**Study of Resolution Power and Intensity Distribution for Optical System Using Synthetic Square Aperture**

**Azhr A. Raheem**

*University of Kerbala/College of science/Department of physics*

**Abstract:**

 In this research effect of square synthetic aperture(apertures are distributed alternately as checker board ) has been studied on intensity of the line object in comparison with square aperture for ideal optical system or contain focus error (W20=0.5λ,1λ,2λ) or spherical aberrations (W40=0.5λ,1λ) or coma aberrations (W31=0.5λ) with rotation angle (ψ=0,90,180) .

The total number of sub apertures are depending on number of rows and columns in square synthetic aperture and the numbers of rows and columns have been chosen are similar which values are )T=2,4,6,8,10(

The results showed that increasing the number of synthetic sub apertures lead to increasing resolution power in ideal optical system and increasing intensity whatever the aberrations amount was high in the optical system and so work to decrease the secondary peaks in the intensity distribution of line image.

**الخلاصة:**

 تم في هذا البحث دراسة تأثير الفتحة المربعة المركبة (الفتحات موزعة بشكل متناوب مثل لوحة الشطرنج )على الشدة لصورة جسم خطي ومقارنتها بالفتحة المربعة لنظام بصري مثالي أو يحتوي على خطأ بؤري مقداره (W20=0.5λ,1λ,2λ) أو زيغ كروي (W40=0.5λ,1λ) أو زيغ هالي (W31=0.5λ) بزاوية دوران (Ψ =0,90,180).إن عدد الفتحات الثانوية يعتمد على عدد الصفوف والأعمدة في الفتحة المركبة وقد تم اختيار عدد الصفوف مساوي لعدد الأعمدة وبقيم (T=2,4,6,8,10) .بينت النتائج إن زيادة عدد الفتحات الثانوية المركبة يؤدي إلى زيادة قدرة التحليل للنظام البصري المثالي وزيادة الشدة حتى وان كانت كمية الزيوغ عالية في النظام البصري وكذلك تعمل على تقليل القمم الثانوية في توزيع الشدة لصورة الجسم الخطي.

**Introduction:**

The image that is formed by the optical systems will be approximately corrected from the aberrations, but there isn't ideal image correspond to the object dimensions because of the wave nature of the light diffraction which almost affects with several factors like the type of illumination that be used (incoherent, coherent and partial coherent), the object shape and the aperture shape [K. H. Al-Lamy, 2008].

The diffraction is a phenomena or effect resulting from the interaction of the radiation waves with the limiting edges of the aperture stop of an optical system [M. J. Riedl, 2001, R.E.Fischer and B. Tadic-Galeb, 2000] .

 The rims of the lenses present in an optical system restrict the light cone and hence the brightness of the image which formed by the optical system, in addition, stop apertures are deliberately introduced either outside or within the optical system to minimize aberrations, to identify the aperture which ultimately limits the cone of the light rays emanating from an on-axis object and making its way through the optical system, each aperture or stop present in the optical system is imaged in the object space by the lenses which lie on the object side of it, the aperture image which subtends the smallest angle at the axial object is the entrance pupil and the physical aperture which gives rise to this image (real or virtual) is the aperture Stop or the limiting iris[K. K. Sharma, 2006].

The synthetic aperture is technique used in field of effect aperture shape on the image formed by optical beams entering the aperture ,a synthetic (square) aperture has been used which is appear when optical system consists of a number of relatively small-aperture systems that work together to achieve a resolution comparable to that of a single(square) aperture system, the resolution enhancement is only achieved if the optical path lengths for rays traveling through each aperture are equal [Samuel T. Thurman and James R. Fienup, 2003].

Thus, multi-aperture systems typically have some mechanism for controlling the optical path delay of each aperture, such mechanisms also can be used to intentionally introduce optical path differences (or time delays) between apertures and perform Fourier transform spectroscopy[Samuel T. Thurman and James R. Fienup, 2003].

The researchers have been evaluated the performance of optical system by the variation the aperture shape and found the effect on image formed by different object for optical system ideal or contain aberrations, Kendrick et. al. [R. L. Kendrick, E. H. Smith, and A.L.Duncan,2003] demonstrated such fourier transform spectroscopy experimentally with a multi aperture telescope, the researcher Reddy [K. L. Reddy ,1995] indicates that the resolution limits increased with presence lanczos filters, Alqazzaz [ Alqazaz H.H.,1995] has studied the line spread function of optical system has obstructed square aperture for different values ​​of obstruction, in the field of synthetic aperture Aljebory [Aljebory.A.F, 2007] performed the effect of the Gaussian filter on the synthetic aperture, as well as (Raheem) [Azhr.A.Raheem,2009] studied the effect of synthetic aperture on line object.

 In this research take an interest in study the effect of a aperture shape on the line object image by studying synthetic square aperture on line spread function (LSF), the number of rows and columns which used (T=2,4,6,8,10),the system has been used ideal or containing the different values ​​of the focus error or aberrations.

**Theoretical:**

There are several means of evaluation the performance of a complete system, such as signal-to-noise ratio (S/N) and intensity distribution on formed image and resolving power of optical system, the line spread function is one of the most functions for describing the performance of an optical system and can be extended to include the effects of an obstructed aperture, apodization and any factor external to the optical system, line spread function is the distribution of light produced on the imaging it is usually measured by recording the amount of light entering a thin slit parallel to the image of the line as a function of slit position.

The intensity of line image for incoherent line source located at the coordinates (u,v) in the image plane can found by using fourier transformations [Born M.,wolf E. ,1984] as :

where: G(u,v) is intensity of point image which can be expressed by multiplying of the amplitude function and its complex form [Born M.,wolf E. ,1984, K.D. M¨oller, 2003] thus :

 : is the pupil function which equal to[Born M.,wolf E. ,1984] :

k: wave number which is equal to (2π/λ), λ: a wavelength of light used,: function permeability its equal to (1) of the symmetric aperture, while the other term of equation represents serial aberration which for spherical and com aberrations is [J.D.Gaskill,1970]:

Where w20,w40,w31,w51: represents the spherical aberrations and coma aberrations terms respectively, Ψ: represent rotation angle .

The boundary of each sub aperture can be found from location of each sub aperture which the equation of them is:

……..(4)

Thus the eq.(2) can be write :

or:

 or :

Let ) ,(,we can study the intensity distribution on one axis thus we can put) m=0 (,then the eq.(7) becomes :



Where (NOR): a constant normalization of function is calculated by make ((L(0)=1) for the free system[Born M.,wolf E. ,1984].

The integration limits given by the aperture shape, then the equation (8) for square aperture (figure 1) with area (A=π) and side ( ) will be :



The suggested synthetic aperture (figure 2) come out of divide the square aperture into sub apertures work to allow passing light at sub aperture and don’t passing light at neighboring sub aperture by alternation continually (as checker board), base on the number of row and column inside synthetic aperture, thus location of sub apertures depend on angle between center of each sub aperture and center of original square aperture, then the values of  for sub aperture at position with number of row (n) and number of column (m) are :

 … (10)

Where: (T): represents total number of rows or columns, (G): represents number equal to (1) when we want to make the first up sub square from left side allow light and when (G=-1) the reverse case happen.

The limits of integration for each sub aperture depend on side of this sub aperture will be similar to others, so that when the area of origin square aperture is (π) and length of side  , so the area of combined sub square apertures are equal to (π) and the side of each one will equal to :

But the number of sub square aperture that allow to light passing are the half, therefore the area of all sub apertures equal to (π/2) , so we let sub side equal to which make the total area of opening apertures is (π) ,then the equation (9) will be as finally form :

The term (T1,T2) depend on number of rows and columns which is define as :



Where D: digit when D=1 the number of rows and columns will be odd, if D=0 the number of rows and columns will be even.

The last equation (11)go back to equation (9) when number of rows and columns (T=1), and the length of sub side is , in this case the result agreement with the previous research [Azhr.A.Raheem,2009] .

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 Figure (1)

Square aperture

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 Figure (2)

 Synthetic square apertures (T=8)

**Results and Discussion:**

 The line spread function has been solved by using the program (MathCAD) for the ideal optical system or contains focus error (W20=0.5λ,1λ,2λ) and when contains spherical aberration (W40=0.5λ,1λ) or coma aberration (W31=0.5λ) with different value of rotation angle (Ψ=0,90,180).

The apertures which taking are square aperture and synthetic square aperture, the number of rows and columns are choosing (T=2,4,6,8,10),that mean the apertures allow light to pass through it are (2,8,18,32,50) .

The two cases (G=1,G=-1) are solved , the result appear similar in two cases at ideal system or contain focus error and aberrations which are chosen in research.

Figure (3) show the intensity distribution of line object for ideal system, we notice the all curve of intensity begin from point with value (1),this case agree with Airy disk for distribution of intensity ,but increasing the number of sub apertures lead to increment resolution power of optical system because of decrease in half width of the curves whenever number of rows and columns are increase.

The figures (4,5,6) describe the effect of the focus error (W20=0.5λ, 1λ, 2λ) respectively in the optical system ,this state cause to lessening of the central intensity so that it reach to (0.57016, 0.10622, 0.09291), while the intensity for optical system use synthetic square aperture stay at the best values and whenever increase number of sub apertures lead to increase the intensity at the image plane in spite of the effect of focus error, where the intensity reach to (0.99978, 0.99913, 0.99654) when the focus error (W20=0.5λ, 1λ, 2λ) respectively at (T=10) .

The effect of focus error (W20=1λ) with spherical aberration (W40=0.5λ, 1λ) is explaining in figures (7,8), the intensity despite effect this case of aberration on optical system the sub apertures try to stay the central intensity as the ideal system and it reach to (0.99911, 0.99909) when the aberrations are (W20=1λ ,W40=0.5λ, 1λ) respectively, while it reach at one square aperture to (0.14163, 0.11738) in the same cases .

The effect of sub apertures dexterous on system when the optical system have many kind of aberrations such as spherical and coma aberrations and focus error as shown in figures (9,10,11) when optical system contain focus error and spherical aberration (w20=w40=1) and coma aberration (w31=0.5) with rotation angle (Ψ=0,90,180) respectively, the LSF be (0.0662,0.04927,0.0662) for square aperture at (Ψ=0,90,180) respectively and the shape of image appear deformed ,while the intensity when the synthetic aperture used become (0.99909) approximate at any value of rotationangle when (T=10) ,therefore the synthetic square aperture stay the image clear, sharp and this specifications increase whenever increase the number of rows and columns .

In all figures the synthetic square aperture woke very well at high number of rows and columns specially (T=10) ,but the case (T=2) cannot get over effect of aberration rather be good in comparison with square aperture but isn't the better from other case in search.

The increase intensity and resolution power at the image plane for optical system which uses synthetic aperture goes back to the distribution field of Fraunhofer diffraction pattern for any distribution of aperture similar in shape and direction itself will equal to Fourier transformation for any function aperture individually (that is the distribution of diffraction field) multiplied by the outcome of the output total diffraction pattern of the source points, therefore intensity appear centering and high .

Also noticing at all figures the increasing (T) lead to decrease the secondary peak in LSF for all states aberrations presence in the optical system which make to increasing clearness of line image and lessening of noise in image .

Figure (3)

Intensity distribution of line object image for ideal optical system

Figure (4)

Intensity distribution of line object image for optical system contain focus error (w20=0.5)

Figure (5)

Intensity distribution of line object image for optical system contain focus error (w20=1)

Figure (6)

Intensity distribution of line object image for optical system contain focus error (w20=2)

Figure (7)

Intensity distribution of line object image for optical system contain focus error (w20=1) and spherical aberration (w40=0.5)

Figure (8)

Intensity distribution of line object image for optical system contain focus error (w20=1) and spherical aberration (w40=1)



Figure (9)

Intensity distribution of line object image for optical system contain focus error and spherical aberration (w20=w40=1) and coma aberration (w31=0.5) with rotation angle (Ψ=0)

Figure (10)

Intensity distribution of line object image for optical system contain focus error and spherical aberration (w20=w40=1) and coma aberration (w31=0.5) with rotation angle (Ψ=90)

Figure (11)

Intensity distribution of line object image for optical system contain focus error and spherical aberration (w20=w40=1) and coma aberration (w31=0.5) with rotation angle (Ψ=180)

**Conclusions**:

1. The synthetic square aperture are excellent technique for high resolution power in line image compared with square aperture .
2. Increasing number of sub aperture lead to increasing the resolution power in ideal optical system
3. Increasing number of sub aperture cause high and explicit central intensity and good apodization despite the effect of aberrations in optical system .
4. Increasing number of sub aperture lead to decreasing of secondary peak intensity of image.
5. From the above Conclusions we choose (T=10) as better from other cases in this search .

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