known (value and direction), $V_B$ is known (direction).
- Select SFa $\Rightarrow oc = SFa \cdot ac$, $c\hat{b} = SFa \cdot a_{BCn}$.
- Start from point of zero acceleration such as o.
- Draw oc in direction of $a_c$.
- From c draw $c\hat{b}$ // link directed into point c (on the link).
- From o draw a line in direction of $a_B$, and from $\hat{b}$ draw a line $\perp$ link (to be $a_{Bct}$), they are intersected at b.
- If $\alpha$ is known (value and direction).
- Find $a_{Bct} = \alpha \cdot BC$ then $\hat{b}b = SFa \cdot a_{Bct}$.
- Start from $\hat{b}$ to draw $\hat{b}b \perp$ link.
- Connect ob
  - Find $a_A$, $a_D$.
- Specify bc such that $\frac{ba}{cb} = \frac{BA}{CB}$.
- Measure $oa \Rightarrow a_A = \frac{oa}{SFa}$.
- Specify cd such that $\frac{ba}{cb} = \frac{BA}{CB}$.
- Measure $od \Rightarrow a_D = \frac{cd}{SFa}$.
- To find $\alpha$ (value and direction) in unknown:-
  - Measure $\hat{b}b \Rightarrow a_{Bct} = \frac{\hat{b}b}{SFa}$
  - $\Rightarrow \alpha = \frac{a_{Bct}}{BC}$

Note:- $\alpha$ is the same for all points of the link.
Example (1):- For the crank- connecting rod mechanism shown: OA=10 cm, AB=30 cm, AC=10 cm, it’s single degree of freedom, coordinate is $\theta$. If $\omega_{OA}=30$ rad/sec, $\alpha_{OA}=100$ rad/s$^2$. Find $\omega_{AB}$, $\alpha_{AB}$, $V_B$, $a_B$, $a_C$, $V_C$ at $\theta=30^\circ$.

$$SFP = \frac{2 \text{ cm}}{10 \text{ cm}} = 0.2 \text{ cm/cm}$$

$oa = 2\text{ cm}$, $ab = 6\text{ cm}$, $ac = 2\text{ cm}$.

to draw velocity diagram.

$$V_A = (OA)(\omega_{OA}) = (0.1)(30) = 3 \text{ m/s}.$$ 

$$SFV = \frac{3 \text{ cm}}{3 \text{ m/s}} = 1 \text{ cm/m/s}$$

$O$ point of zero velocity.

$$oa = (1)(3) = 3 \text{ cm}.$$ 

to specify $ac$ 

$$\Rightarrow ac = \left(\frac{AC}{AB}\right)(ab) = 8.7 \text{ mm}.$$ 

measure $ab = 2.6 \text{ cm} \Rightarrow V_{AB} = \frac{2.6}{1} = 2.6 \text{ m/sec}.$$

measure $oc = 2.45 \text{ cm} \Rightarrow V_c = 2.45 \text{ m/s}.$$

to find $V_B$: measure $ob = 2 \text{ cm} \Rightarrow V_B = 2 \text{ m/s}.$

To find $\omega_{AB}$:

$$\omega_{AB} = \frac{V_{AB}}{AB} = \frac{2.6}{0.3} = 8.7 \text{ rad/s}$$

to draw acceleration diagram:

$$a_{An} = (OA)(\omega_{OA}^2) = (0.1)(30)^2 = 90 \text{ m/s}^2$$
\[ a_{At} = (OA)(a_{OA}) = (0.1)(100) = 10 \text{ m/s}^2 \]

\[ a_{ABn} = (AB)(\omega_{AB}^2) = (0.3)(8.7)^2 = 22.7 \text{ m/s}^2 \]

\[ SFa = \frac{5 \text{ cm}}{90 \text{ m/s}^2} = 0.0556 \text{ cm/m/s}^2 \]

\[ \Rightarrow o\dot{a} = 5 \text{ cm} , \ \dot{a}a = (0.0556)(10) = 0.55 \]

\[ a\dot{b} = (0.0556)(22.7) = 1.26 \text{ cm} \]

\[ \text{to find } a_B , \text{measure } ob = 4.8 \text{ cm } \Rightarrow \]

\[ a_B = \frac{4.8}{0.0556} = 86.3 \text{ m/sec}^2 \leftarrow. \]

\[ \text{to specify } ac \Rightarrow ac = \frac{AC}{AB} ab = \left(\frac{10}{30}\right) (32) \approx 10.7 \text{ mm}. \]

\[ \text{measure } oc = 4.76 \text{ cm } \Rightarrow \]

\[ a_c = \frac{4.76}{0.0556} = 85.611 \text{ m/sec}^2. \]

\[ \text{to find } a_{AB}: \]

\[ \text{measure } b\dot{b} = 2.9 \text{ cm } \Rightarrow \]

\[ a_{ABt} = \frac{2.9}{0.0556} = 52.2 \text{ m/sec}^2 \perp AB. \]

\[ \Rightarrow a_{AB} = \frac{a_{ABt}}{AB} = \frac{52.2}{0.3} = 173.9 \text{ rad/s}^2 \uparrow \]
**Example (2):** In the mechanism shown in Fig. below, the link AB rotates with a uniform angular velocity of 30 rad/s. Determine the velocity and acceleration of G for the configuration shown. The length of the various links are, AB=100 mm; BC=300 mm; BD=150 mm; DE=250 mm; EF=200 mm; DG=167 mm; angle CAB=30.

To draw velocity diagram:

\[ V_B = (30)(0.1) = 3 \ m/s \]

\[ SFv = \frac{40\text{mm}}{3\text{m/s}} = 13.33 \ \text{mm/m/s} \]

\[ ad = 29\text{mm} \Rightarrow V_D = \frac{29}{13.33} = 2.17 \ m/s \]

\[ ag = 8 \text{mm} \Rightarrow V_G = \frac{8}{13.33} = 0.6 \ m/s \]

\[ ae = 6\text{mm} \Rightarrow V_D = \frac{6}{13.33} = 0.45 \ m/s \]

\[ bc = 35\text{mm} \Rightarrow V_D = \frac{35}{13.33} = 2.6 \ m/s \]

\[ \Rightarrow \omega_{BC} = \frac{V_{BC}}{BC} = \frac{2.6}{0.3} = 8.7 \ \text{rad/s} \]

\[ de = 33\text{mm} \Rightarrow V_{DE} = 2.5 \ m/s \]

\[ \Rightarrow \omega_{DE} = \frac{V_{DE}}{DE} = \frac{2.5}{0.25} = 10 \ \text{rad/s} \]

\[ \Rightarrow \omega_{EF} = \frac{V_E}{EF} = \frac{0.45}{0.2} = 2.25 \ \text{rad/s} \]
To draw the acceleration diagram:

\[ a_{Bn} = (AB)(\omega_{AB})^2 = (0.1)(30)^2 = 90 \frac{m}{s^2} \]
\[ a_{BCn} = (BC)(\omega_{BC})^2 = (0.3)(8.7)^2 = 22.7 \frac{m}{s^2} \]
\[ a_{DEn} = (DE)(\omega_{DE})^2 = (0.25)(10)^2 = 25 \frac{m}{s^2} \]
\[ a_{EFn} = (EF)(\omega_{AB})^2 = (0.2)(2.25)^2 = 1 \frac{m}{s^2} \]
\[ SFa = \frac{80 \text{ mm}}{90 \text{ mm/s}^2} = 0.778 \frac{\text{mm}}{\text{m/s}^2} \]
\[ ac = 73 \text{ mm} \Rightarrow a_c = \frac{73}{0.778} = 93.9 \frac{m}{s^2} \]
\[ ad = 68.8 \text{ mm} \Rightarrow a_d = \frac{68.8}{0.778} = 88.43 \frac{m}{s^2} \]
\[ ag = 49.5 \text{ mm} \Rightarrow a_g = \frac{49.5}{0.778} = 63.62 \frac{m}{s^2} \]
\[ ae = \text{mm} \Rightarrow a_e = \frac{57}{0.778} = 73.3 \frac{m}{s^2} \]
\[ c\dot{c} = 32.5 \text{ mm} \Rightarrow a_{CBt} = 41.3 \frac{m}{s^2} \]

\[ \Rightarrow a_{CB} = \frac{a_{CBt}}{CB} = \frac{41.8}{0.3} = 139.3 \text{ rad/s}^2 \text{ c.w } \bigcup \]
\[ \dot{e}_1 e = 74.5 \text{ mm} \Rightarrow a_{EDt} = \frac{74.5}{0.778} = 95.8 \frac{m}{s^2} \bigvee \]

\[ \Rightarrow a_{ED} = \frac{a_{EDt}}{ED} = \frac{95.8}{0.25} = 383 \text{ rad/s}^2 \text{ c.w } \bigcup \]
\[ a_{Eft} = 73.3 \frac{m}{s^2} \downarrow, a_E = 73.3 \frac{m}{s^2} \downarrow. \]

\[ \Rightarrow a_{EF} = \frac{a_{Eft}}{EF} = \frac{73.8}{0.2} = 367 \text{ rad/s}^2 \text{ c.w } \bigcup \]
Example (3):- Figure below shows the mechanism of a moulding press in which OA=80 mm, AB=320 mm, BC=120 mm, BD=320 mm. The vertical distance of OC is 240 mm and horizontal distance of OD is 160 mm. When the crank OA rotates at 120 r.p.m. anticlockwise, determine: the acceleration of D and angular acceleration of the link BD.

Solution:

\[ SFP = \frac{6 \text{ cm}}{32 \text{ cm}} = 0.1875 \text{ cm/cm} \]

\[ oa = 1.5\text{ cm}, ab = bd = 6\text{ cm}, \]

\[ bc = 2.25\text{ cm}, oc = 4.5\text{ cm}, od = 3\text{ cm}. \]

to draw velocity diagram.

\[ \omega_{OA} = 120 \times \frac{2\pi}{60} = 12.57 \frac{\text{rad}}{\text{sec}} \]

\[ V_A = (OA)(\omega_{OA}) = (0.08)(12.57) = 1 \text{ m/s}. \]

\[ SFV = \frac{3 \text{ cm}}{1 \text{ m/s}} = 3 \text{ cm/m/s} \]

O, C point of zero velocity.

\[ oa = (1)(3) = 3 \text{ cm} \uparrow. \]

measure cb = 3.2 cm \uparrow

⇒ \[ V_b = \frac{3.2}{3} = 1.066 \text{ m/sec.} \]

\[ \omega_b = \frac{V_b}{CB} = \frac{1.066}{0.12} = 8.8833 \text{ rad/s} \uparrow. \]

measure od = 3.35 cm \downarrow

⇒ \[ V_d = \frac{3.35}{3} = 1.116 \text{ m/sec} \downarrow. \]
measure $ab = 0.8 \text{ cm} \Rightarrow V_{ab} = \frac{0.8}{3} = 0.266 \text{ m/sec}$.

$\omega_{ab} = \frac{V_b}{AB} = \frac{0.266}{0.32} = 0.83125 \text{ rad/s}$

measure $bd = 0.93 \text{ cm} \Rightarrow V_{bd} = \frac{0.93}{3} = 0.32 \text{ m/sec}$.

$\omega_{bd} = \frac{V_{bd}}{BD} = \frac{0.32}{0.32} = 1 \text{ rad/s}$

measure $bc = 3.13 \text{ cm} \Rightarrow V_{bc} = \frac{3.13}{3} = 0.266108 \text{ m/sec}$.

$\omega_{bc} = \frac{V_{bc}}{BC} = \frac{1.08}{0.12} = 9 \text{ rad/s}$

to draw acceleration diagram:

$a_{OAn} = (OA)(\omega_{OA}^2) = (0.08)(12.57)^2 = 12.64 \text{ m/s}^2$.

$a_{BAn} = (AB)(\omega_{AB}^2) = (0.32)(0.83125)^2 = 0.221125 \text{ m/s}^2$.

$a_{BCn} = (BC)(\omega_{BC}^2) = (0.12)(9)^2 = 9.72 \text{ m/s}^2$.

$a_{BDn} = (BD)(\omega_{BD}^2) = (0.32)(1)^2 = 0.32 \text{ m/s}^2$.

$SFa = \frac{10 \text{ cm}}{12.64 \text{ m/s}^2} = 0.7911 \text{ cm/m/s}^2$
Home Work:

Q1/ The diagram of a linkage is given in Fig. below. Find the velocity and acceleration of the slider D and the angular velocity of DC when the crank O₁A is in the given position and the speed of rotation is 90 rev/min in the direction of the arrow. O₁A = 24mm, O₂B = 60mm, CD = 96mm, AB = 72mm, CB = 48mm.

Q2/ In the mechanism shown in Fig. below the crank AOB rotates uniformly at 200 rev/min, in clockwise direction, about the fixed centre O. Find the velocity and acceleration of slider F.
Q3/ In the toggle mechanism, as shown in Fig. below, D is constrained to move on horizontal path. The dimensions of various links are: AB = 200 mm; BC = 300 mm; OC = 150 mm; BD = 450 mm.

The crank OC is rotating in a counter clockwise direction at a speed of 180 r.p.m., increasing at the rate of 50 rad/s². Find, for the given configuration (a) velocity and acceleration of D, and (b) angular velocity and angular acceleration of BD.

Q4/ In a mechanism as shown in Fig. below, the crank OA is 100 mm long and rotates in a clockwise direction at a speed of 100 r.p.m. The straight rod BCD rocks on a fixed point at C. The links BC and CD are each 200 mm long and the link AB is 300 mm long. The slider E, which is driven by the rod DE is 250 mm long. Find the velocity and acceleration of E.
Q5/ The mechanism of a warpping machine, as shown in Fig. below, has the dimensions as follows:

\[ O_1A = 100 \text{ mm}; \ AC = 700 \text{ mm}; \ BC = 200 \text{ mm}; \ BD = 150 \text{ mm}; \ O_2D = 200 \text{ mm}; \ O_2E = 400 \text{ mm}; \ O_3C = 200 \text{ mm}. \]

The crank \( O_1A \) rotates at a uniform speed of \( 100 \text{ rad/sec} \). For the given configuration, determine:

1- linear velocity of the point \( E \) on the bell crank lever,
2- acceleration of the points \( E \) and \( B \), and
3- angular acceleration of the bell crank lever.

Q6/ In the mechanism shown in Fig. below, the crank \( AB \) is 75 mm long and rotate uniformly clockwise at 8 rad/sec. Given that \( BD = DC = DE \);
\( BC = 300 \text{ mm} \), draw the velocity and acceleration diagrams. State the velocity and acceleration of the pistons at \( C \) and \( E \).